Recent results on $|V_{cb}|$ and $|V_{ub}|$ at the Belle II experiment

Racha Cheaib, DESY

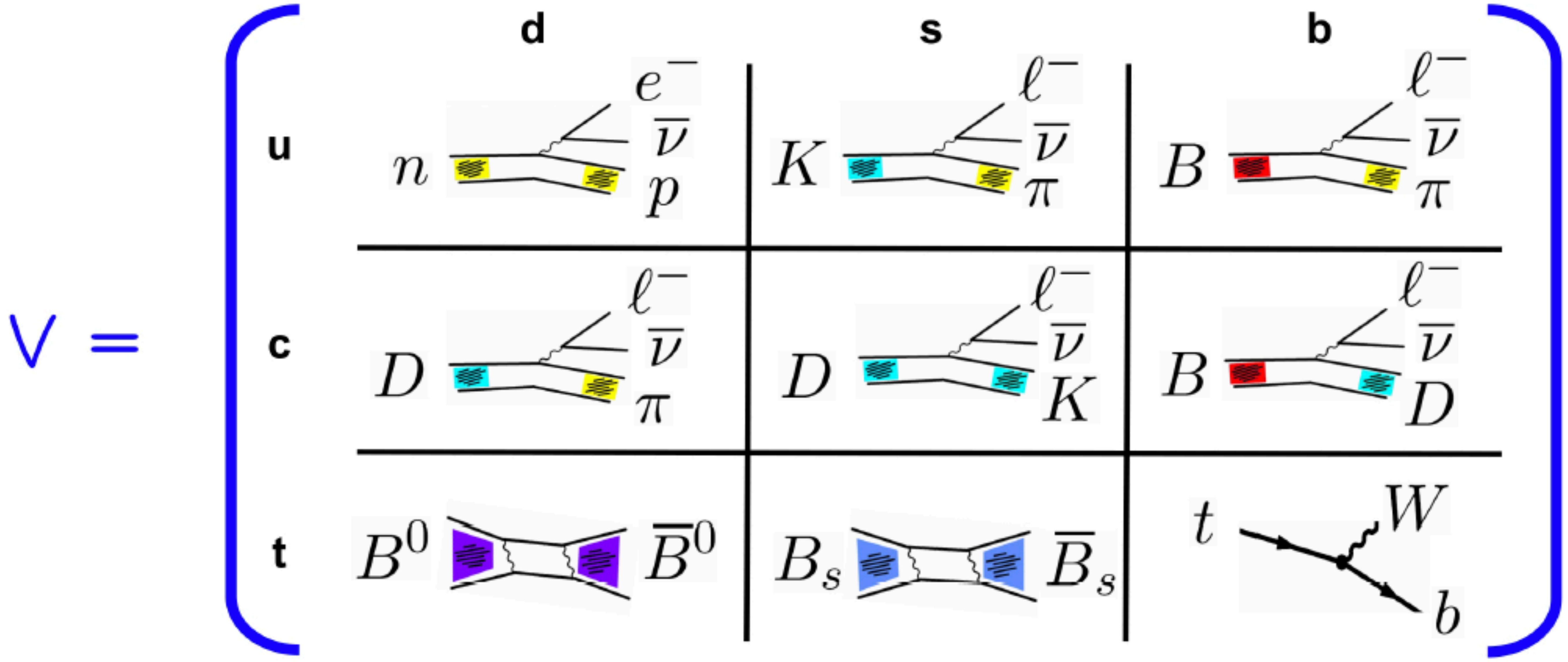
EPS

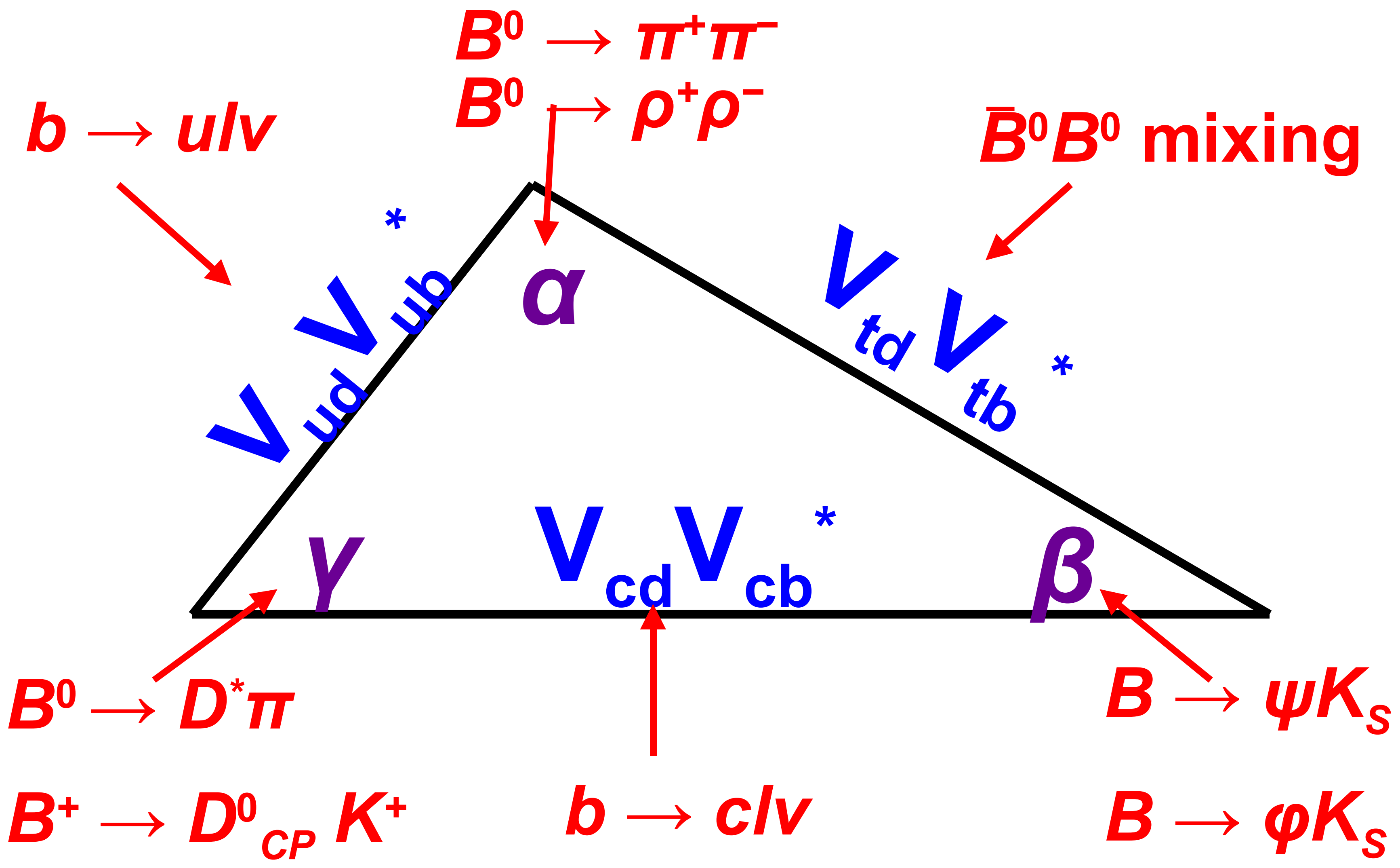
Aug 22, 2023

On behalf of the Belle II Experiment

$|V_{cb}|$ and $|V_{ub}|$

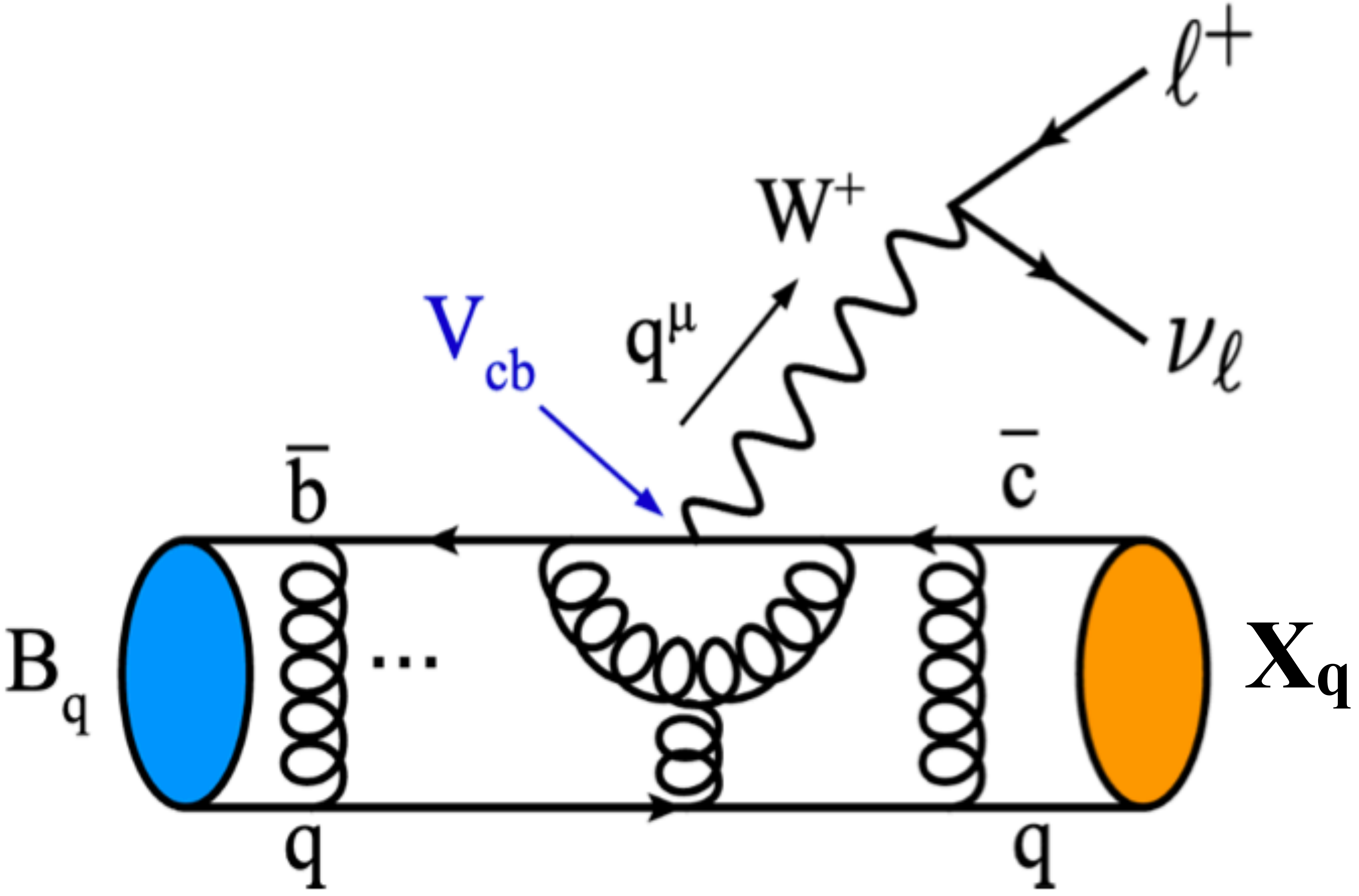
Precision measurements of CKM matrix at the core of the physics program at Belle II





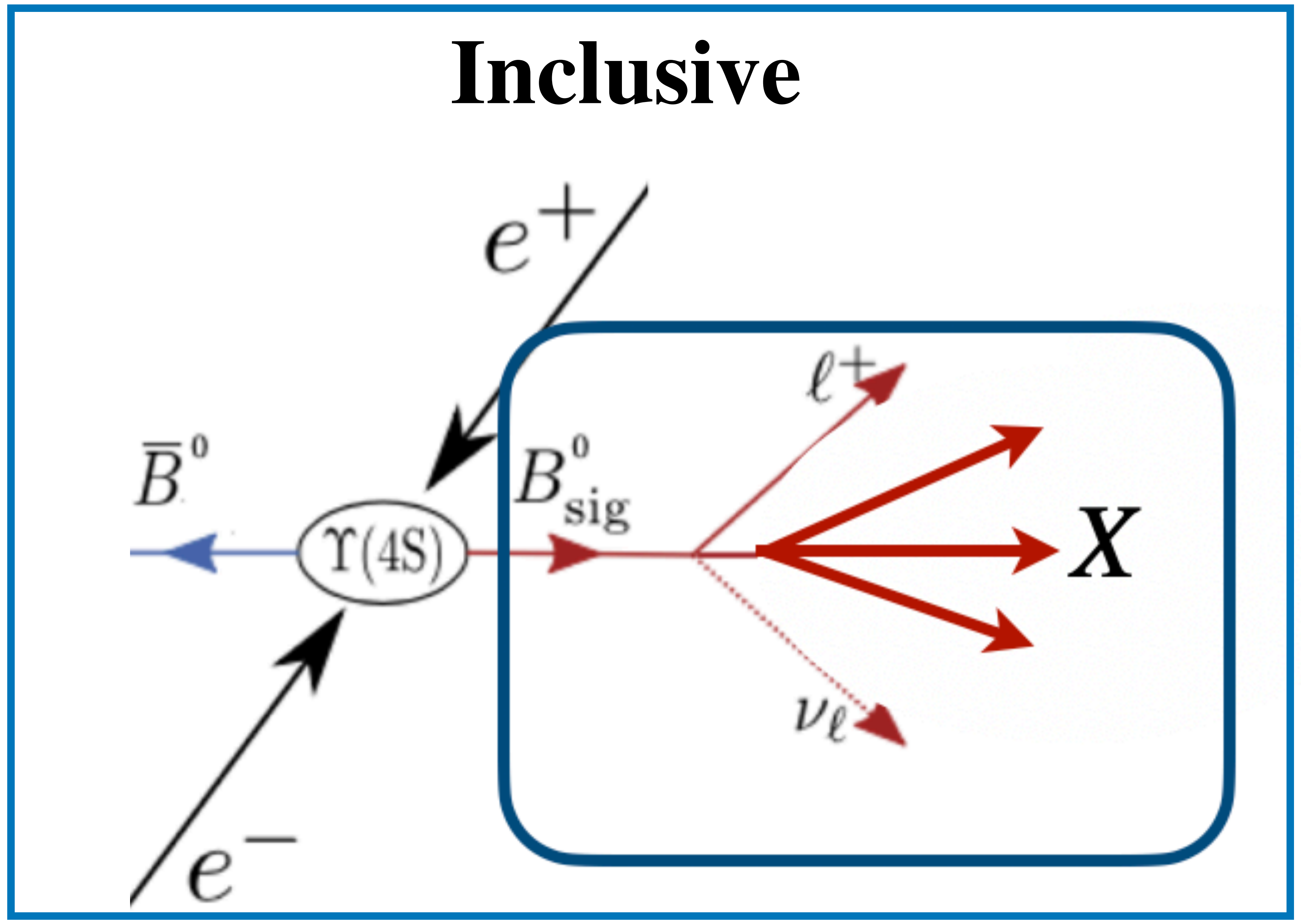
$|V_{cb}|$ and $|V_{ub}|$

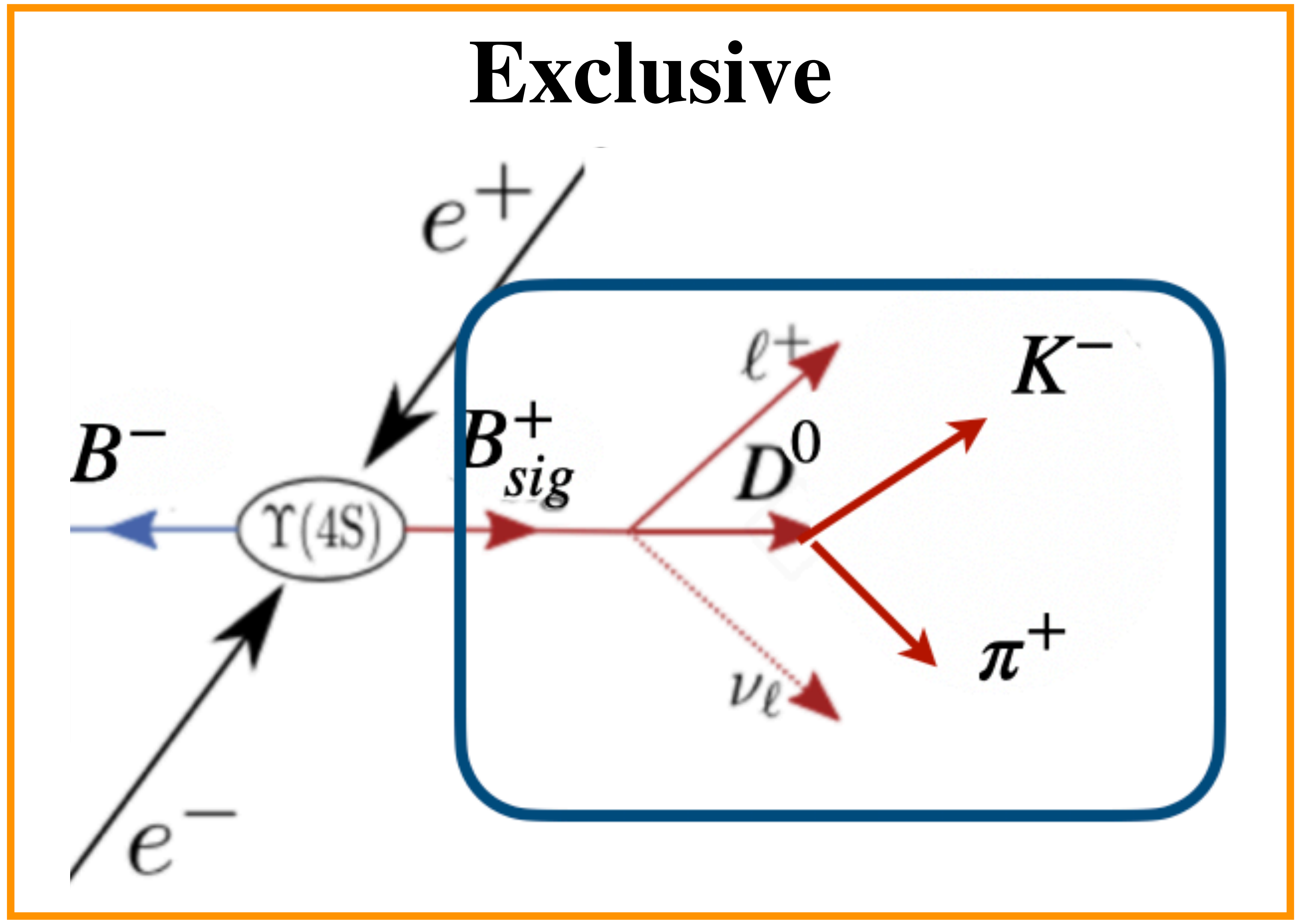
$|V_{ub}|$ and $|V_{cb}|$ are determined mainly from semileptonic decays of *B* mesons



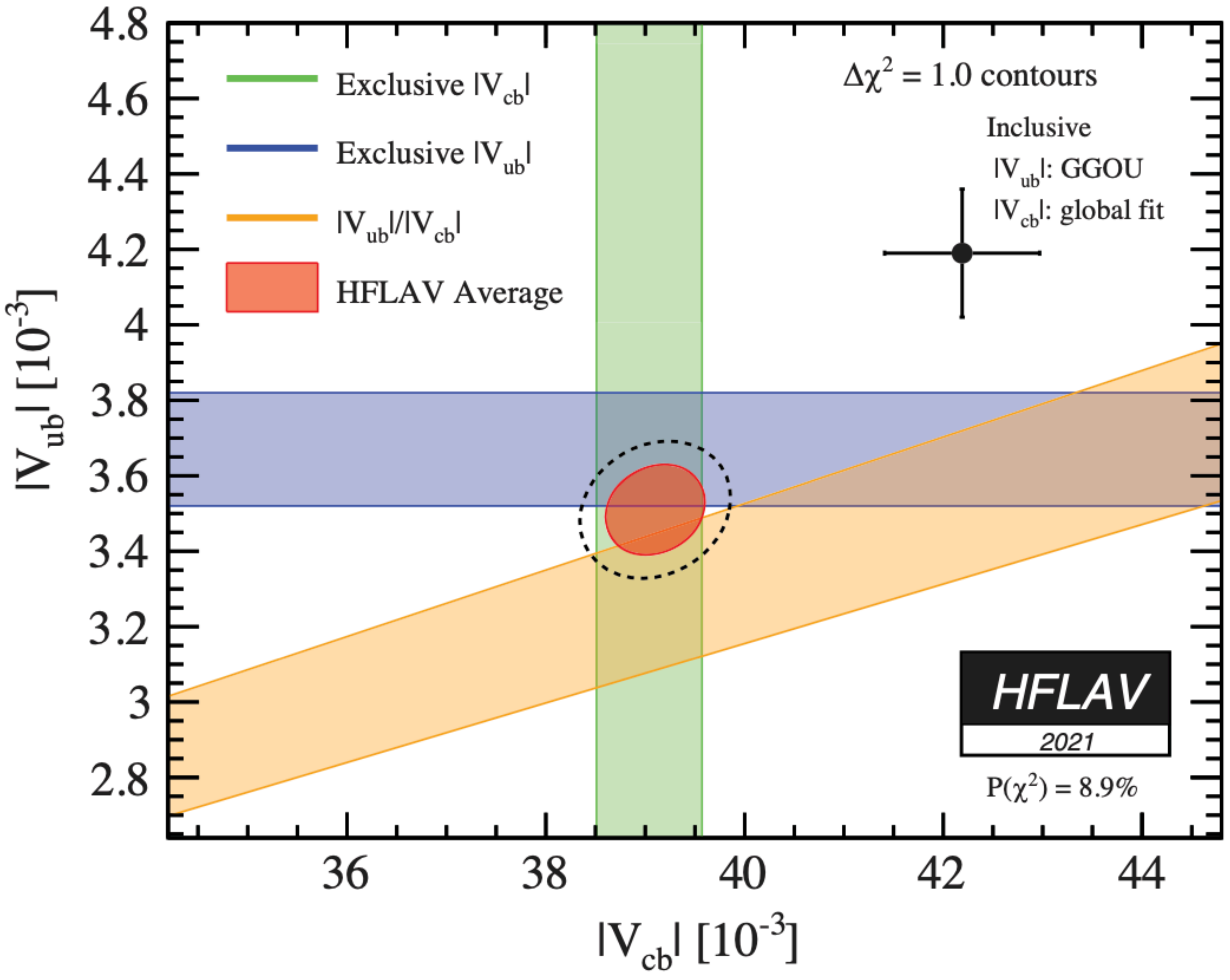
$|V_{cb}|$ and $|V_{ub}|$

Two main approaches for $|V_{ub}|$ and $|V_{cb}|$ measurements





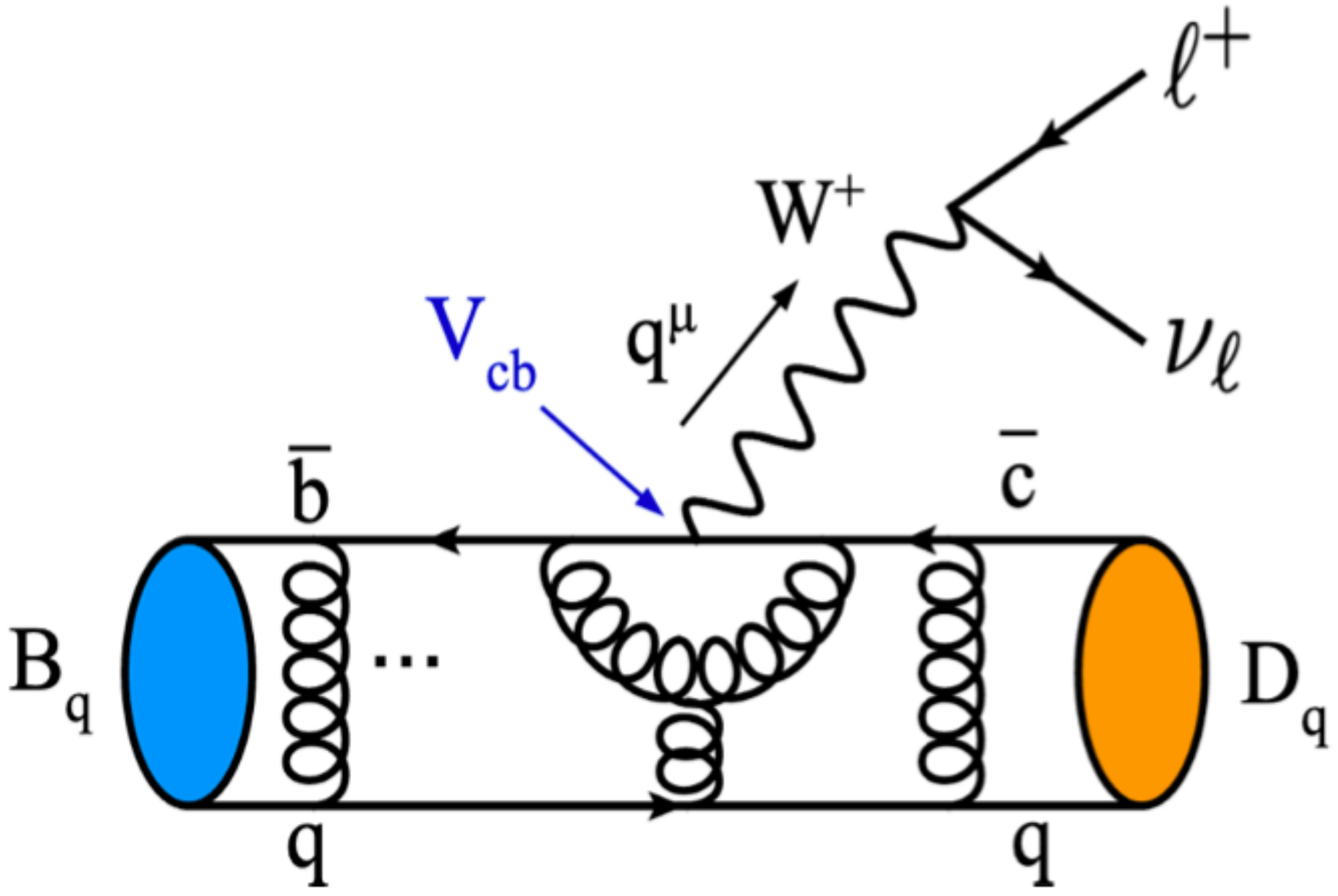
$|V_{cb}|$ and $|V_{ub}|$



Tension between exclusive and inclusive $|V_{ub}|$ and $|V_{cb}|$ measurements along with other related B-anomalies .

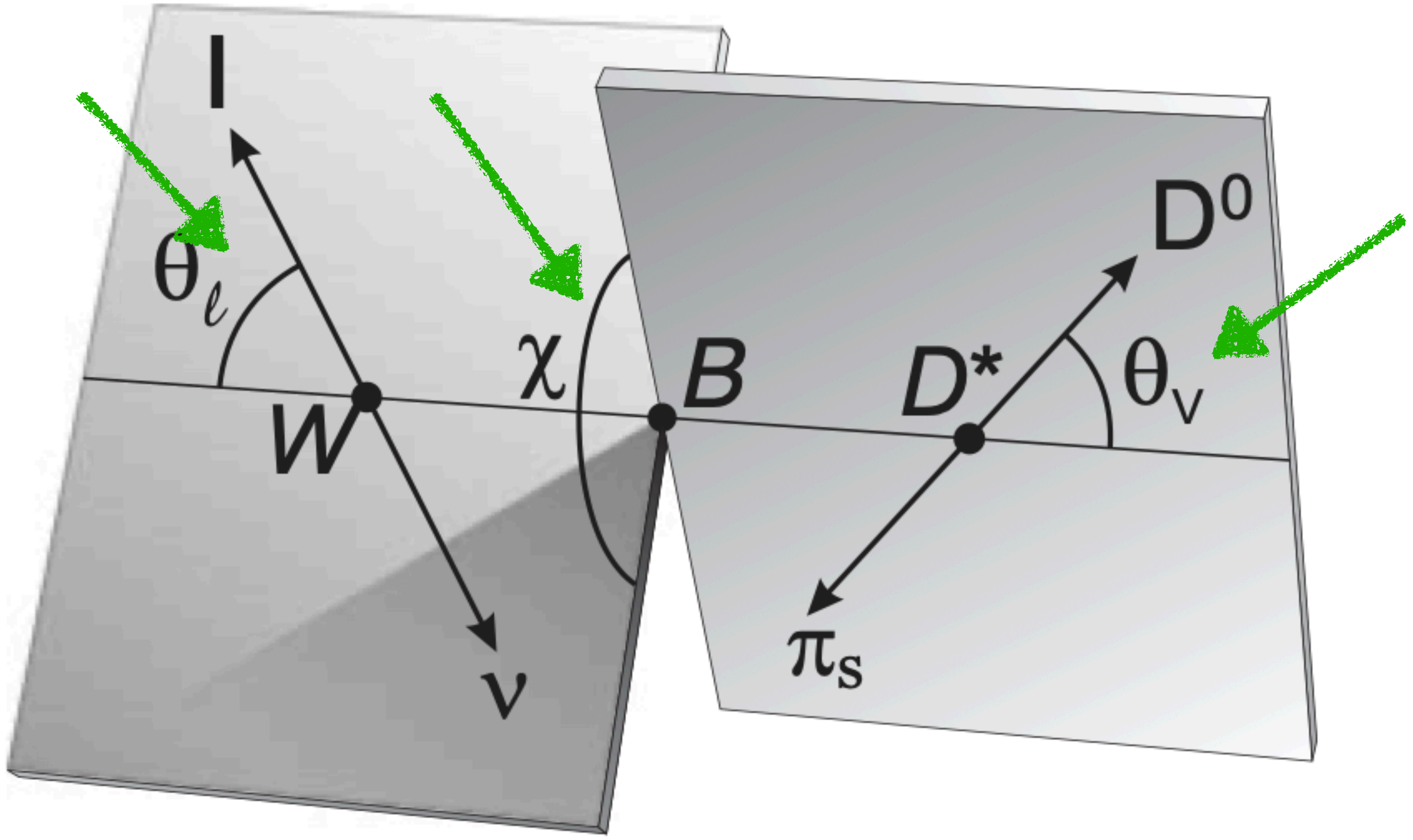
So how do we do this at Belle II ?

Exclusive $|V_{cb}|$



$B \to D^* \ell \nu$

- Clean mode with a good handle on efficiency and backgrounds.
- Measure the differential rate as a function of 3 angular distributions and w.



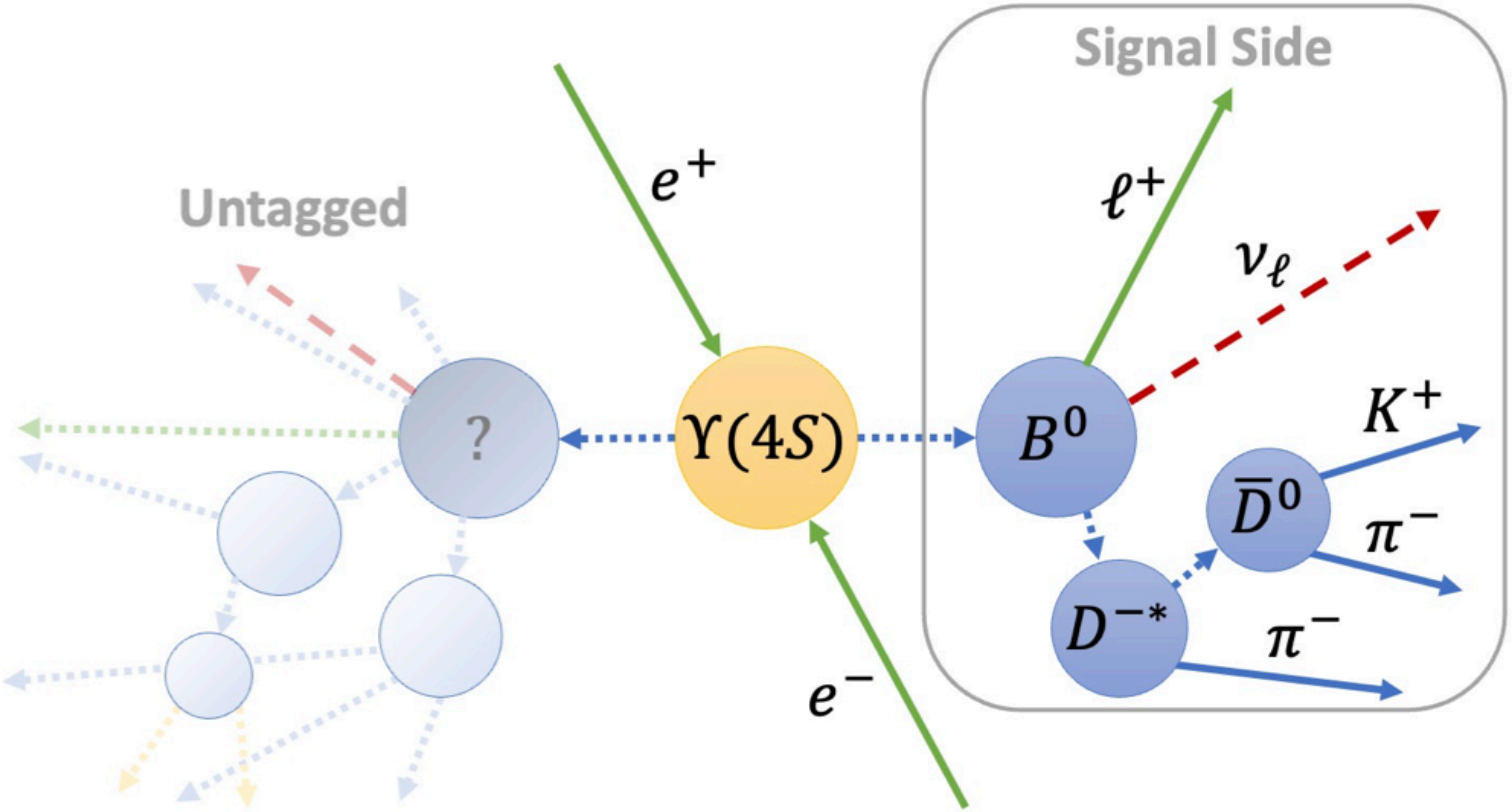
$$\frac{d\Gamma}{dw} = \frac{\eta_{\mathrm{EW}}^2 G_F^2}{48\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 g(w) F^2(w) |V_{cb}|^2$$

- Extract $|V_{cb}|$ using averaged differential rate and input from Lattice QCD calculations.

$$w = \frac{m_B^2 + m_D^2 - q^2}{2 m_B m_D}$$

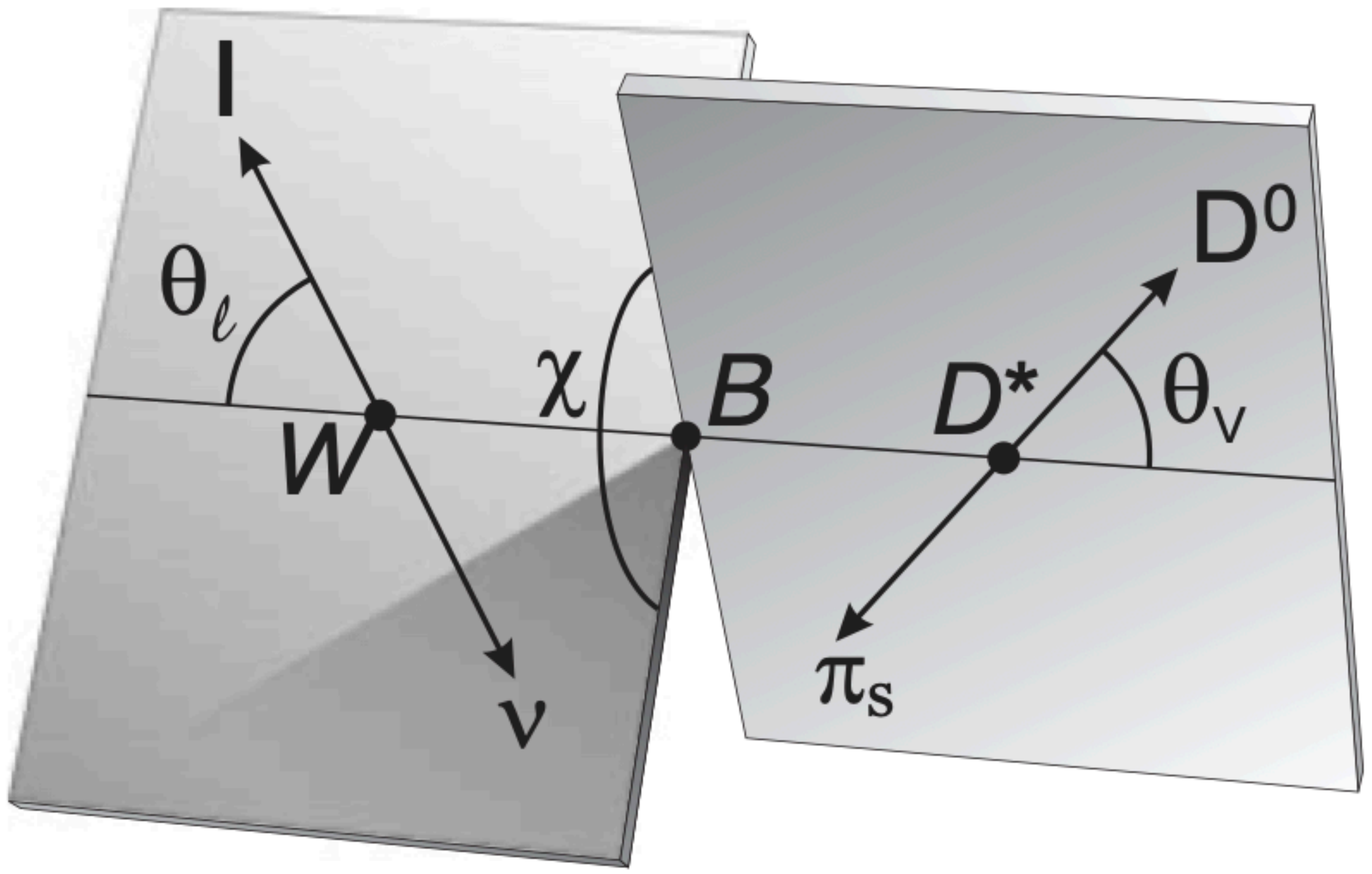
$\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$

- Untagged analysis using 2019-2021 Belle II dataset: 189 fb^{-1}
- Reconstruct $D^{*+} \to [D^0 \to K^-\pi^+]\pi^+$ and identify lepton candidate

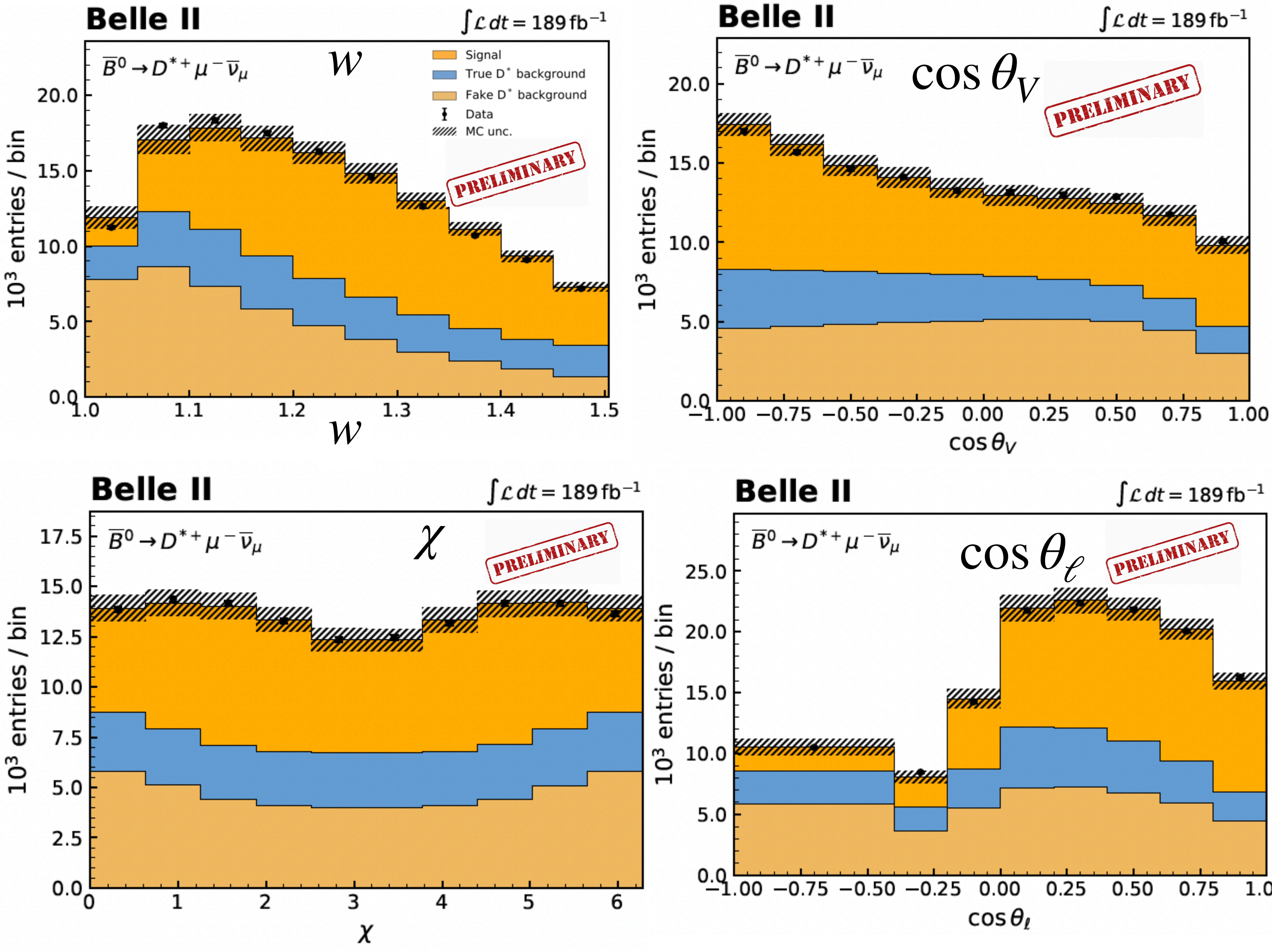


$\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$

- Need the direction of signal B:
  - Use the direction of the $D^{*+}\ell^-$ (Y) system to constrain the signal B direction on a cone with opening angle θ_{BY}.
  - Examine residual tracks and clusters not used in the signal reconstruction to determine other B direction.

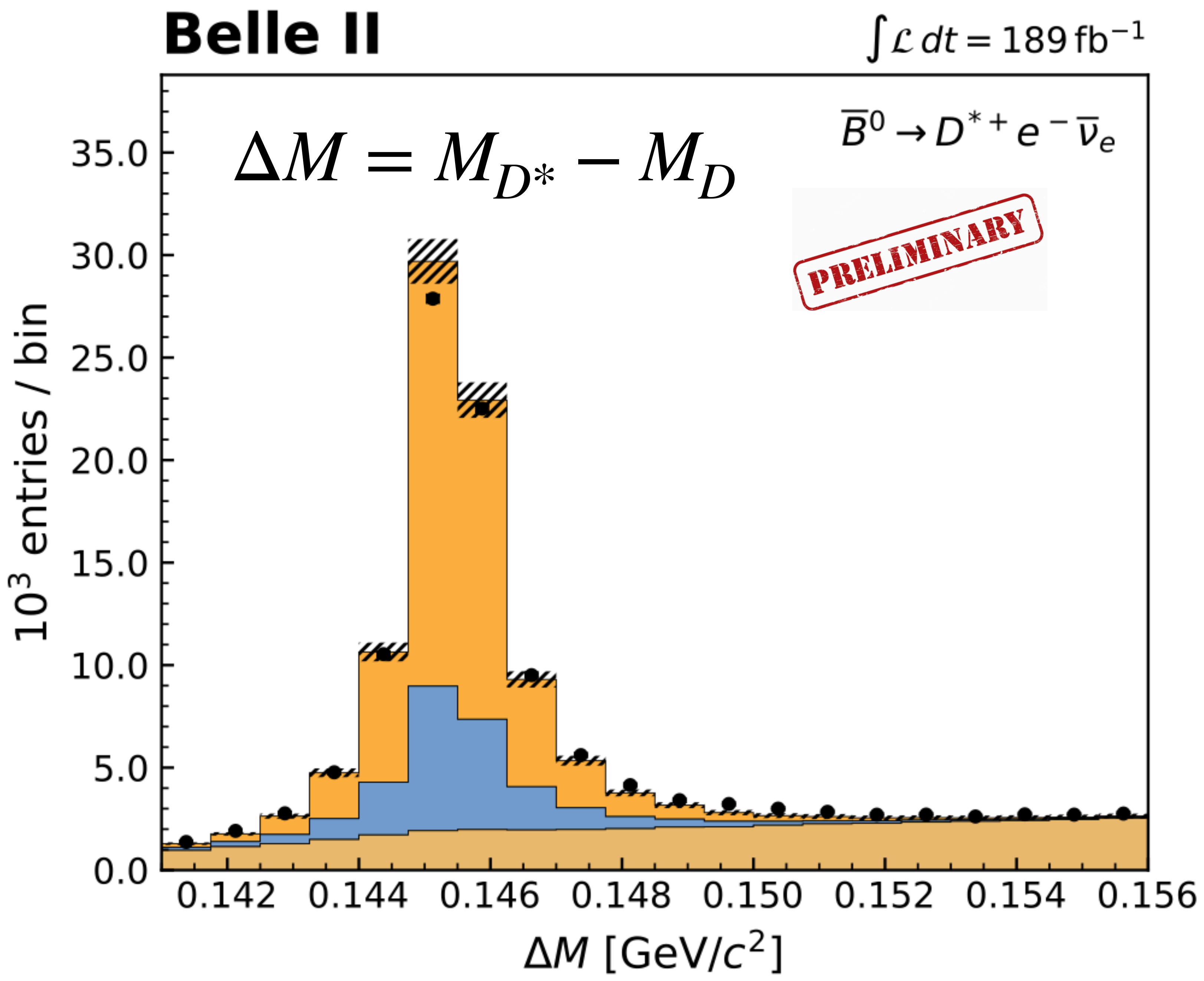


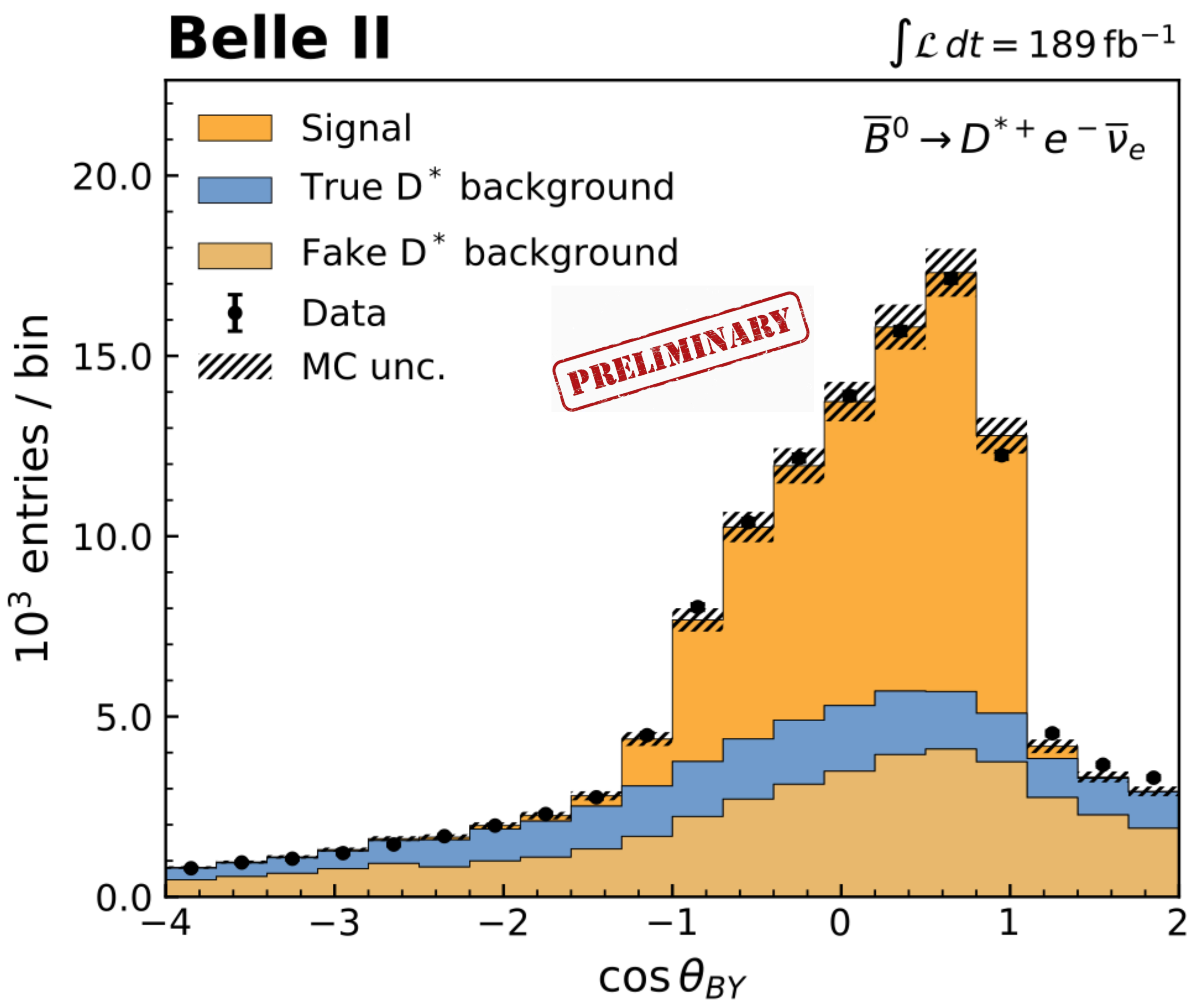
$$\cos\theta_{BY} = \frac{2E_B^{\text{c.m.}}E_Y^{\text{c.m.}} - m_B^2c^4 - m_Y^2c^4}{2|\vec{p}_B^{\,\text{c.m.}}||\vec{p}_Y^{\,\text{c.m.}}|c^2},$$



$\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$

- Extract signal yield from binned maximum likelihood fits to ΔM and $\cos\theta_{BY}$ in each bin of the kinematic distributions

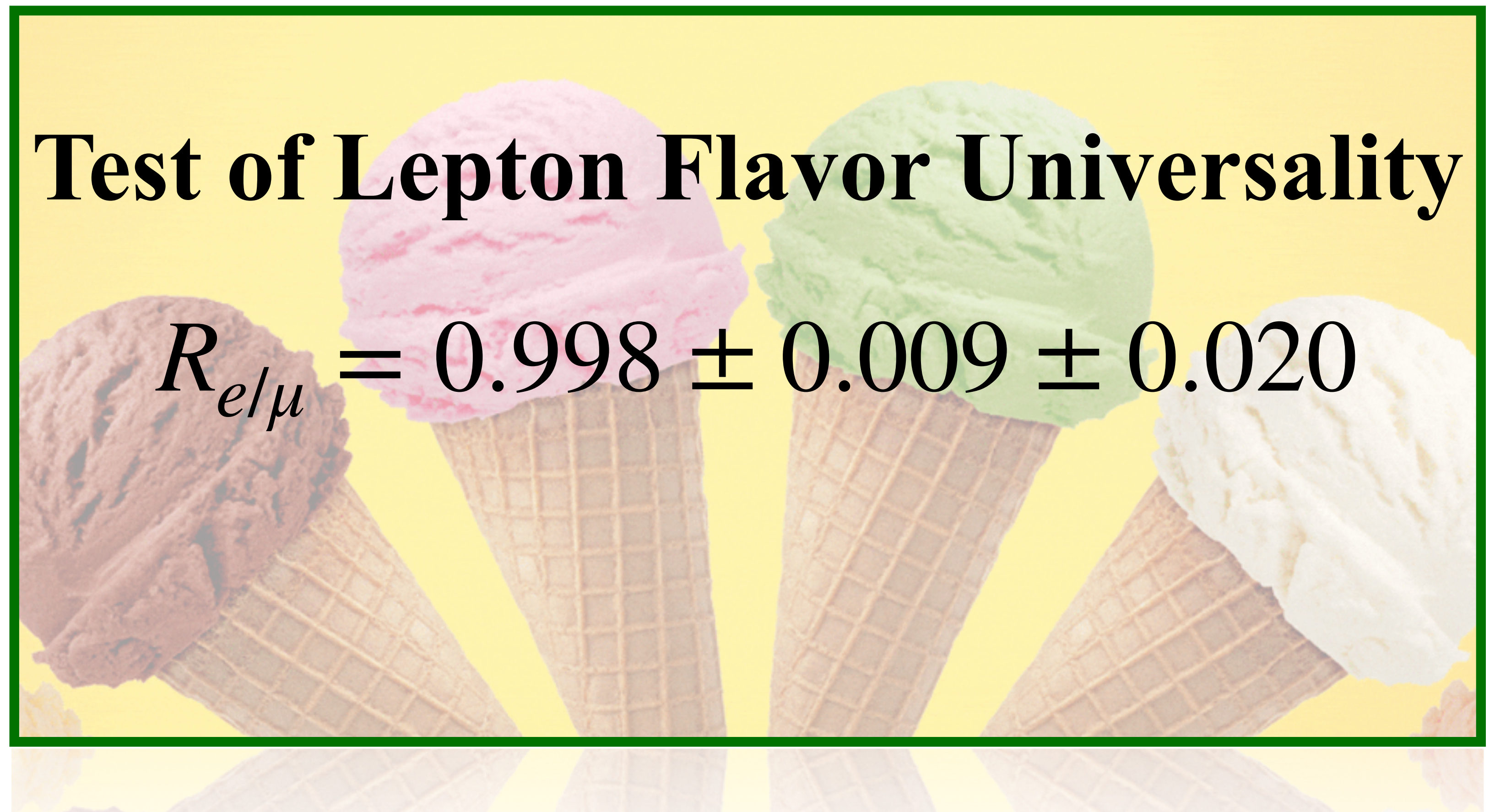




$$\mathcal{B}(\overline{B}^0 \to D^{*+}e^-\bar{\nu}_e) = (4.917 \pm 0.032 \pm 0.216)\%$$

$$\mathcal{B}(\overline{B}^0 \to D^{*+}\mu^-\bar{\nu}_\mu) = (4.926 \pm 0.032 \pm 0.231)\%$$

$$\mathcal{B}(\overline{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell) = (4.922 \pm 0.023 \pm 0.220)\%$$



Belle II

$B \to D^* \ell \nu$

- Take the average of the differential rate from the 4 measured distributions and the two lepton flavours.
- Requires input on form factor parametrization.

parameterize the non-perturbative physics of the $B \to D^*$ transition

$$\frac{d\Gamma}{dw} = \frac{\eta_{\mathrm{EW}}^2 G_F^2}{48\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 g(w) F^2(w) |V_{cb}|^2$$

- Extract $|V_{cb}|$ at zero recoil, $w = 1$.

$$w = \frac{m_B^2 + m_D^2 - q^2}{2 m_B m_D}$$

$$\begin{aligned} h_{A_1}(w) &= h_{A_1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15) z^2 - (231\rho^2 - 91) z^3\right], \\ R_1(w) &= R_1(1) - 0.12(w-1) + 0.05(w-1)^2, \\ R_2(w) &= R_2(1) + 0.11(w-1) - 0.06(w-1)^2, \end{aligned}$$

CLN parametrization

Nucl. Phys. B530, 153(1998)

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{N} a_{i,n} z^n$$

$$z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

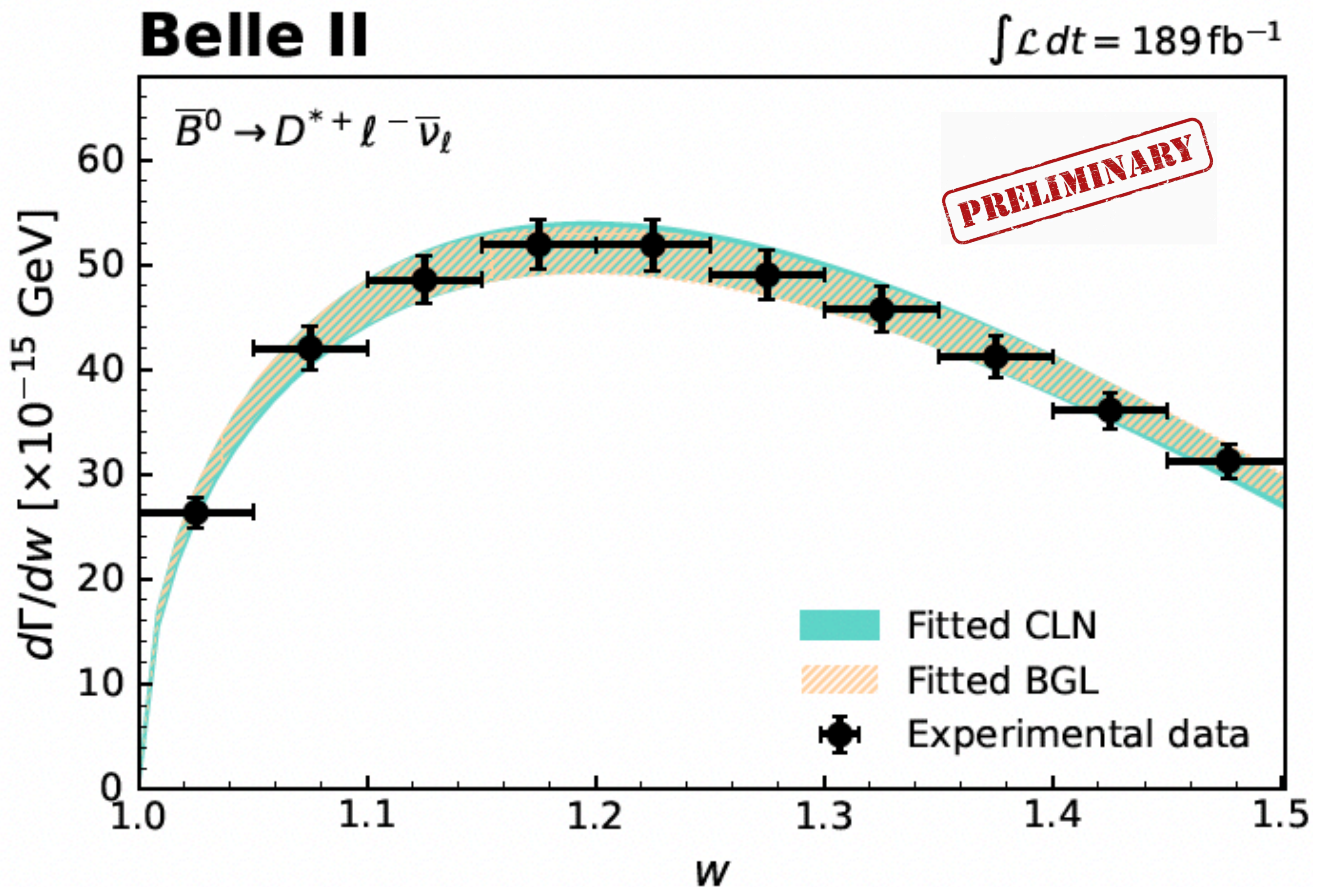
BGL parametrization

Phys. Rev. Lett. 74, 4603 (1995)

$\bar{B}^0 \to D^{*+} \ell^- \bar{\nu}_\ell$

- Use averaged decay rate to determine the form factors and $|V_{cb}|$

$$|V_{cb}|\eta_{EW}\mathcal{F}(1) = \frac{1}{\sqrt{m_B m_{D^*}}}\left(\frac{|\tilde{b}_0|}{P_f(0)\phi_f(0)}\right)$$



Agreement between CLN and BGL fits

$$|V_{cb}|_{CLN} = (40.13 \pm 0.47 \pm 0.93 \pm 0.58) \times 10^{-3}$$

$$|V_{cb}|_{BGL} = (40.57 \pm 0.31 \pm 0.95 \pm 0.58) \times 10^{-3}$$

Using LQCD input: $\mathcal{F}(1) = 0.906 \pm 0.013$ PRD 89,114504 (2014)

In agreement with the world average of exclusive and inclusive approach

albeit with large certainties.

Leading systematic uncertainties are from the limited size of MC sample and the slow pion reconstruction.

$\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$

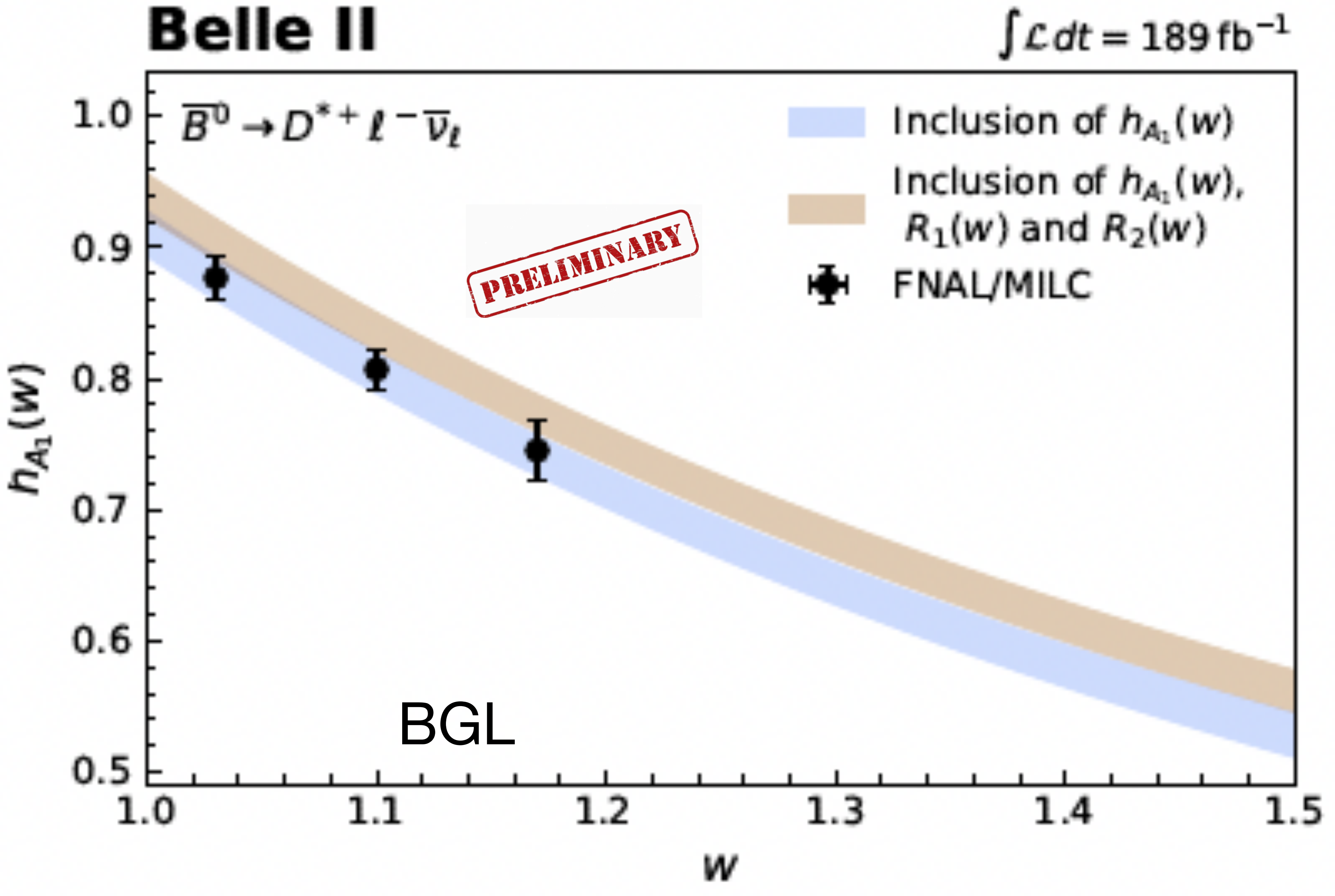
- Looking beyond zero recoil with recent results (Eur. Phys. J. C 82, 1141 (2022))
- LQCD input from Fermilab/MILC at w=[1.03, 1.10, 1.17]

BGL

	Constraints on $h_{A_1}(w)$	Constraints on $h_{A_1}(w)$, $R_1(w)$, $R_2(w)$
$a_0 \times 10^3$	21.7 ± 1.3	25.6 ± 0.8
$b_0 \times 10^3$	13.19 ± 0.24	13.61 ± 0.23
$b_1 \times 10^3$	-6 ± 6	2 ± 6
$c_1 \times 10^3$	-0.9 ± 0.7	0.0 ± 0.7
$\lvert V_{cb}\rvert \times 10^3$	40.3 ± 1.2	38.3 ± 1.1
χ^2/ndf	39/33	75/39
p-value	21%	0.04%

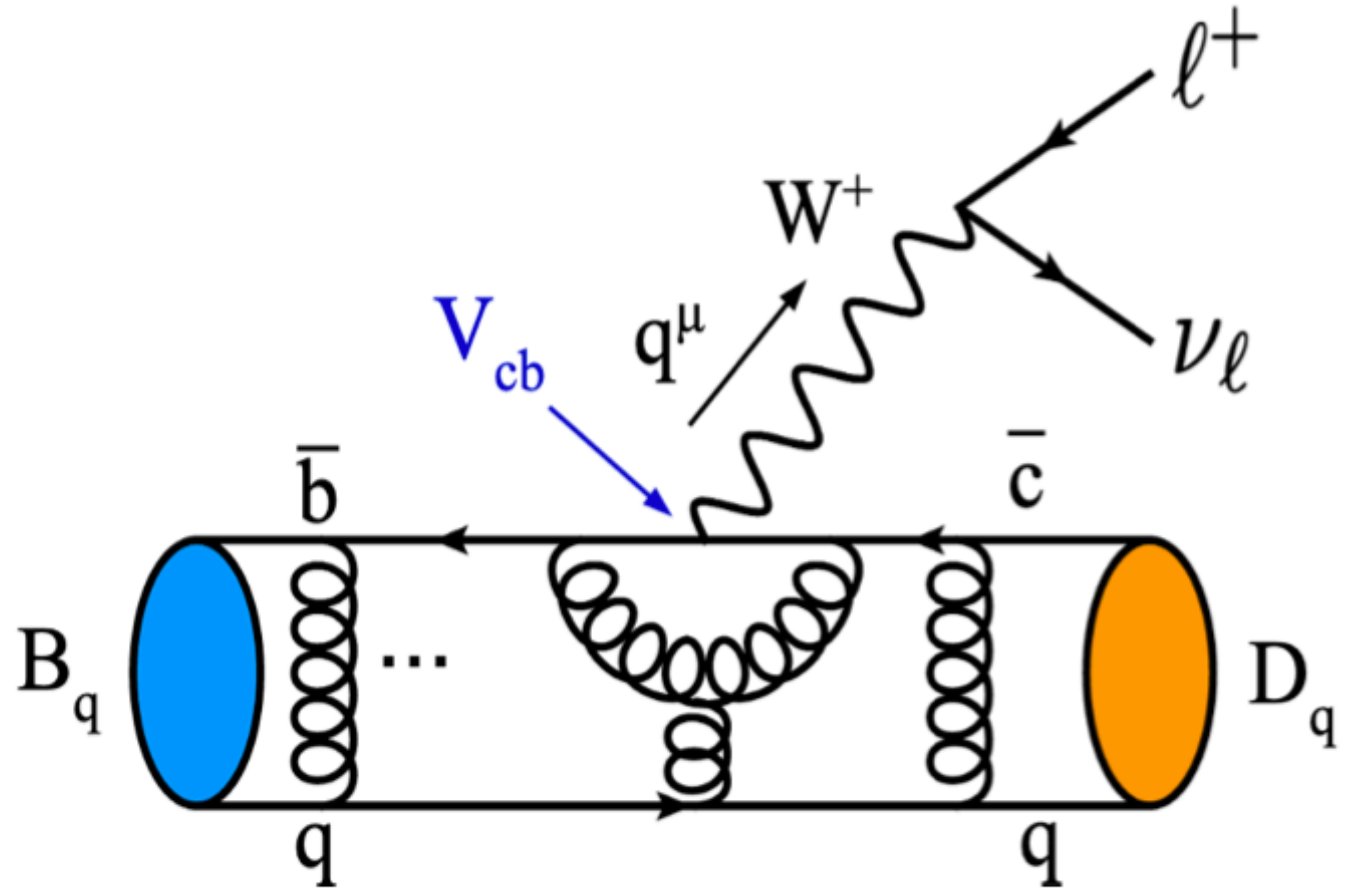
CLN

	Constraints on $h_{A_1}(w)$	Constraints on $h_{A_1}(w)$, $R_1(w)$, $R_2(w)$
$h_{A_1}(1)$	0.91 ± 0.02	0.94 ± 0.02
ρ^2	1.22 ± 0.05	1.21 ± 0.04
$R_1(1)$	1.14 ± 0.07	1.26 ± 0.04
$R_2(1)$	0.88 ± 0.03	0.88 ± 0.03
$\lvert V_{cb}\rvert \times 10^3$	40.3 ± 1.2	38.7 ± 1.1
χ^2/ndf	39/33	70/39
p-value	23%	0.2%



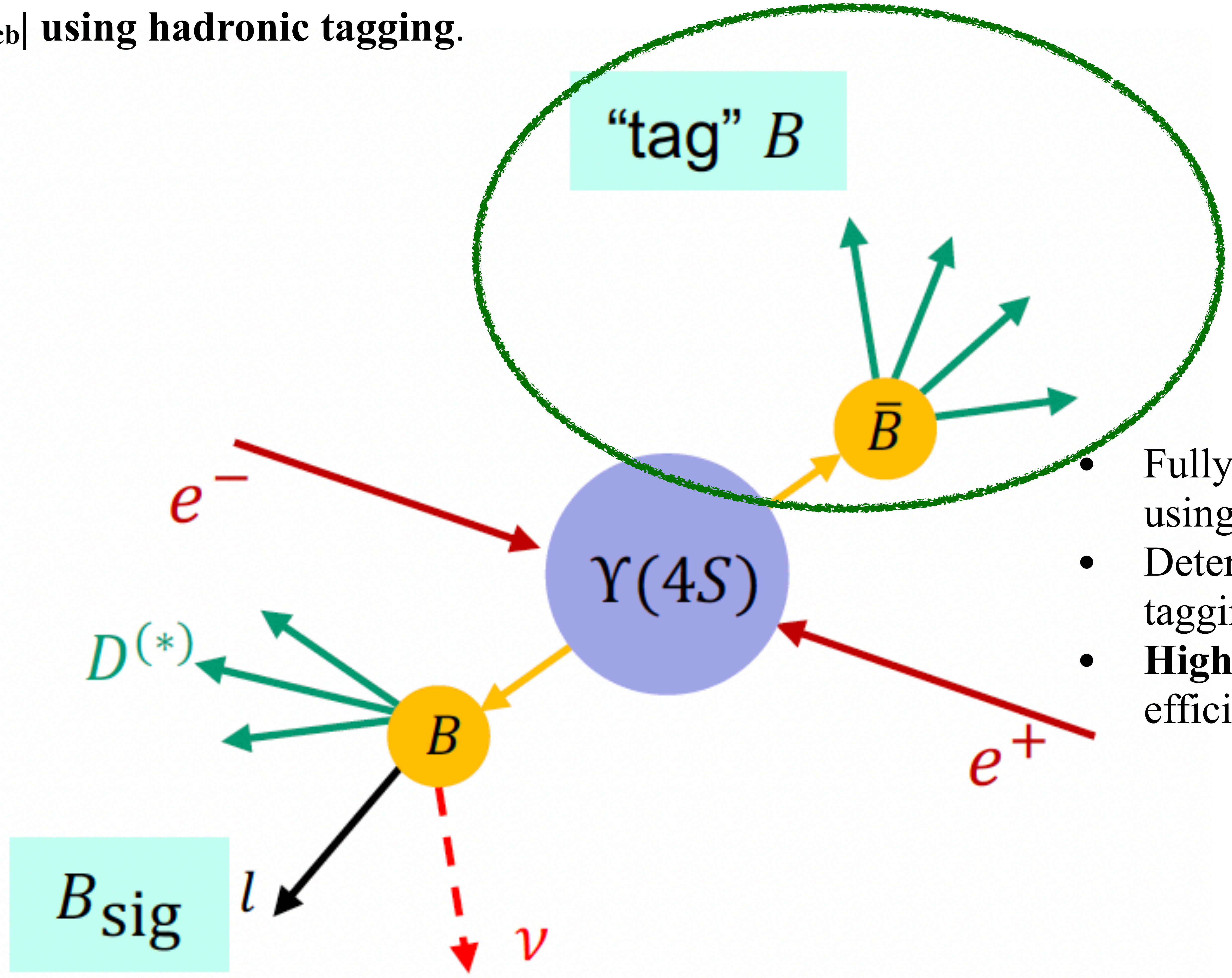
- Agreement between the different parameterizations and the extracted $|V_{cb}|$ values .
- Constraints on $h_{A_1}(w), R_1(w), R_2(w)$ shift $|V_{cb}|$ significantly and lead to tension with predictions from FNAL/MILC.

Tagged Exclusive $|V_{cb}|$



Tagged $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$

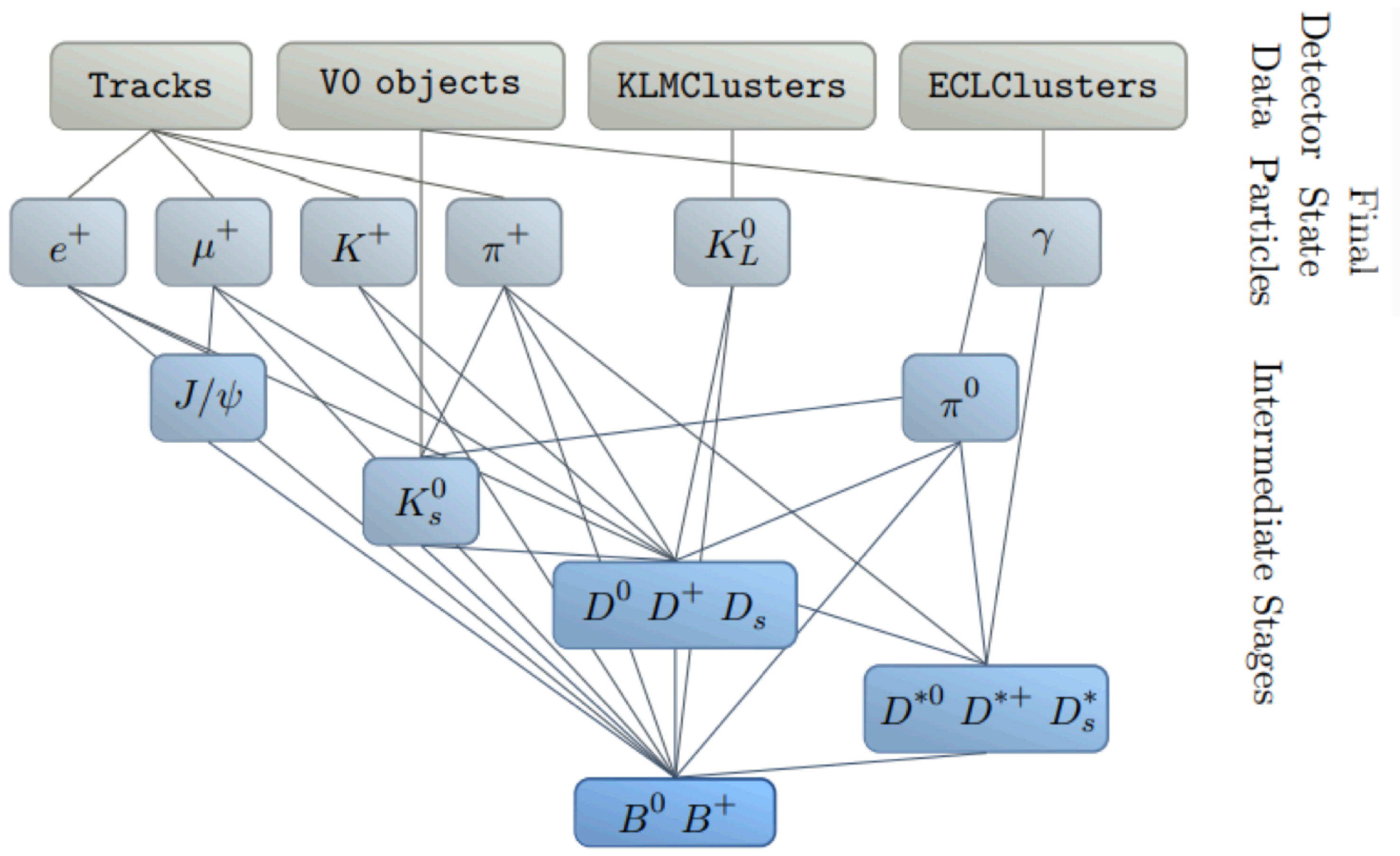
- Another approach to **measure $|V_{cb}|$ using hadronic tagging**.

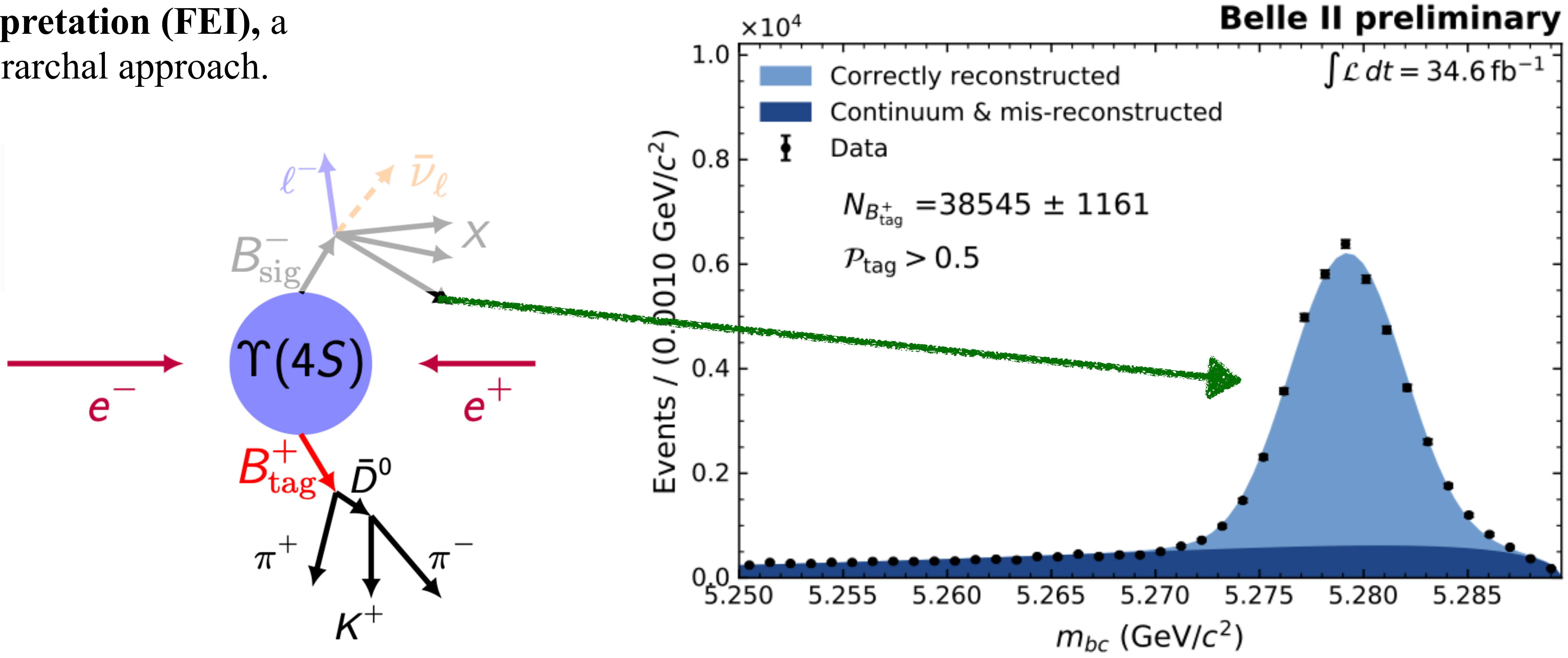


- Fully reconstruct the decay of the other B using hadronic modes.
- Determine exclusively the direction of the tagging B and thus the signal B.
- **High purity** approach albeit lower efficiency.

B-tagging at Belle II

- Exclusive reconstruction of B mesons using hadronic and semi-leptonic modes.
- Achieved using the **Full Event Interpretation (FEI),** a multivariate algorithm based on a hierarchal approach.





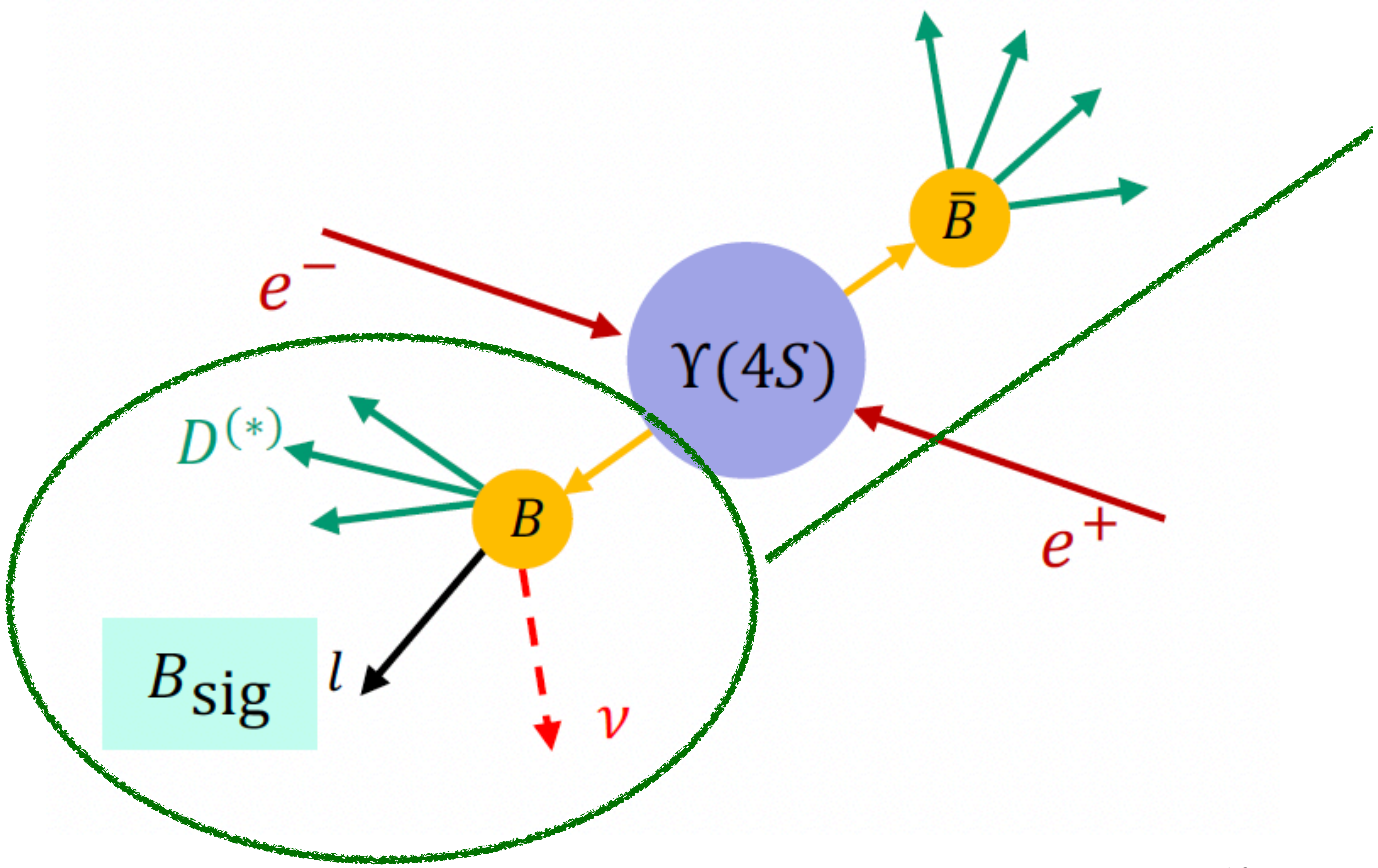
- Employs over 200 Boosted Decision Trees to reconstruct ~10000 B decay chains.
- **30-50% improvement in efficiency** compared to Full Reconstruction at Belle.

Comp. Softw. Big. Sci. 3 (2019)

Tagged $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$

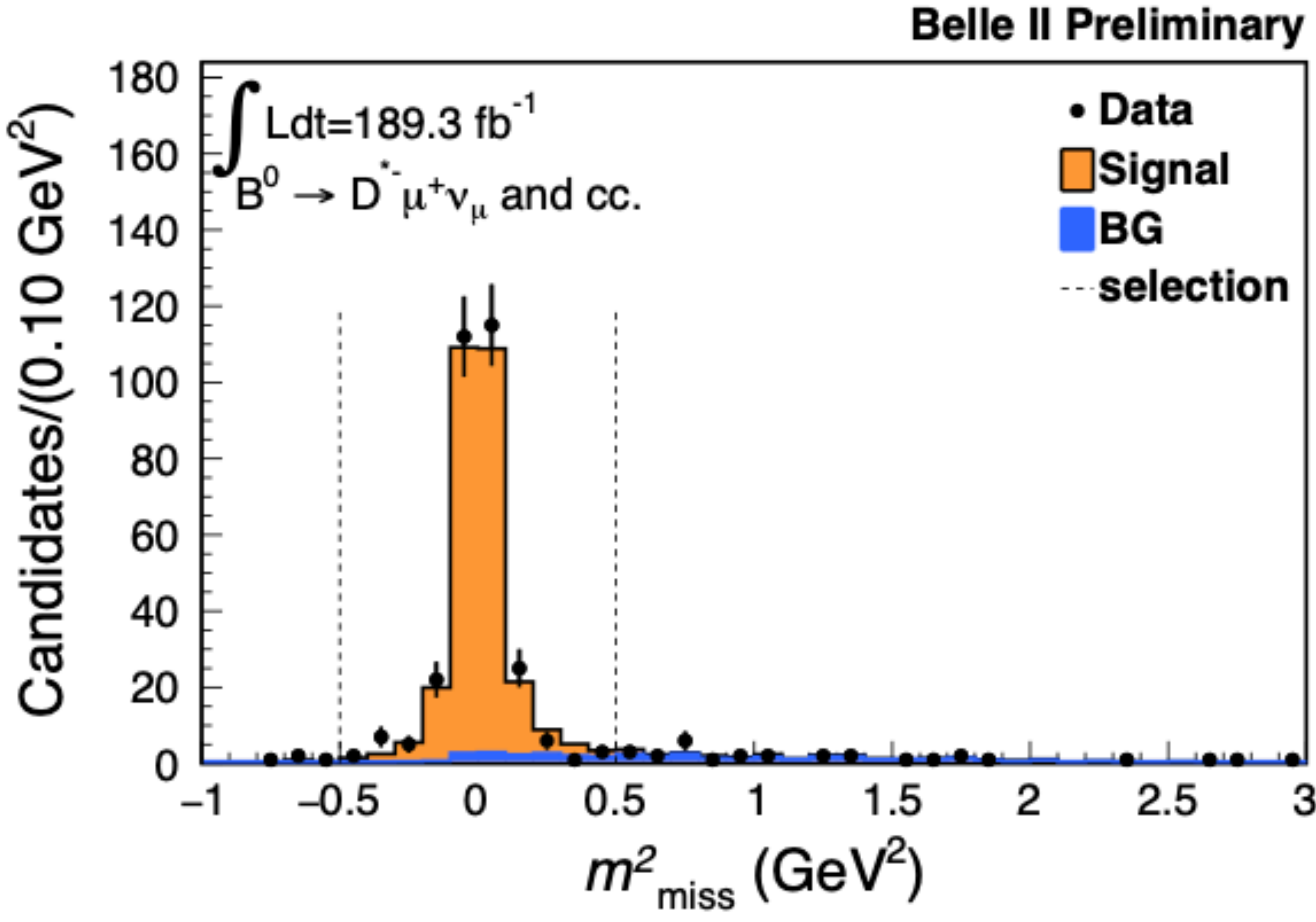
arXiv.2301.04716

- Infer momentum and direction of signal B candidate
- Constrain the kinematics related to the missing neutrino by reconstructing everything else in the event.



- Reconstruct $\bar{B}^0_{sig} \to [D^{*+} \to D^0\pi^+]\ell^-\bar{\nu}_\ell$ and determine m^2_{miss}:

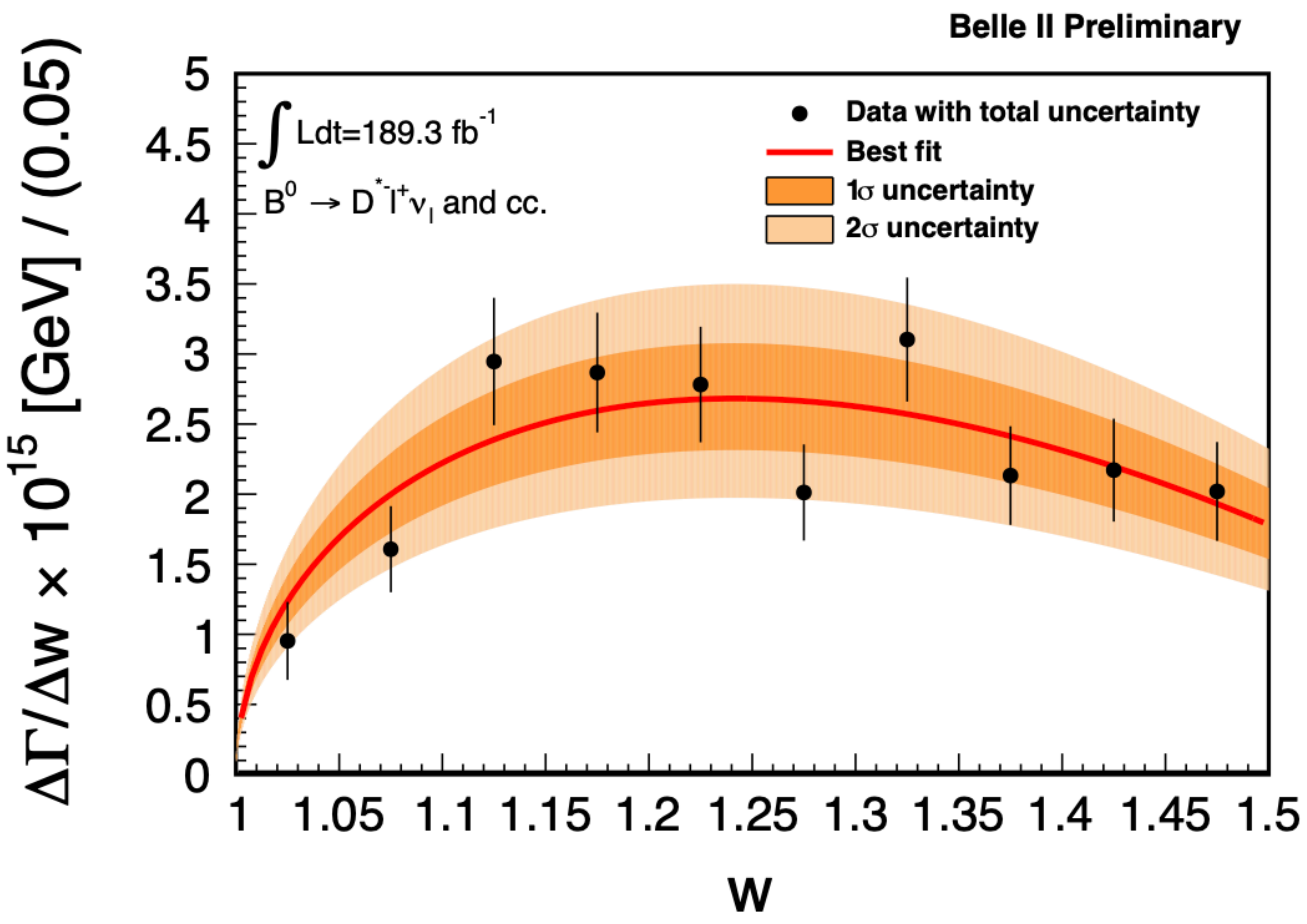
$$m^2_{miss} = (p_{e^+e^-} - p_{B_{\text{tag}}} - p_{D^*} - p_\ell)^2$$



arXiv.2301.04716

Tagged $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$

- Extract $|V_{cb}|$ using fit to the differential distribution and the total decay rate in the CLN parametrisation using input values of the form factors $R_1(1)$ and $R_2(1)$. (Eur. Phys. J. C 81 (2021))



$$|V_{cb}| = 37.9 \pm 2.7$$

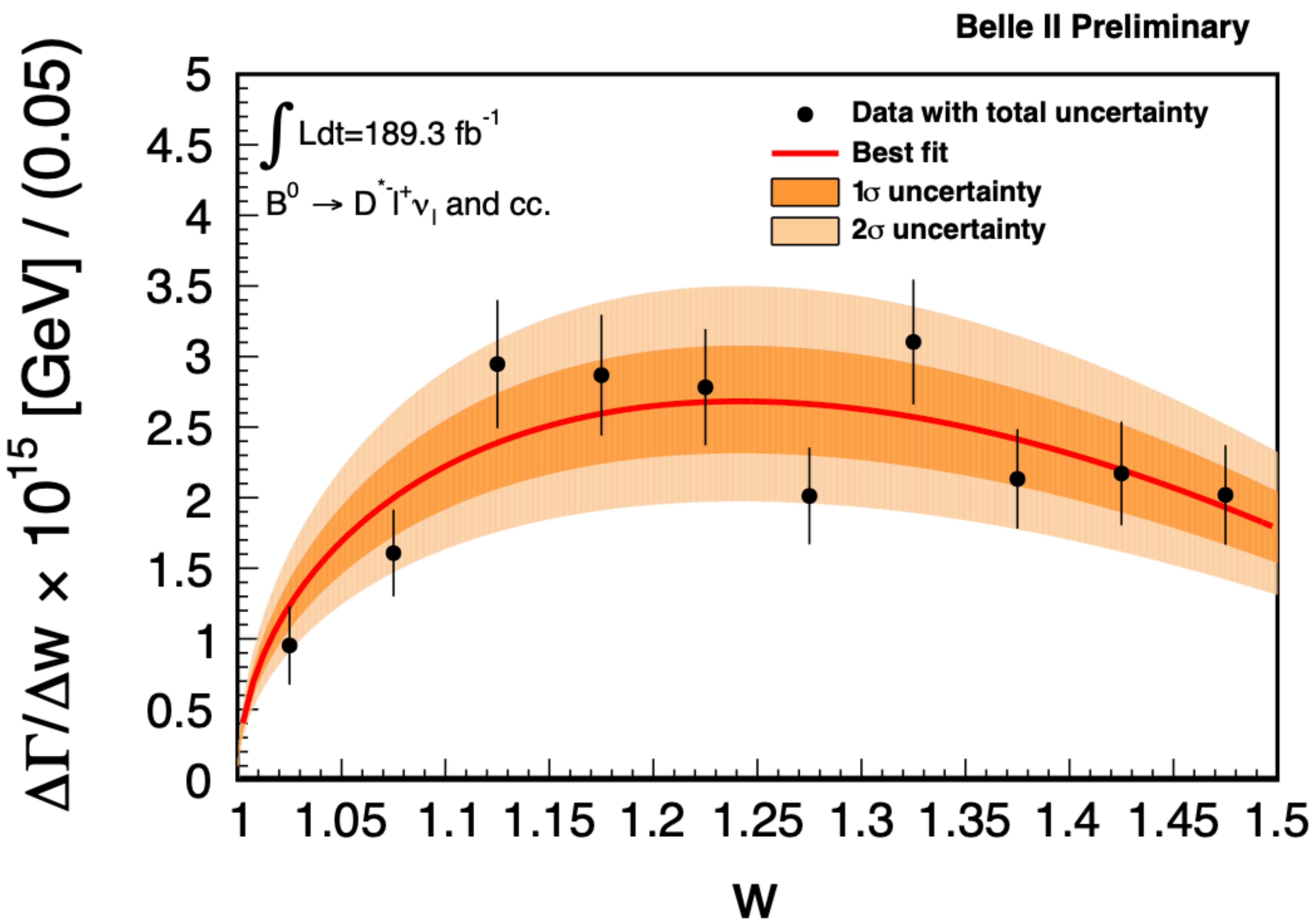
In agreement with the world average of exclusive measurements

Systematic sources	Relative uncertainty (%)
FEI efficiency	3.9
Low momentum π efficiency	4.1
Tracking efficiency	0.9
Lepton particle identification	2.0
Background	1.2
$N_{B\bar{B}}$	2.9
f_{+0}	1.2
Number of mixed $B\bar{B}$	0.9
$\mathcal{B}\left(D^{*-} \to \pi^- \bar{D}^0\right)$	0.7
$\mathcal{B}\left(\bar{D}^0 \to K^+\pi^-\right)$	0.8
ECL energy	1.0
Form factor	0.1
MC sample size	1.8
Total	7.3

Tagged $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$

arXiv.2301.04716

- Extract $|V_{cb}|$ using fit to the differential distribution and the total decay rate in the CLN parametrization using input values of the form factors $R_1(1)$ and $R_2(1)$. (Eur. Phys. J. C 81 (2021))



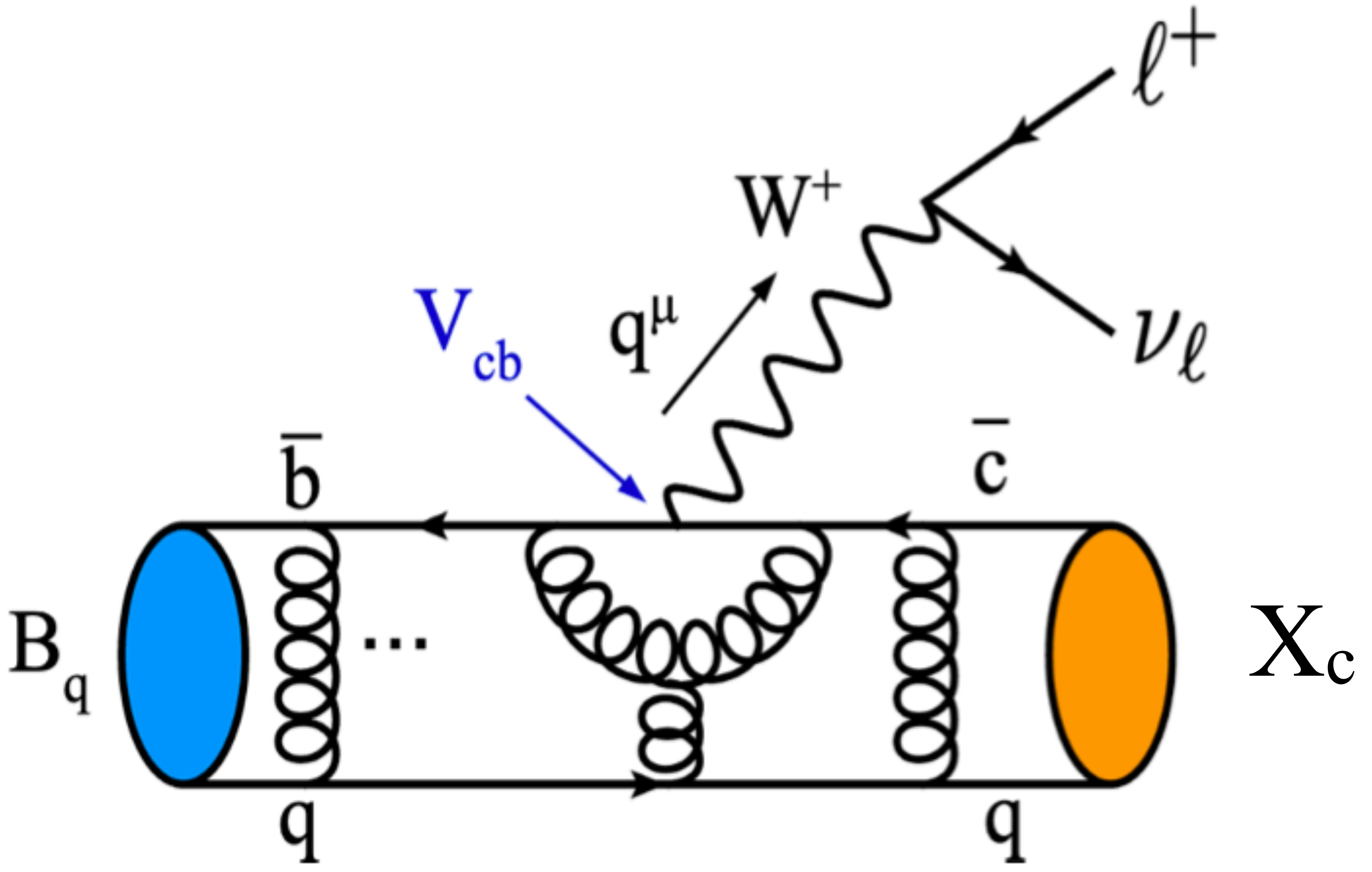
Systematic sources	Relative uncertainty (%)
FEI efficiency	3.9
Low momentum π efficiency	4.1
Tracking efficiency	0.9
Lepton particle identification	2.0
Background	1.2
$N_{B\bar{B}}$	2.9
f_{+0}	1.2
Number of mixed $B\bar{B}$	0.9
$\mathcal{B}\left(D^{*-} \to \pi^- \bar{D}^0\right)$	0.7
$\mathcal{B}\left(\bar{D}^0 \to K^+\pi^-\right)$	0.8
ECL energy	1.0
Form factor	0.1
MC sample size	1.8
Total	7.3

$$|V_{cb}| = 37.9 \pm 2.7$$

In agreement with the world average of exclusive measurements

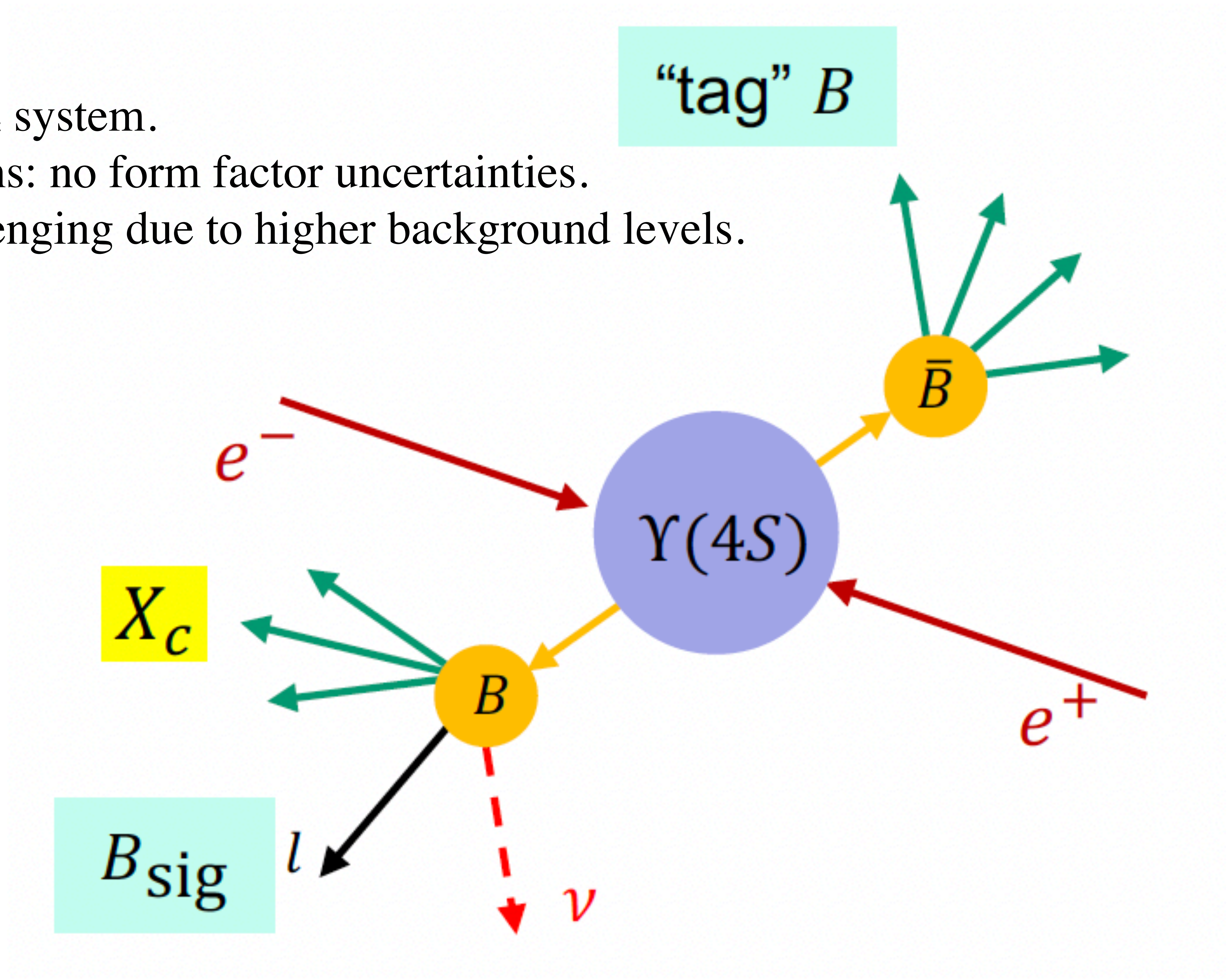
A determination of the form factors using both BGL and CLN parametrizations, along with $|V_{cb}|$, with this tagged approach is in progress with current Belle II dataset.

Inclusive $|V_{cb}|$



Inclusive $|V_{cb}|$

- No reconstruction of the X_c system.
- Clean theoretical predictions: no form factor uncertainties.
- Experimentally more challenging due to higher background levels.



Inclusive $|V_{cb}|$

- HQE in powers of $1/m_b$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B}\left[\Gamma_0 + \Gamma_{\mu_\pi}\frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \Gamma_{\rho_D}\frac{\rho_D^3}{m_b^3}\right]$$

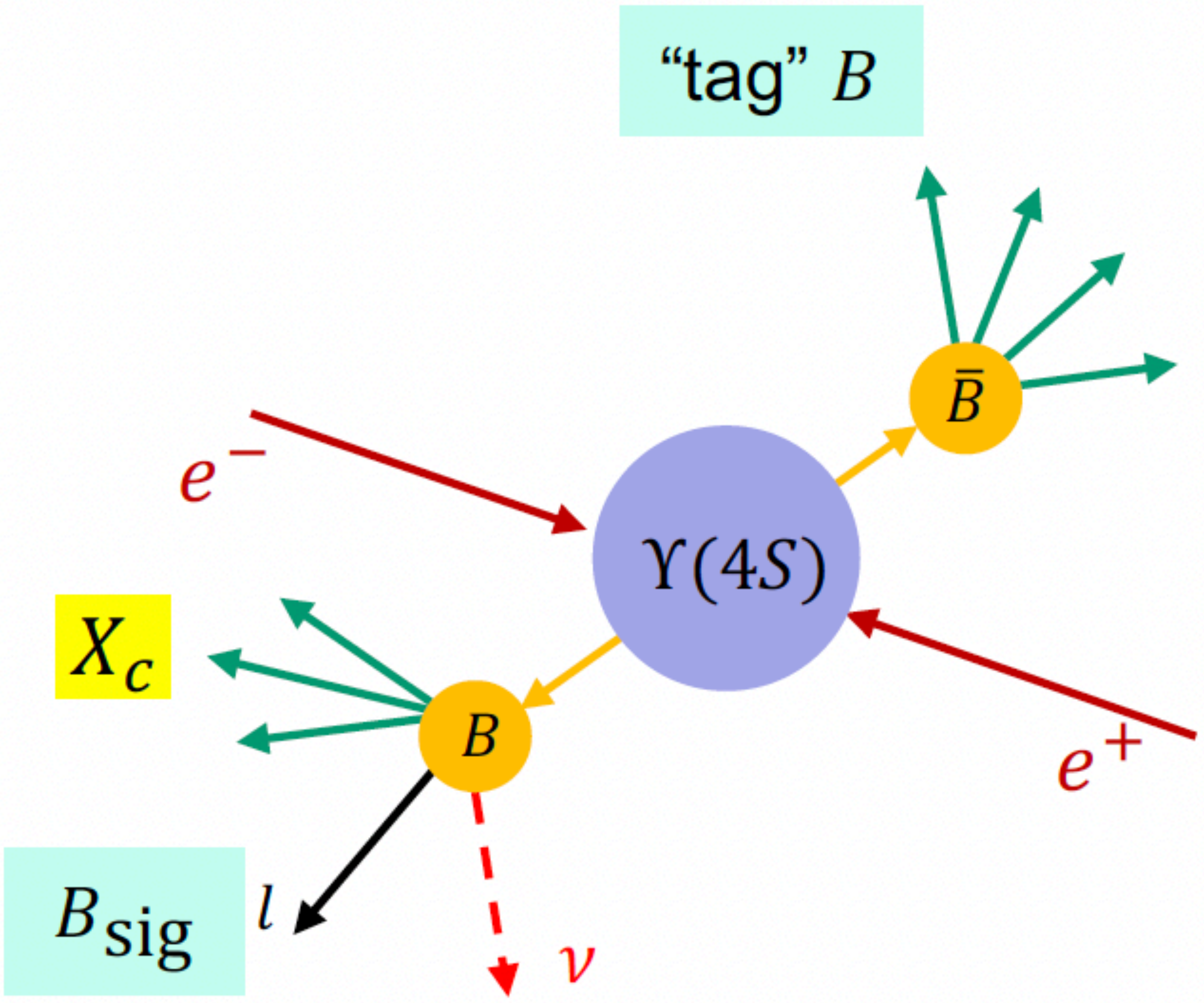
- Determine parameters of HQE using moments of the differential rate.

$$\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, m_b, (m_c)$$

$$\downarrow$$

$$\langle E^n \rangle_{\text{cut}} = \frac{\int_{E_\ell > E_{\text{cut}}} dE_\ell \, E_\ell^n \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\text{cut}}} dE_\ell \frac{d\Gamma}{dE_\ell}}$$

$$\langle (M_X^2)^n \rangle_{\text{cut}} = \frac{\int_{E_\ell > E_{\text{cut}}} dM_X^2 \, (M_X^2)^n \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\text{cut}}} dM_X^2 \frac{d\Gamma}{dM_X^2}}$$



- Determine $|V_{cb}|$ using the total branching fraction as input.

Alternative Inclusive $|V_{cb}|$

- Achieve more precision by including higher order:

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)}\right) + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} \Gamma^{(D,0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \cdots \right)$$

NOVEL APPROACH!!!

- Number of parameters: 4 up to $1/m_b^3$, 13 up to $1/m_b^4$ and 31 up to $1/m_b^5$
- Use reparametrization invariance to link different orders of $1/m_b$ and reduce the number of total parameters
- **Requires RPI observables such as q^2 ⟶ 8 parameters instead of 13 !**

$$\langle (q^2)^n \rangle_{\rm cut} = \int_{q^2 > q^2_{\rm cut}} dq^2 \, (q^2)^n \frac{d\Gamma}{dq^2} \Big/ \int_{q^2 > q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2}$$

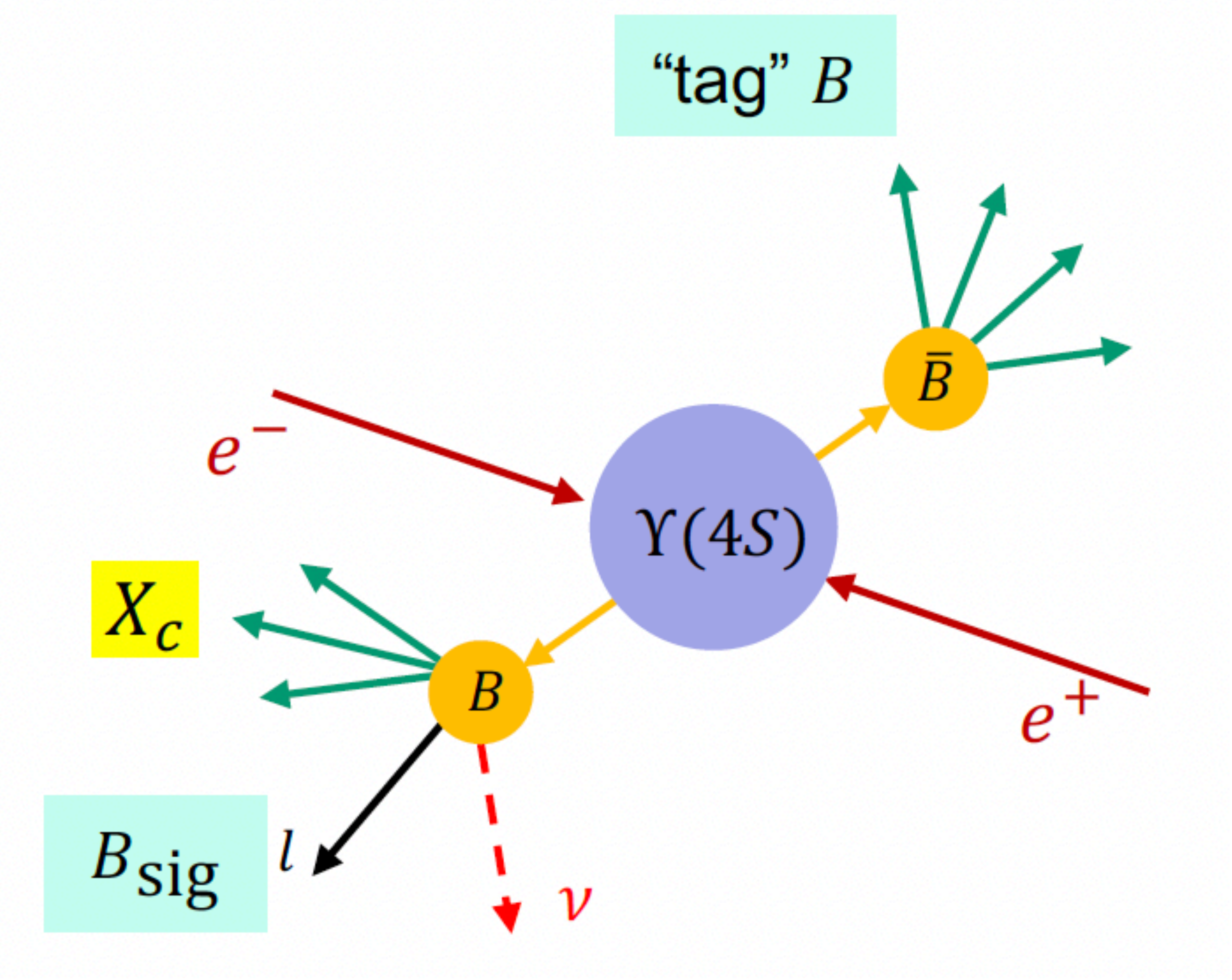
Measure $< q^{2n} >$ with Belle II data to determine inclusive $|V_{cb}|$

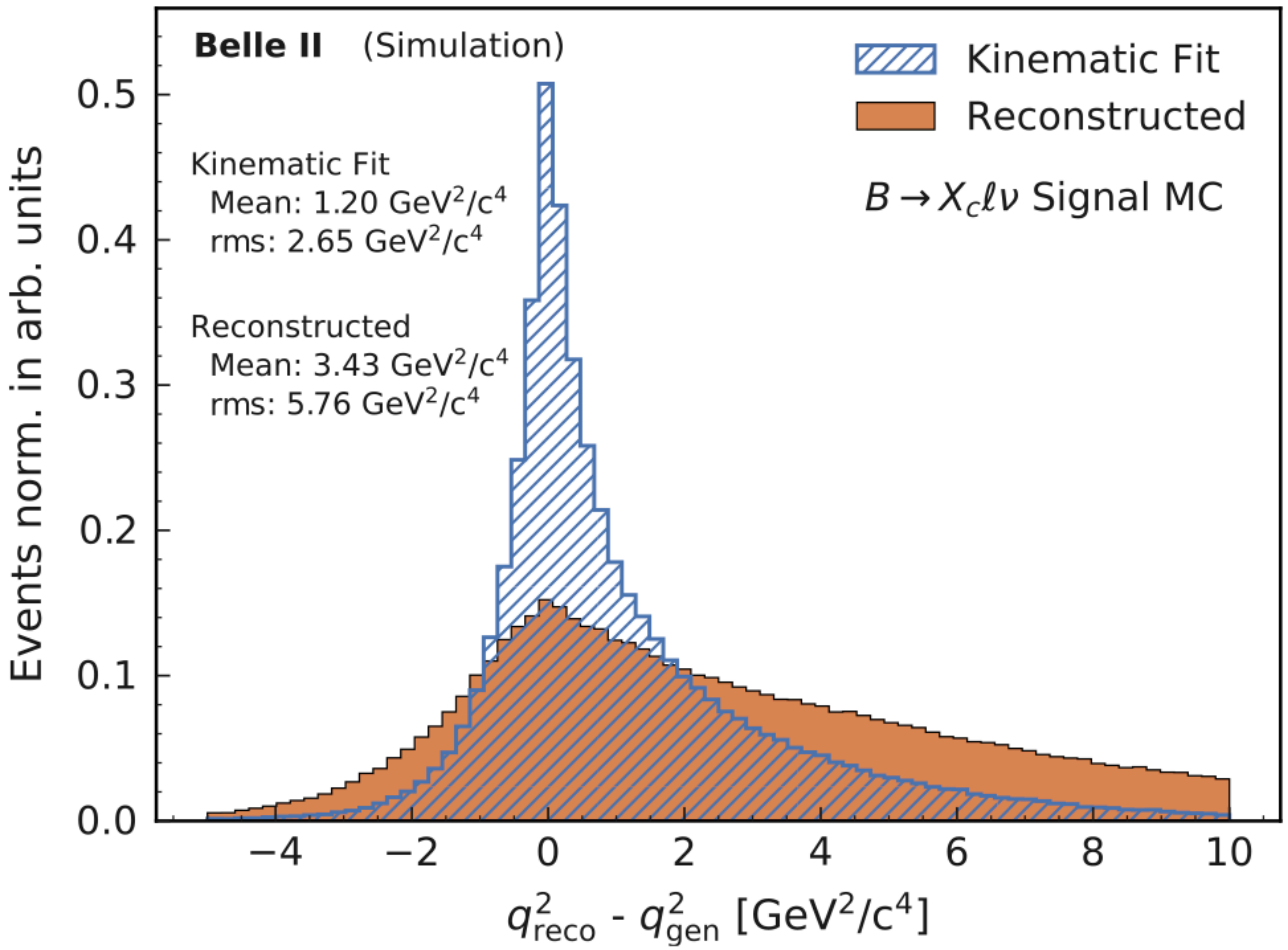
Fael, Mannel, Vos , JHEP 02 (2019) 177

q^2 Moments of $B \to X_c \ell \nu_\ell$

PRD **107**,072002(2023)

- Use **hadronic FEI tagging** with 62.8 fb^{-1} of data and identify one signal-side lepton.
- Identify X_c system using remaining tracks and clusters in the $\Upsilon(4S)$ rest of event.
- Use kinematic fit to improve overall $q^2 = (p^*_{B_{sig}} - p^*_X)^2$, determined in CM frame.

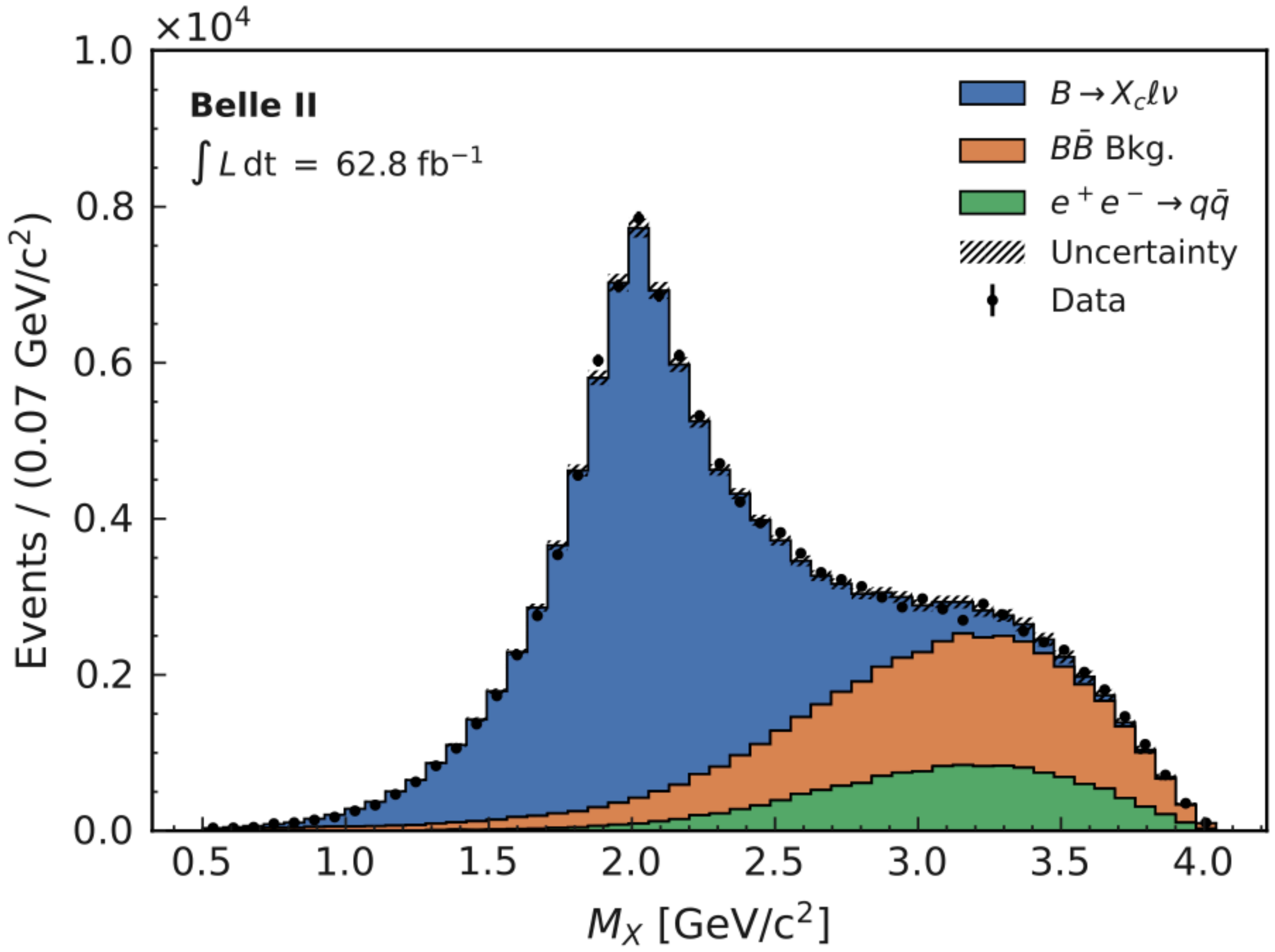


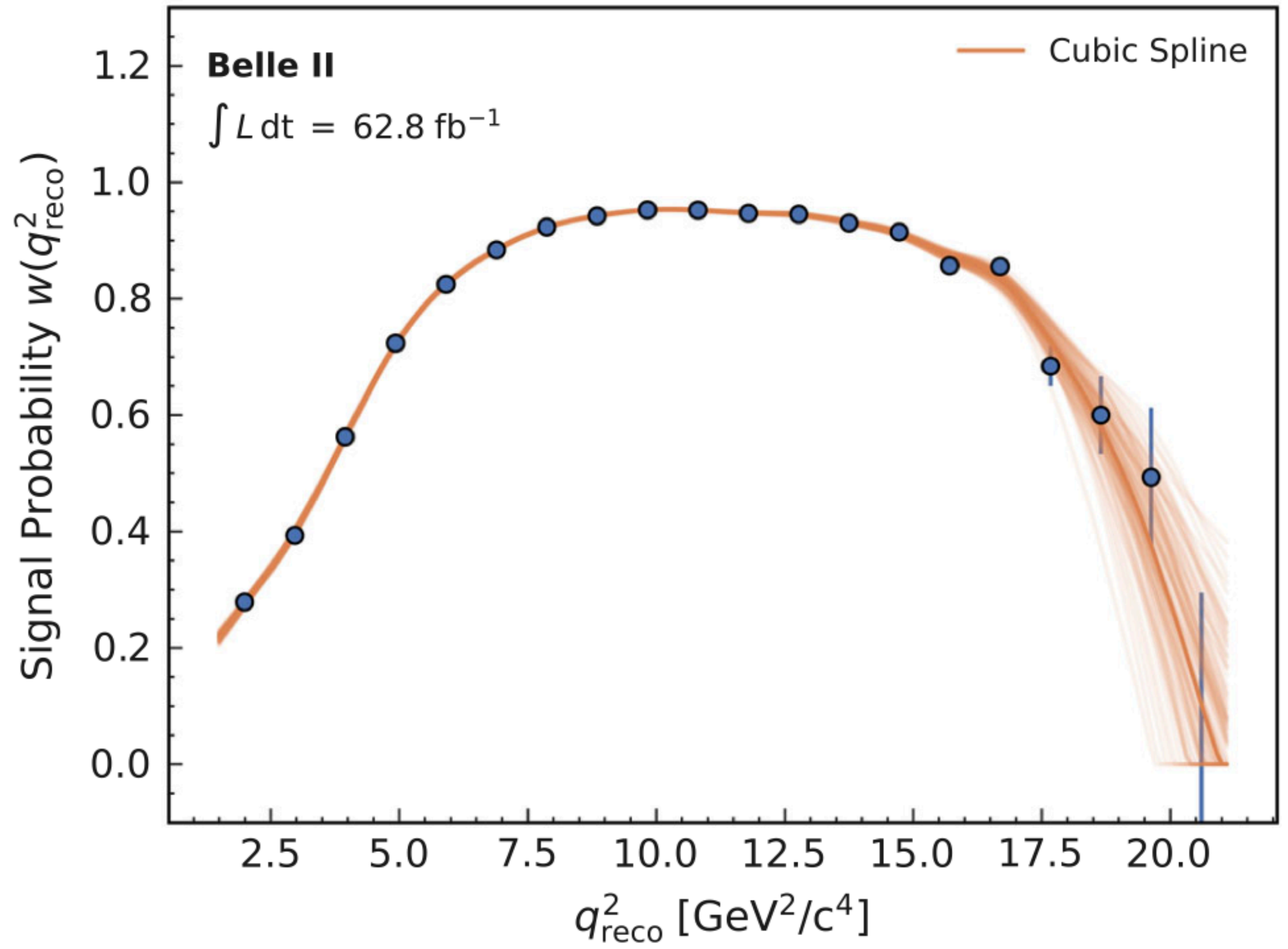


q^2 Moments of $B \to X_c \ell \nu_\ell$

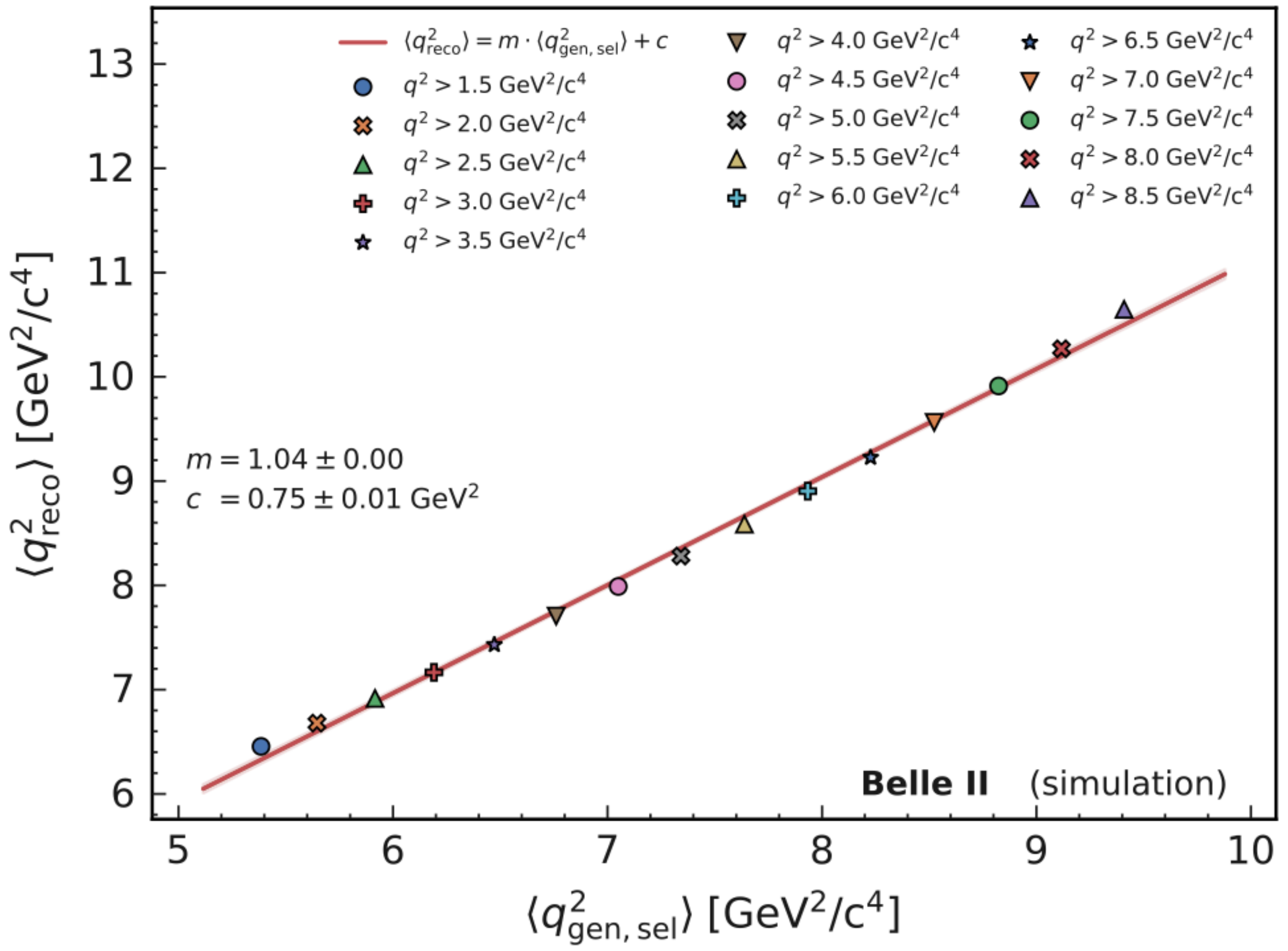
PRD **107**,072002(2023)

- Subtract background from M_X distribution using an event-wise continuous weight function.





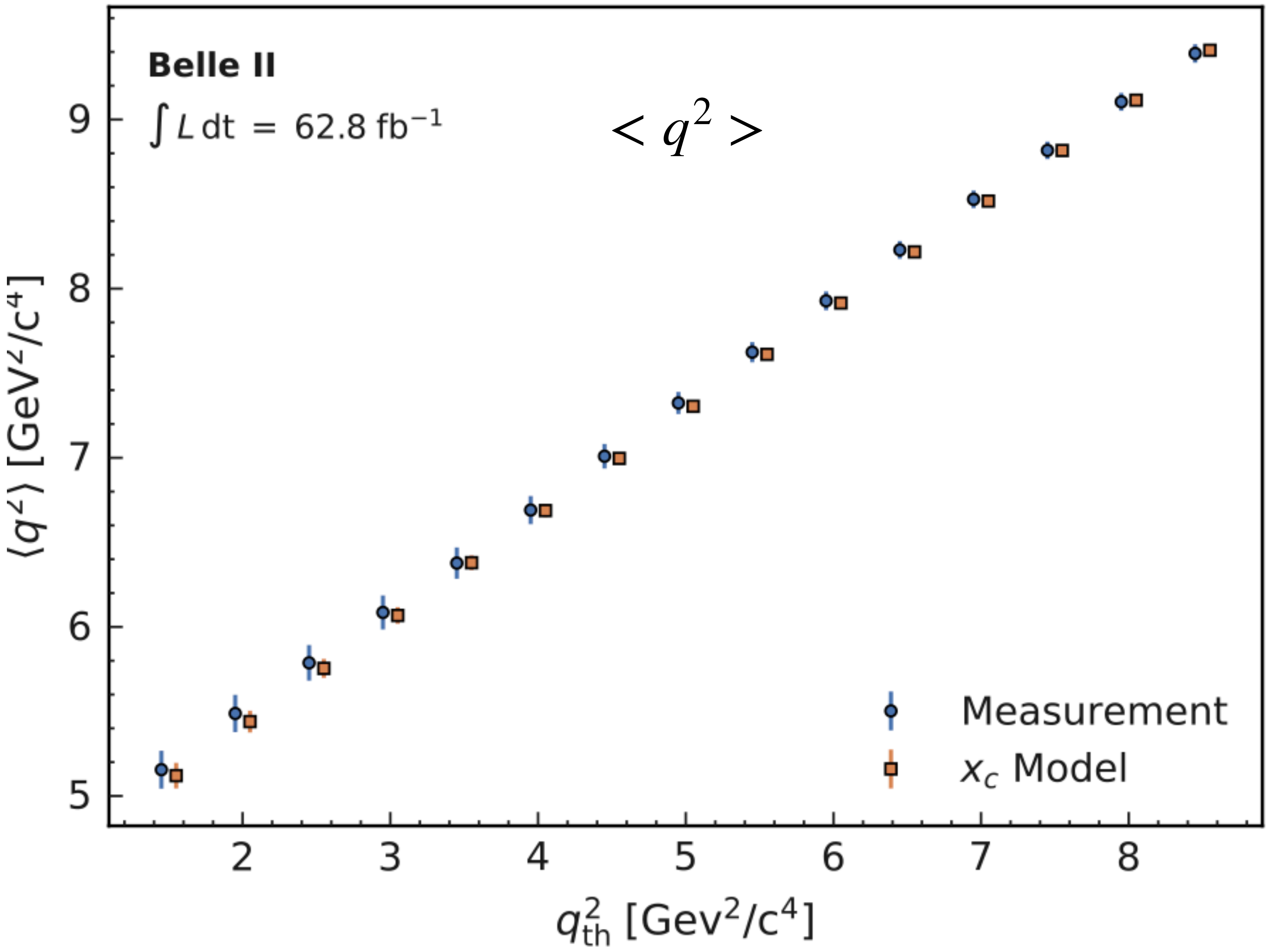
- Calibrate $< q^{2n} >$ using the relationship between the generated and reconstructed moments. $q^{2n}_{\text{calib}} = (q^{2n}_{\text{reco}} - c_n)/m_n$

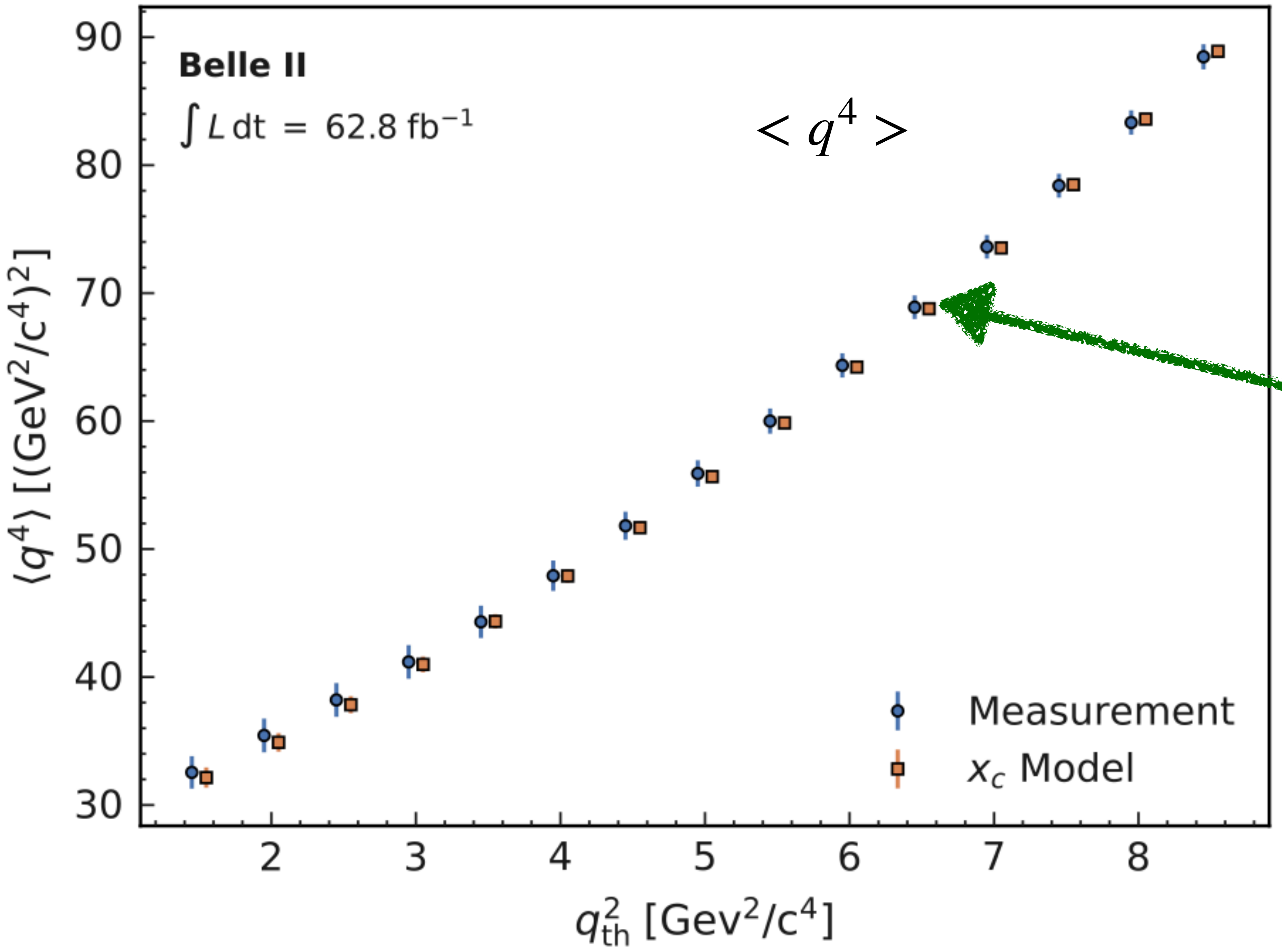


PRD **107**,072002(2023)

q^2 Moments of $B \to X_c \ell \nu_\ell$

- Extract the $< q^{2n} >$, $n = 1 - 4$ moments, in the region $q^2 = 1.5 - 8.0$ GeV$^2/c^4$
- Compare measured moments, after background subtraction and simulation, to moments determined using $B \to X_c \ell \nu$





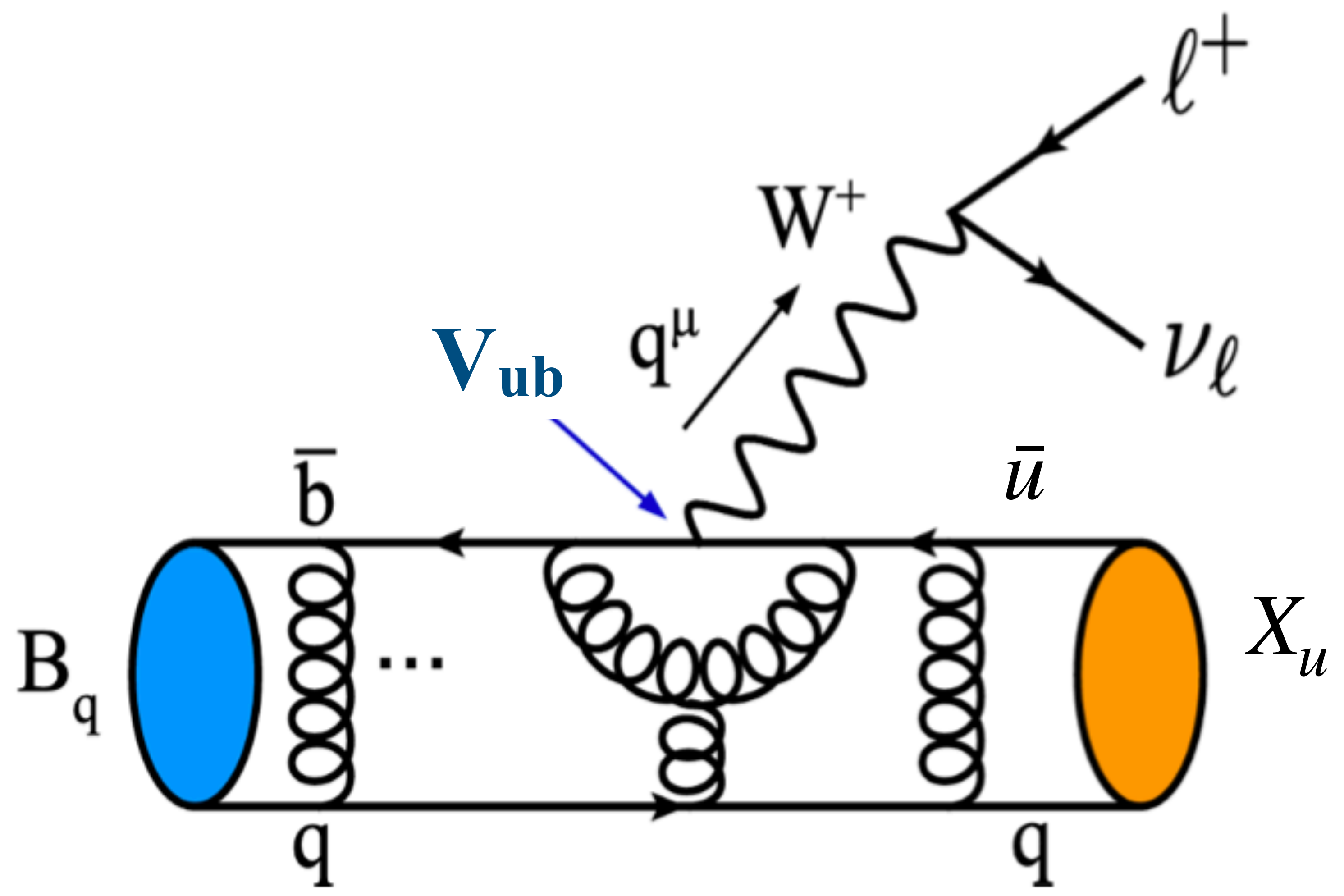
Agreement between measured moments and those extracted using X_c model.

First measurement of $< q^{2n} >$ at a B-factory. Crucial experimental input for the determination of inclusive $|V_{cb}|$.

Combined with Belle data:
$|V_{cb}| = (41.69 \pm 0.63) \times 10^{-3}$
Consistent with inclusive determinations.

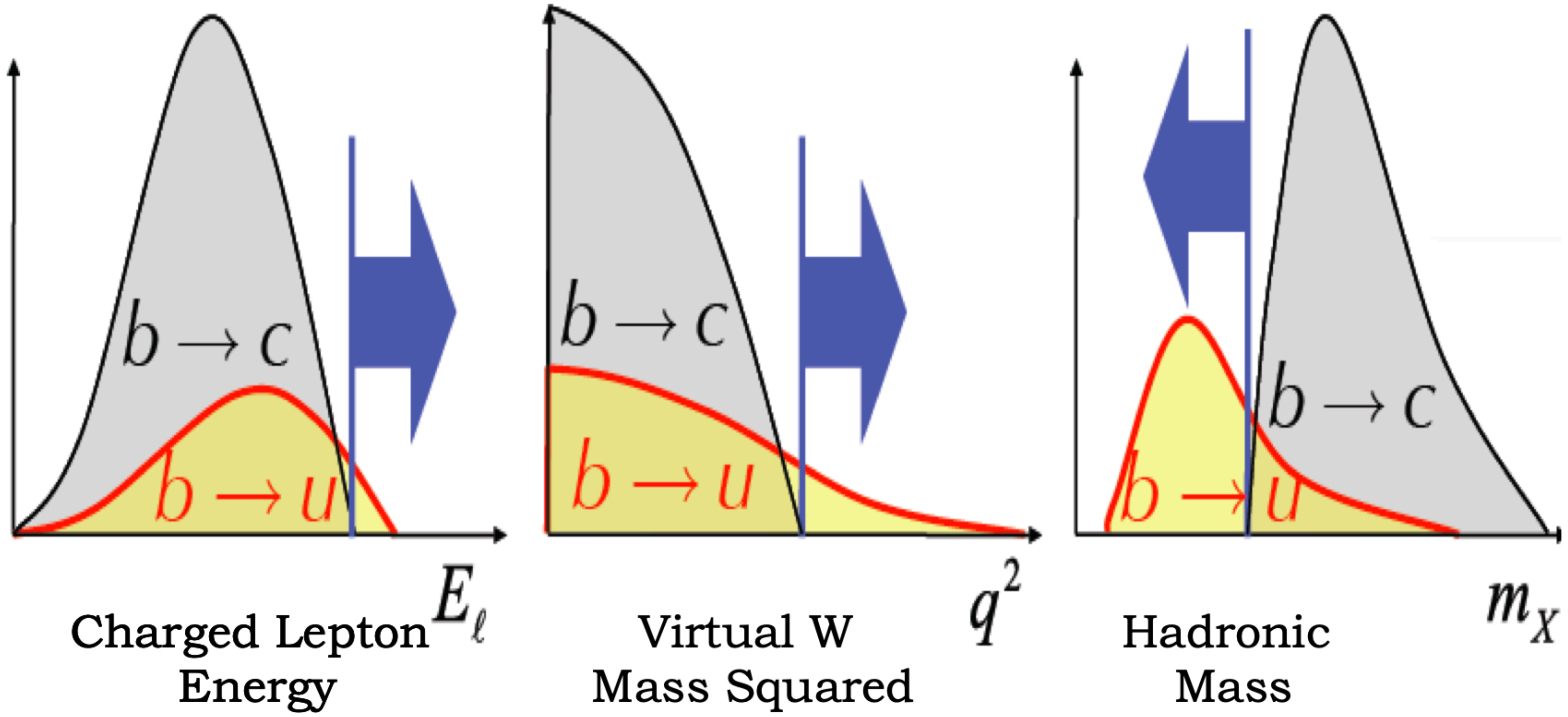
JHEP**10** 068 (2022)

Exclusive $|V_{ub}|$



$|V_{ub}|$

- Roughly an order of magnitude smaller than $|V_{cb}|$
- Experimentally challenging due to dominant $B \to X_c \ell \nu$ background.



- Exclusive via $B \to \pi \ell \nu$
  - Most precise determination of $|V_{ub}|$ (~4%)
  - Form factor determined non-perturbative from lattice QCD (high q^2) or LCSR (q^2~0) .

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f_+(q^2)|^2$$

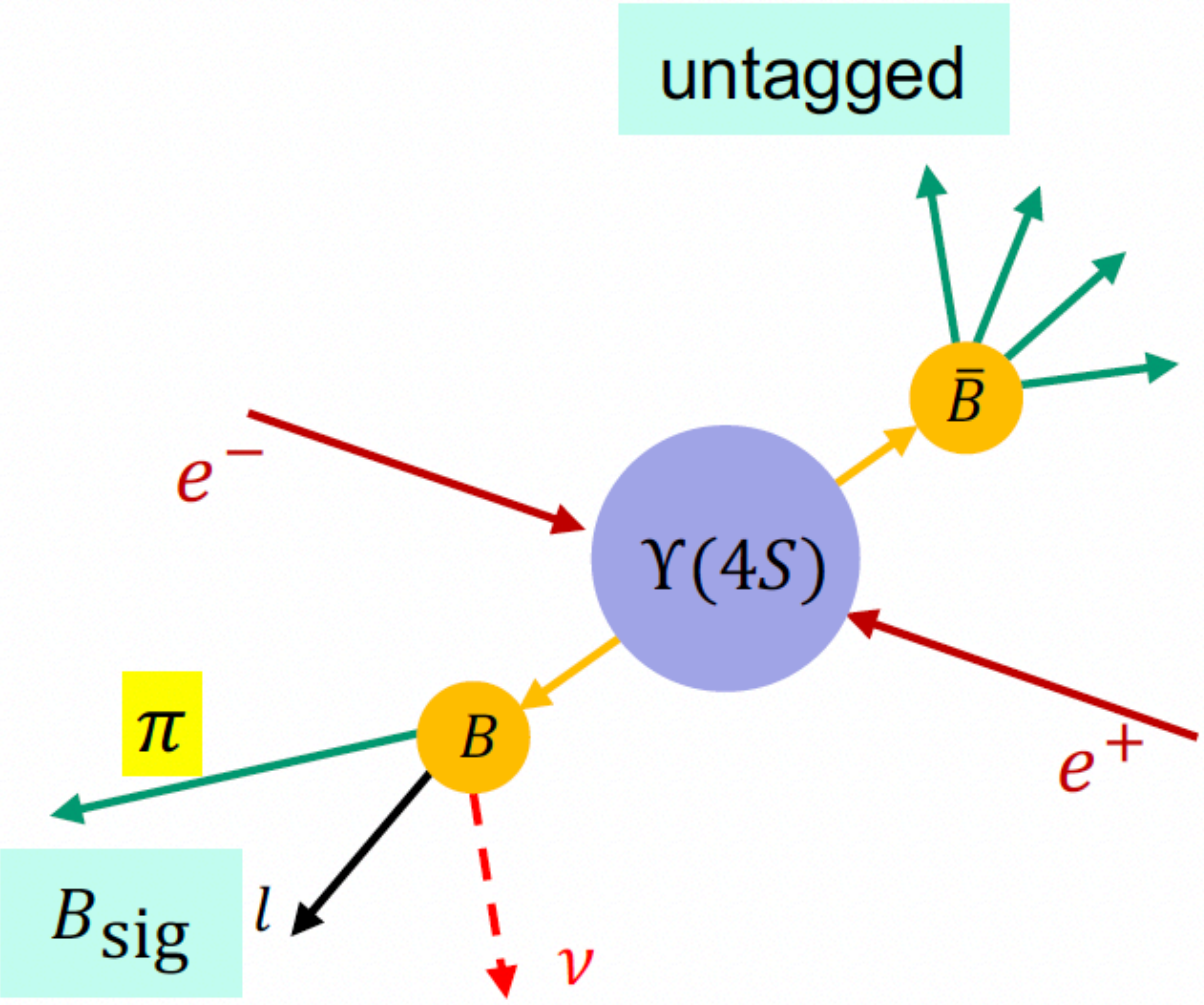
$|V_{ub}| = (4.19 \pm 0.12^{+0.11}_{-0.12}) \times 10^{-3}$ **PDG inclusive**

$|V_{ub}| = (3.67 \pm 0.15) \times 10^{-3}$ **PDG exclusive**

Current ~3σ tension between inclusive and exclusive determinations

Untagged $B^0 \to \pi^- \ell \nu_\ell$

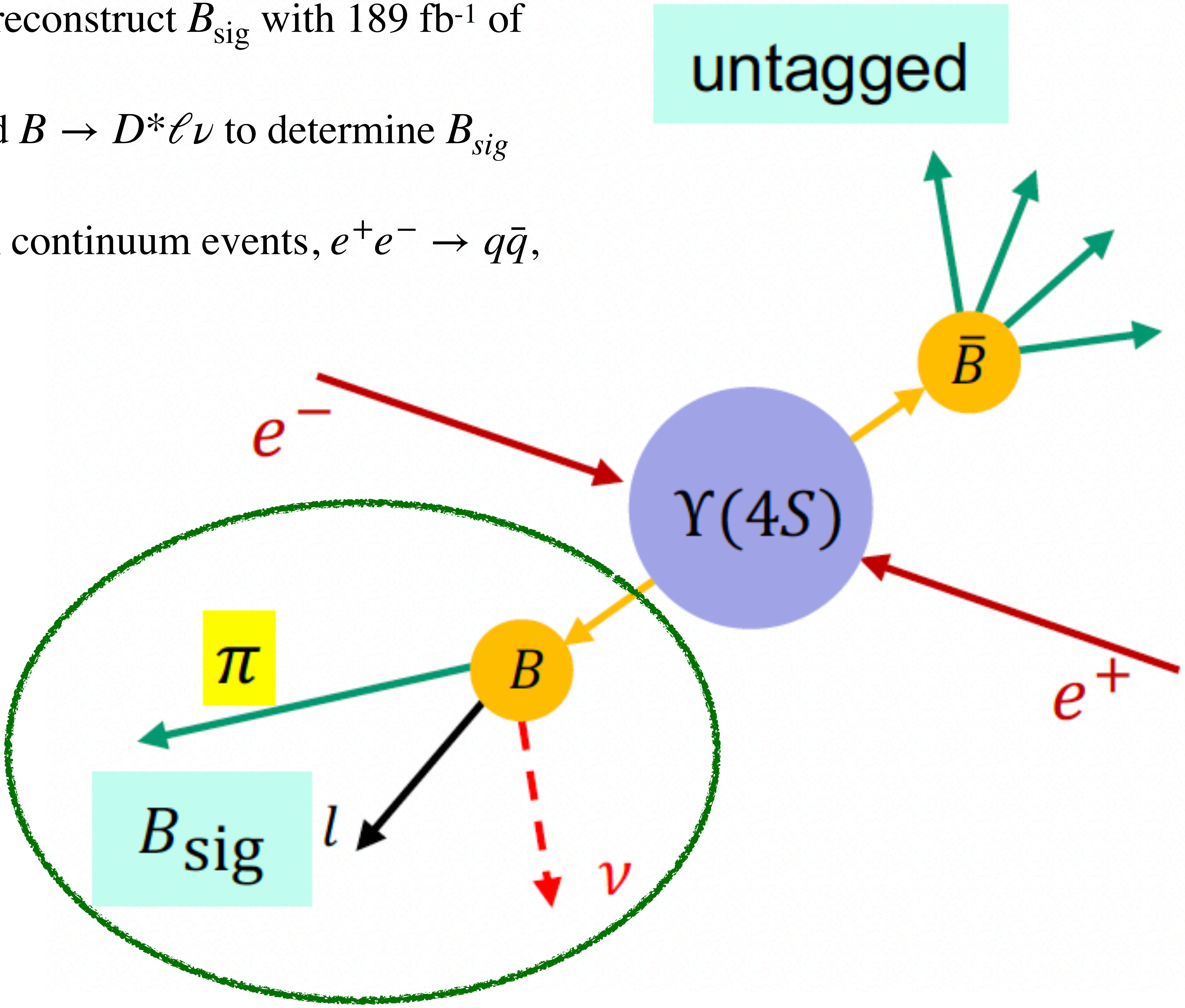
arXiv:2210.04224



Untagged $B^0 \to \pi^- \ell \nu_\ell$

arXiv:2210.04224

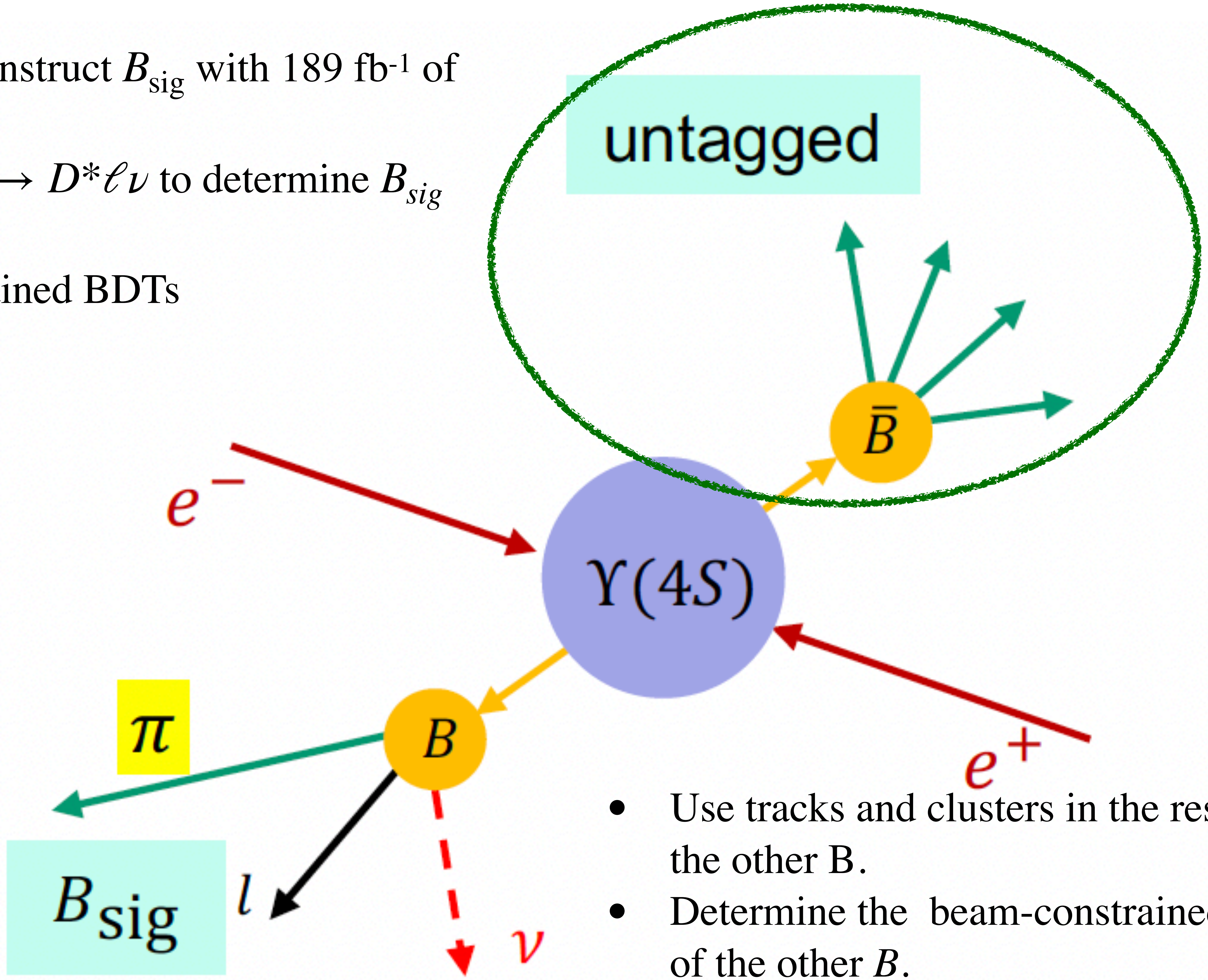
- Identify pion and lepton to reconstruct B_{sig} with 189 fb^{-1} of Belle II data.
- Similar strategy as untagged $B \to D^* \ell \nu$ to determine B_{sig} direction.
- Suppress backgrounds from continuum events, $e^+e^- \to q\bar{q}$, using trained BDTs



Untagged $B^0 \to \pi^- \ell \nu_\ell$

arXiv:2210.04224

- Identify pion and lepton to reconstruct B_{sig} with 189 fb^{-1} of Belle II data.
- Similar strategy as untagged $B \to D^* \ell \nu$ to determine B_{sig} direction.
- Suppress backgrounds using trained BDTs



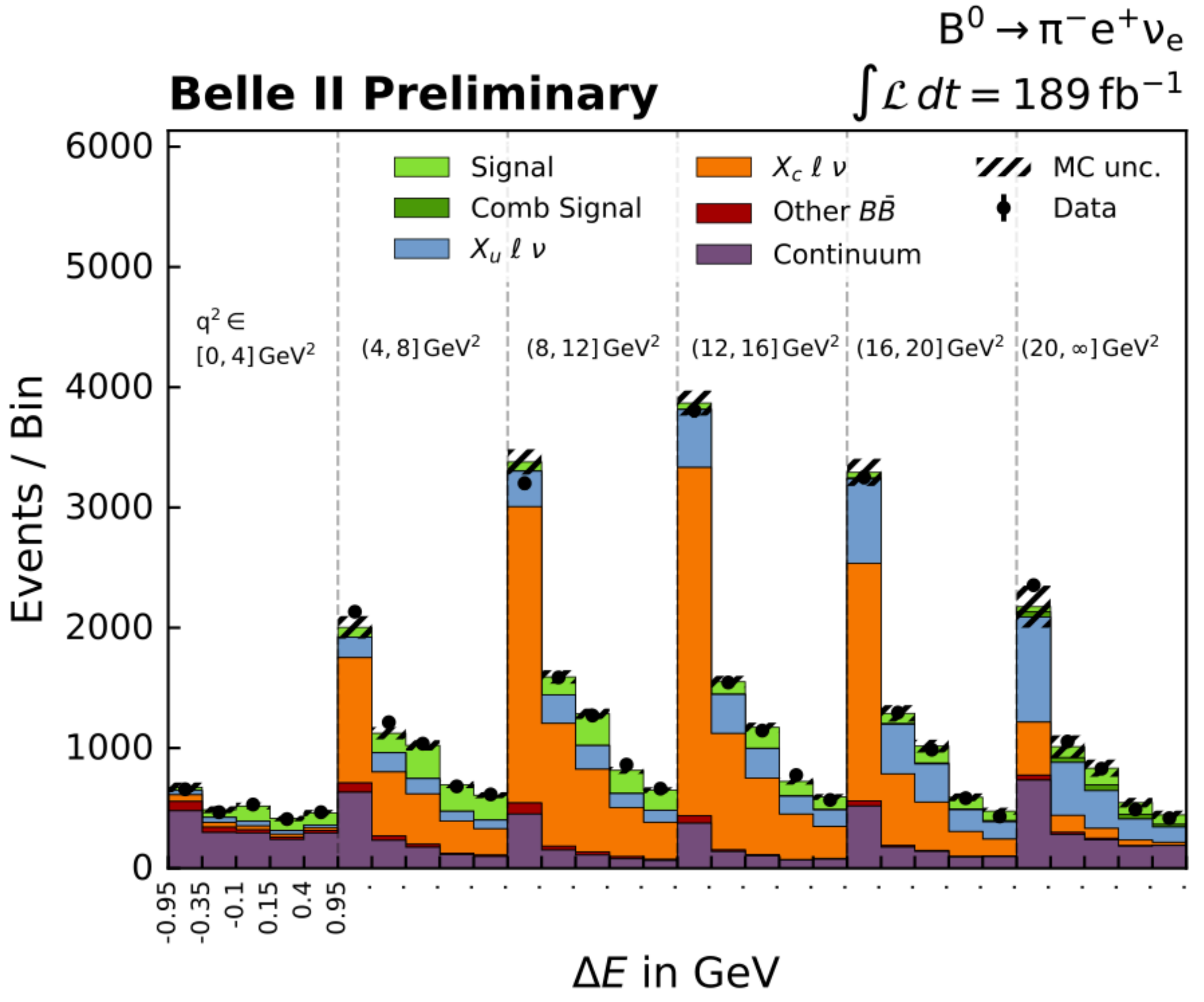
- Use tracks and clusters in the rest of the event to reconstruct the other B.
- Determine the beam-constrained mass, and energy difference of the other B.

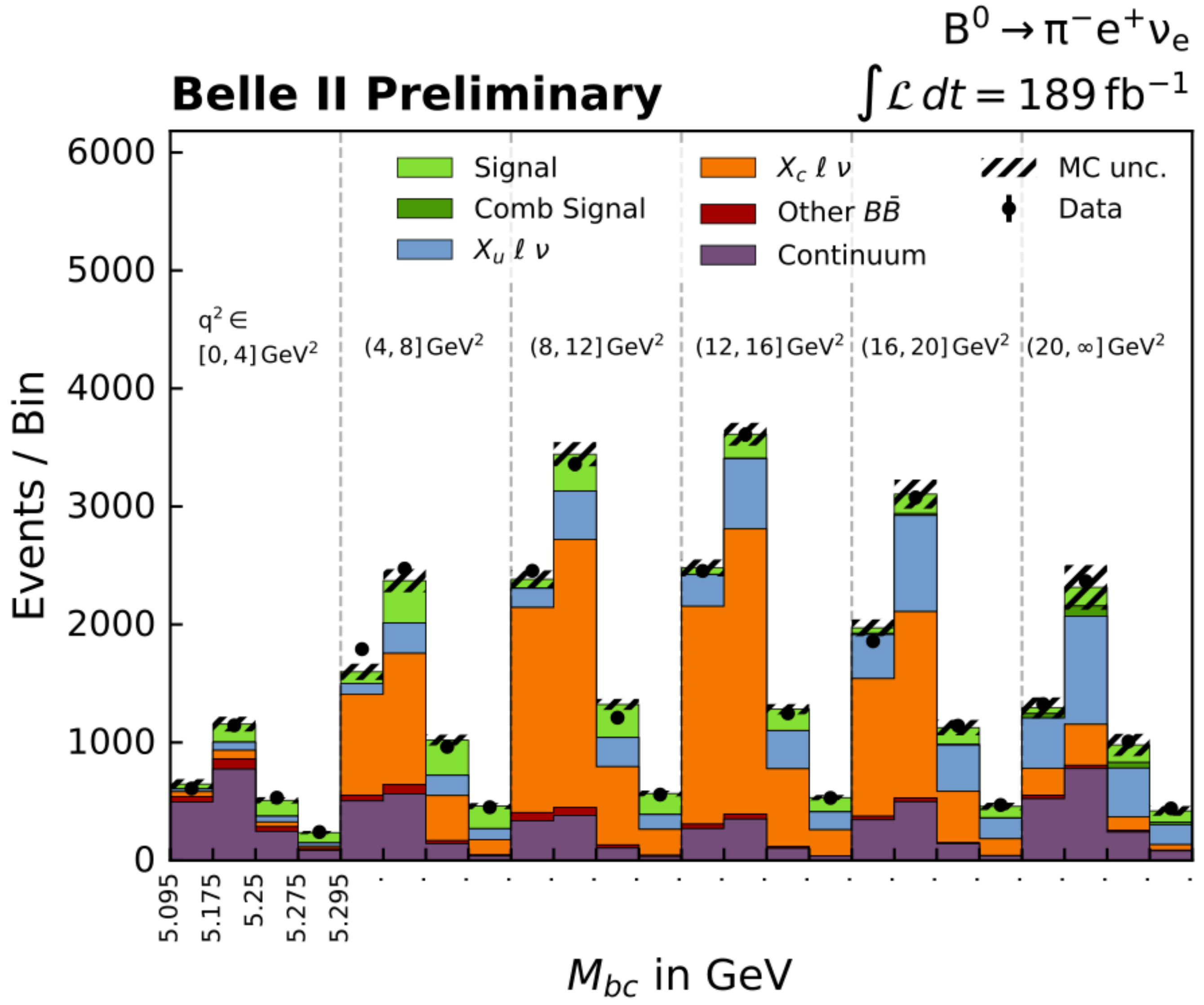
$$M_{bc} = \sqrt{(E^{*2}_{beam} - |\vec{p}^{\,*}_B|^2)} \qquad \Delta E = E^*_B - E^*_{beam}$$

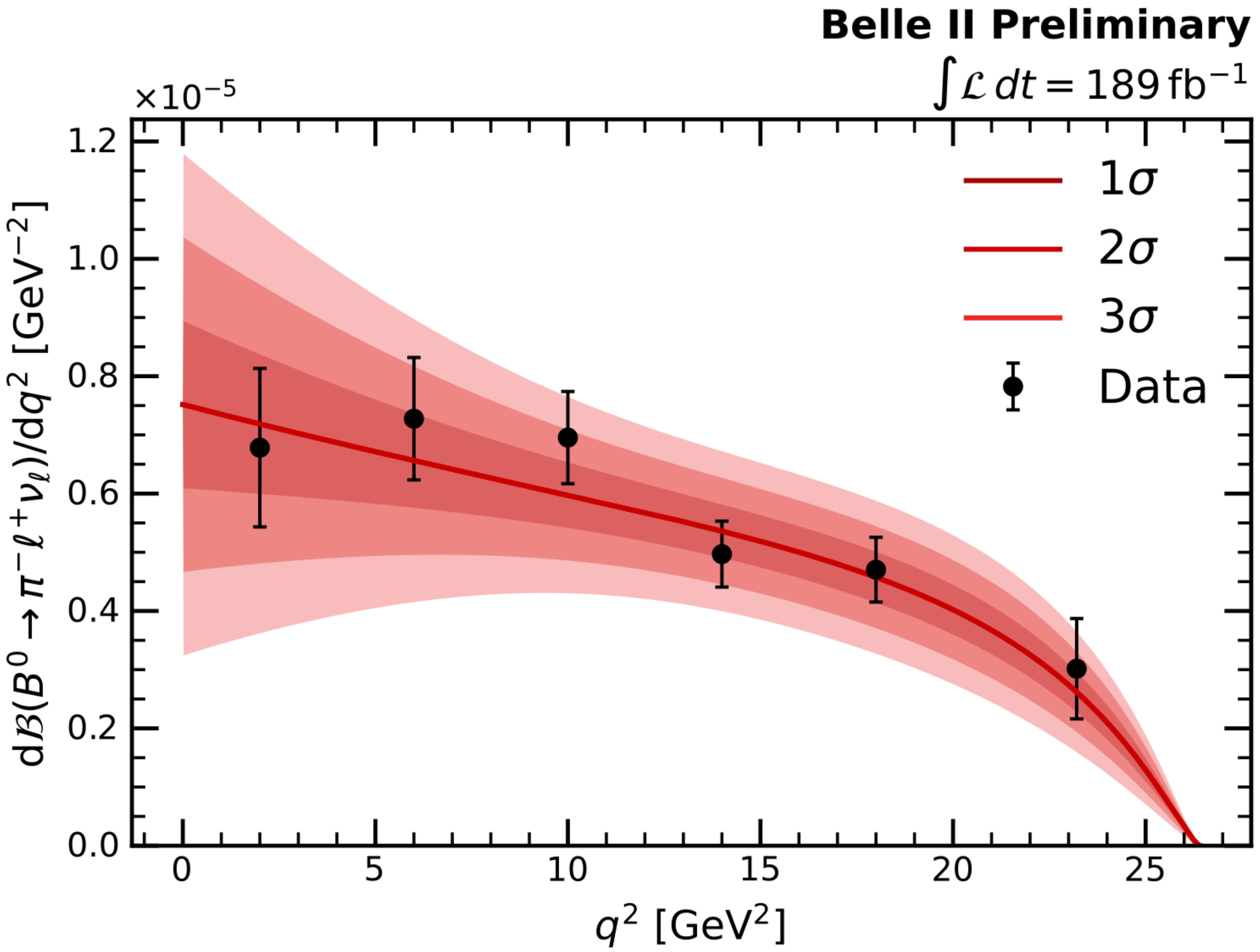
arXiv:2210.04224

Untagged $B^0 \to \pi^- \ell \nu_\ell$

- Extract signal yield from two-dimensional fits to ΔE and M_{bc}.
- Determine partial branching fractions in 6 bins of q^2.
- Extract $|V_{cb}|$ LQCD constraints on the 8 BCL parameters as nuisance parameters. Physical Review D, 92(1), 2015.







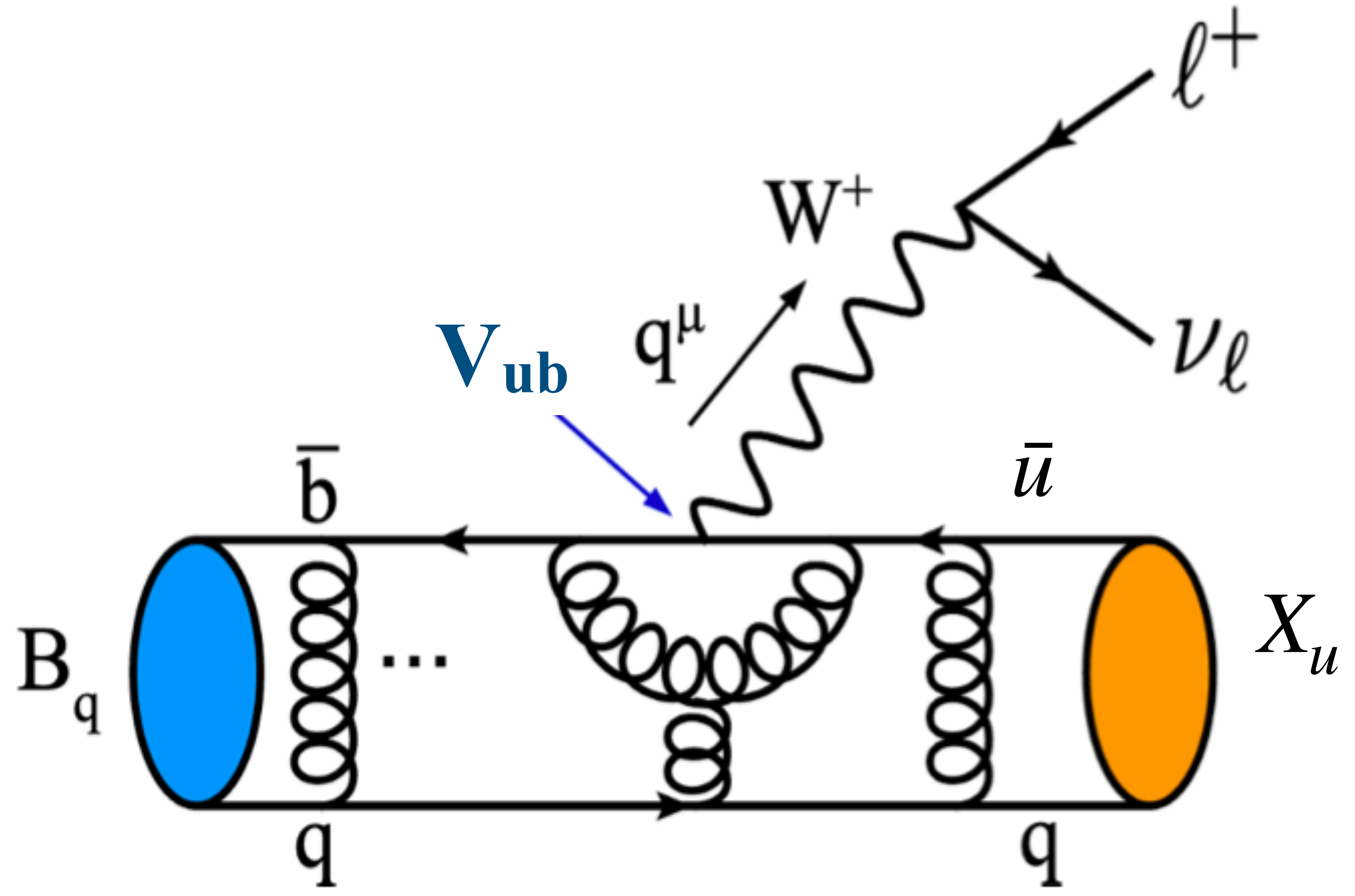
$$\mathcal{B}(B^0 \to \pi^- \ell^+ \nu_\ell) = (1.426 \pm 0.056(\mathrm{stat}) \pm 0.125(\mathrm{syst})) \times 10^{-4}$$

$$|V_{ub}|_{B^0 \to \pi^- \ell^+ \nu_\ell} = (3.55 \pm 0.12(\mathrm{stat}) \pm 0.13(\mathrm{syst}) \pm 0.17(\mathrm{theo})) \times 10^{-3}$$

Leading systematic uncertainty from the estimate of continuum background.

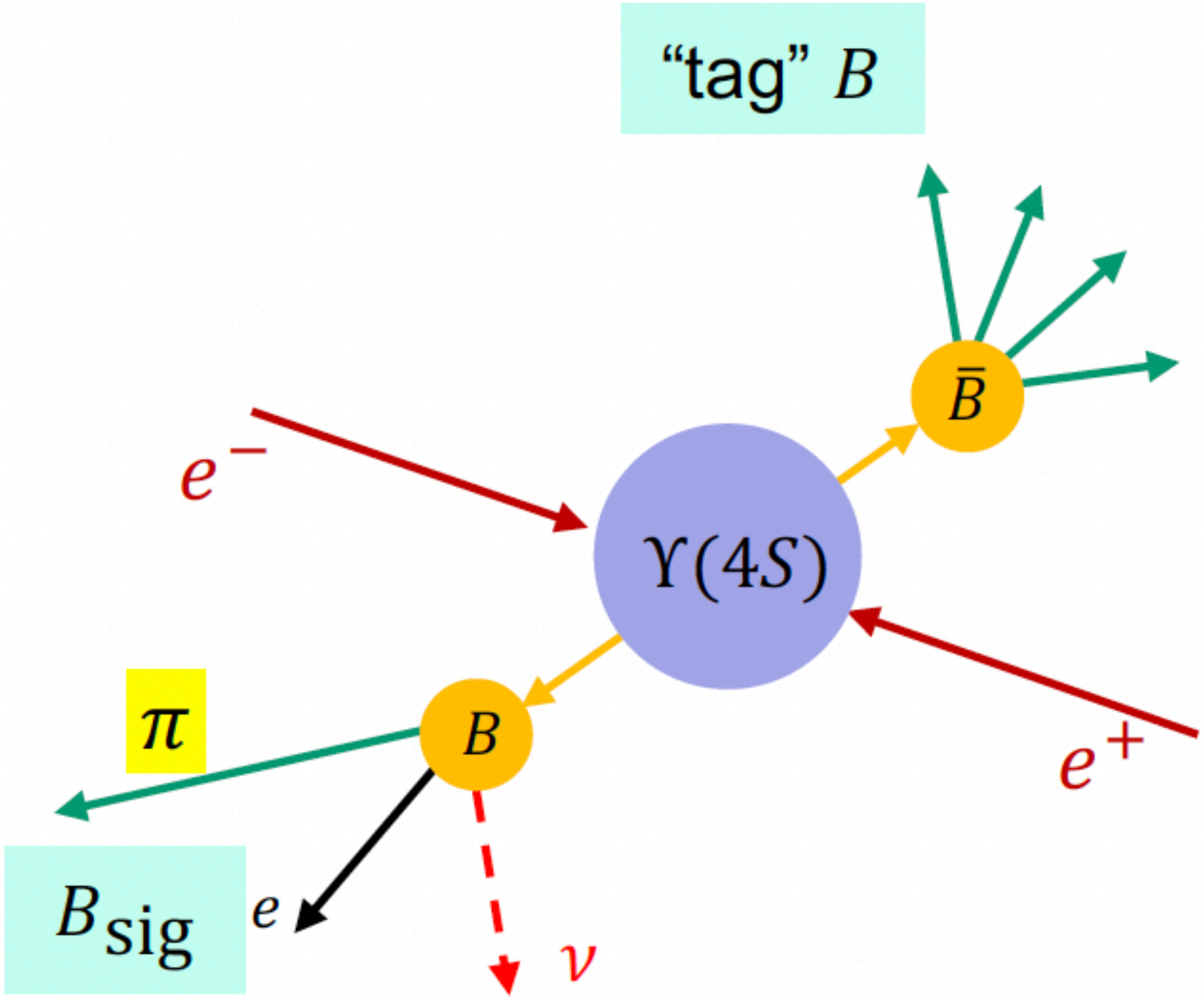
Consistent with world average.

Tagged Exclusive $|V_{ub}|$



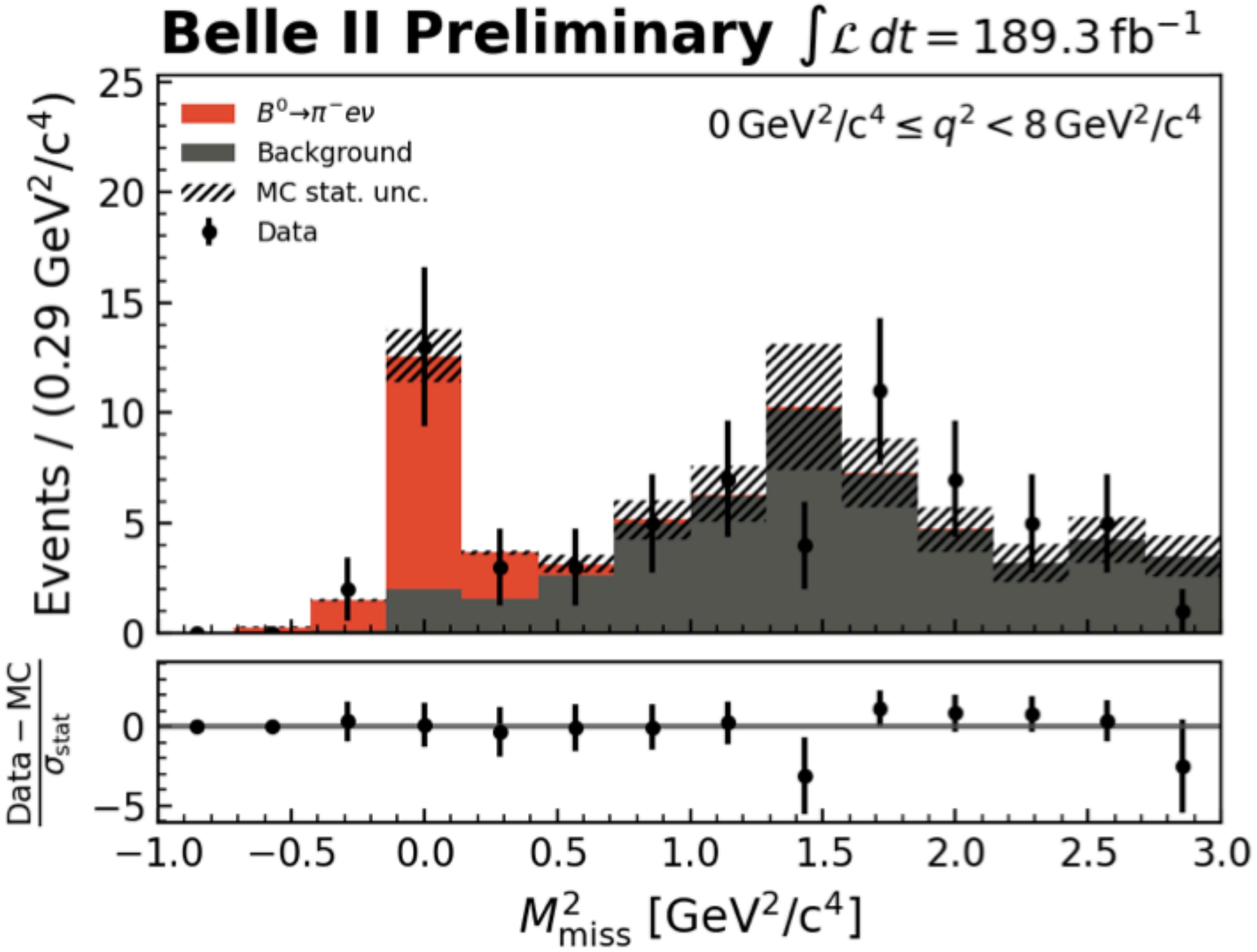
Tagged $B \to \pi e \nu_e$

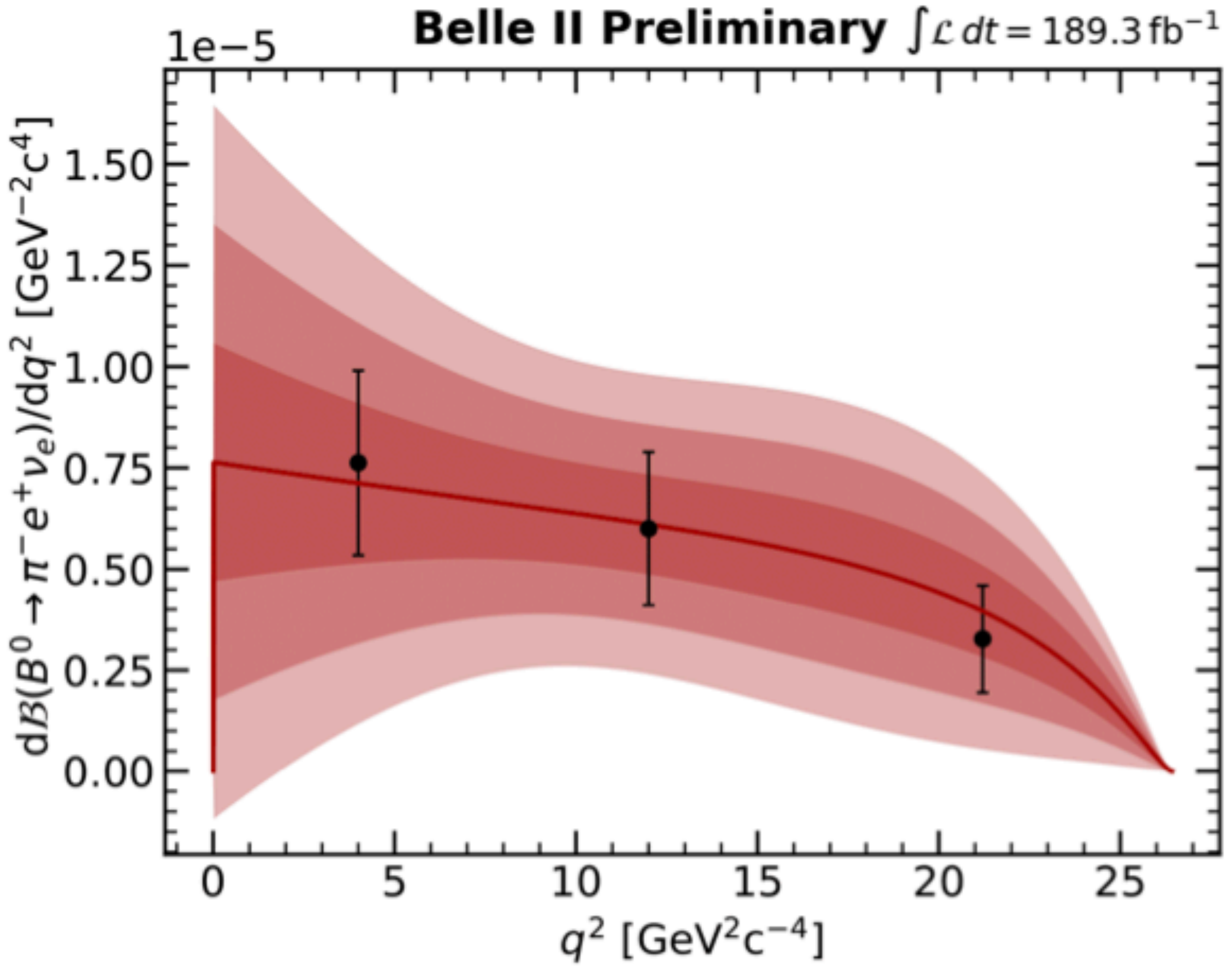
arXiv:2206.08102

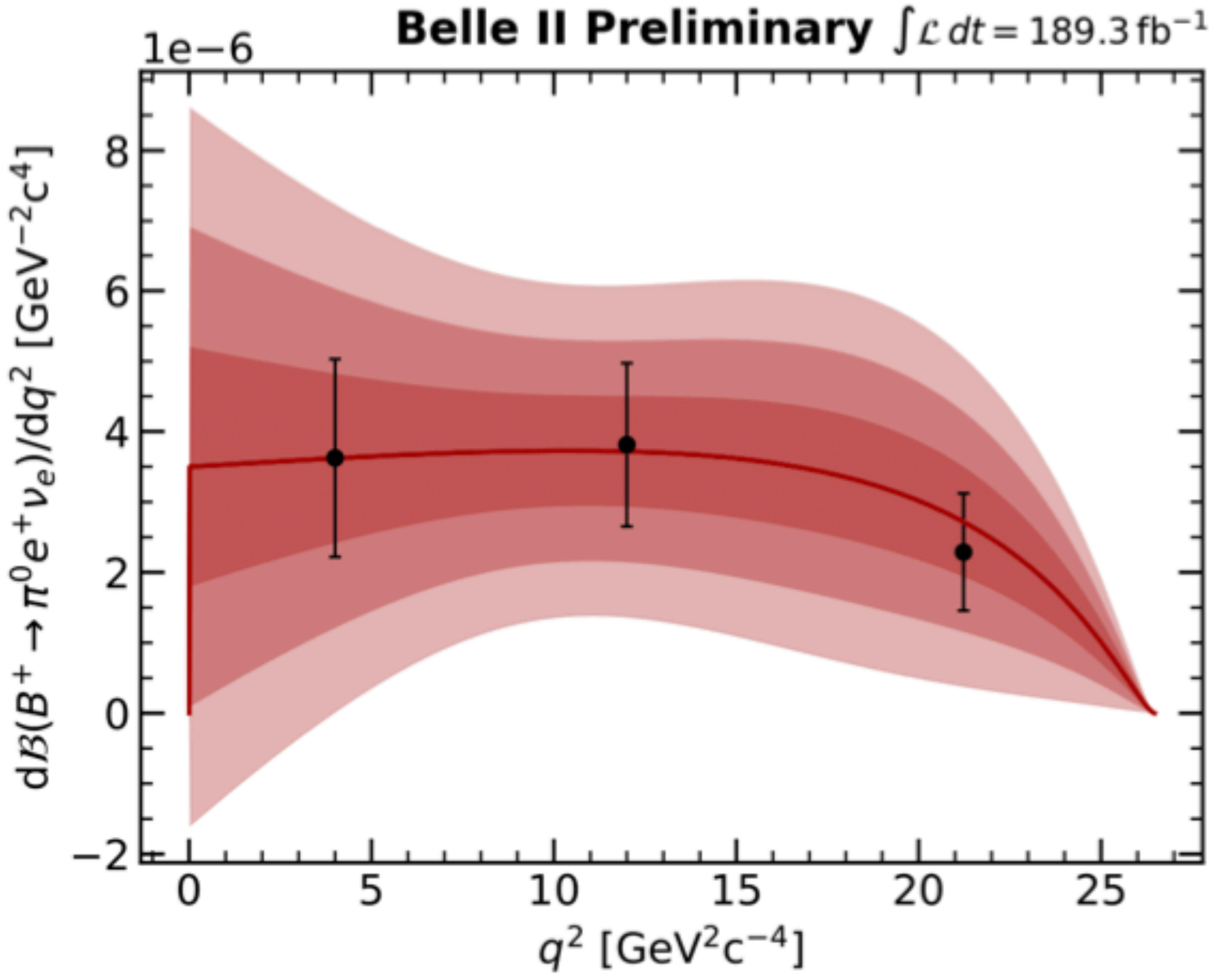


Tagged $B \to \pi e \nu_e$

- FEI hadronic tagging to measure $\mathscr{B}(B \to \pi e \nu)$ with
- Identify oppositely charged lepton and pion using PID algorithms.
- Fit M^2_{miss} in 3 bins of q^2.







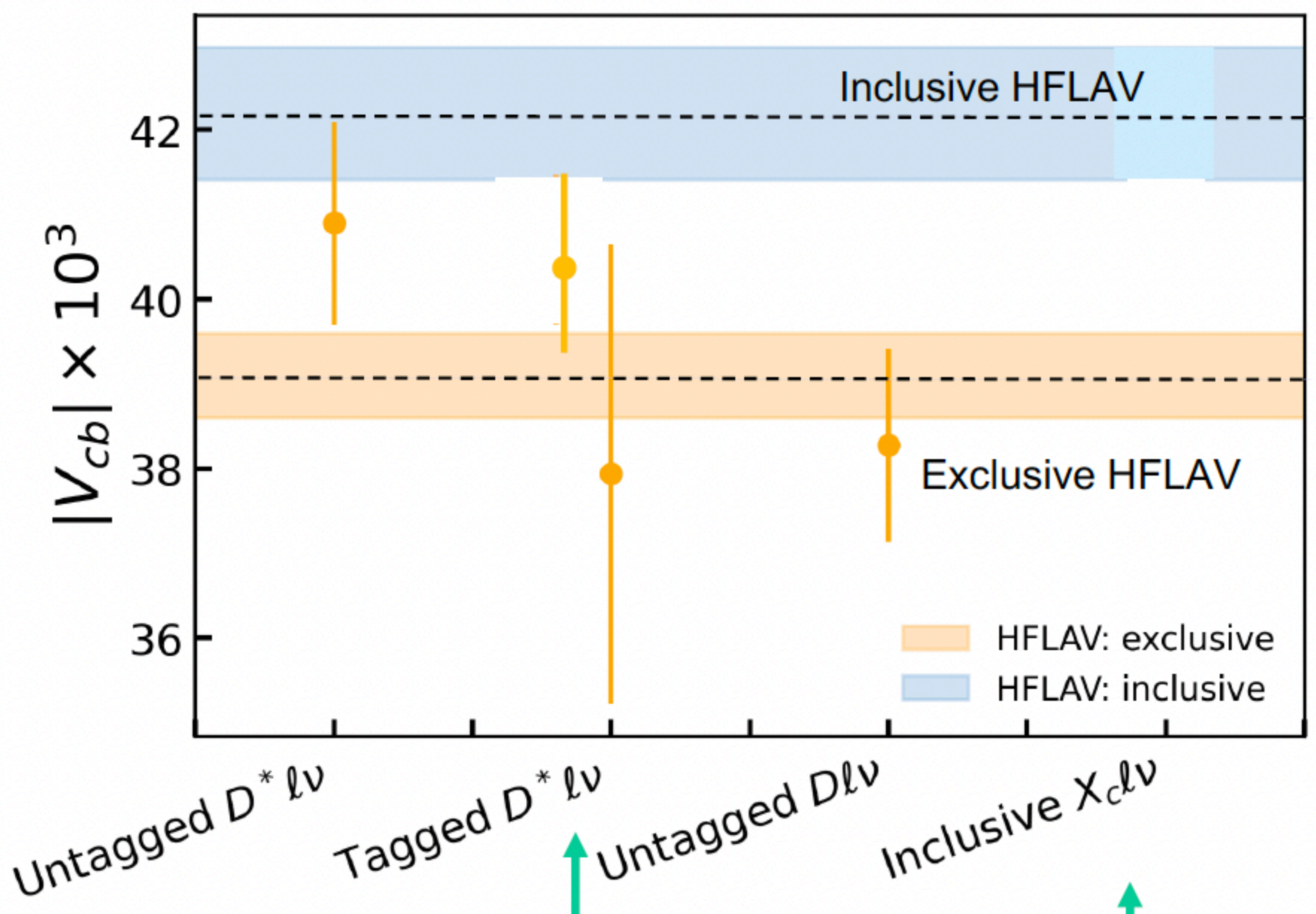
$$\mathcal{B}(B^0 \to \pi^- e^+ \nu_e) = (1.43 \pm 0.27(\mathrm{stat}) \pm 0.07(\mathrm{syst})) \times 10^{-4}$$

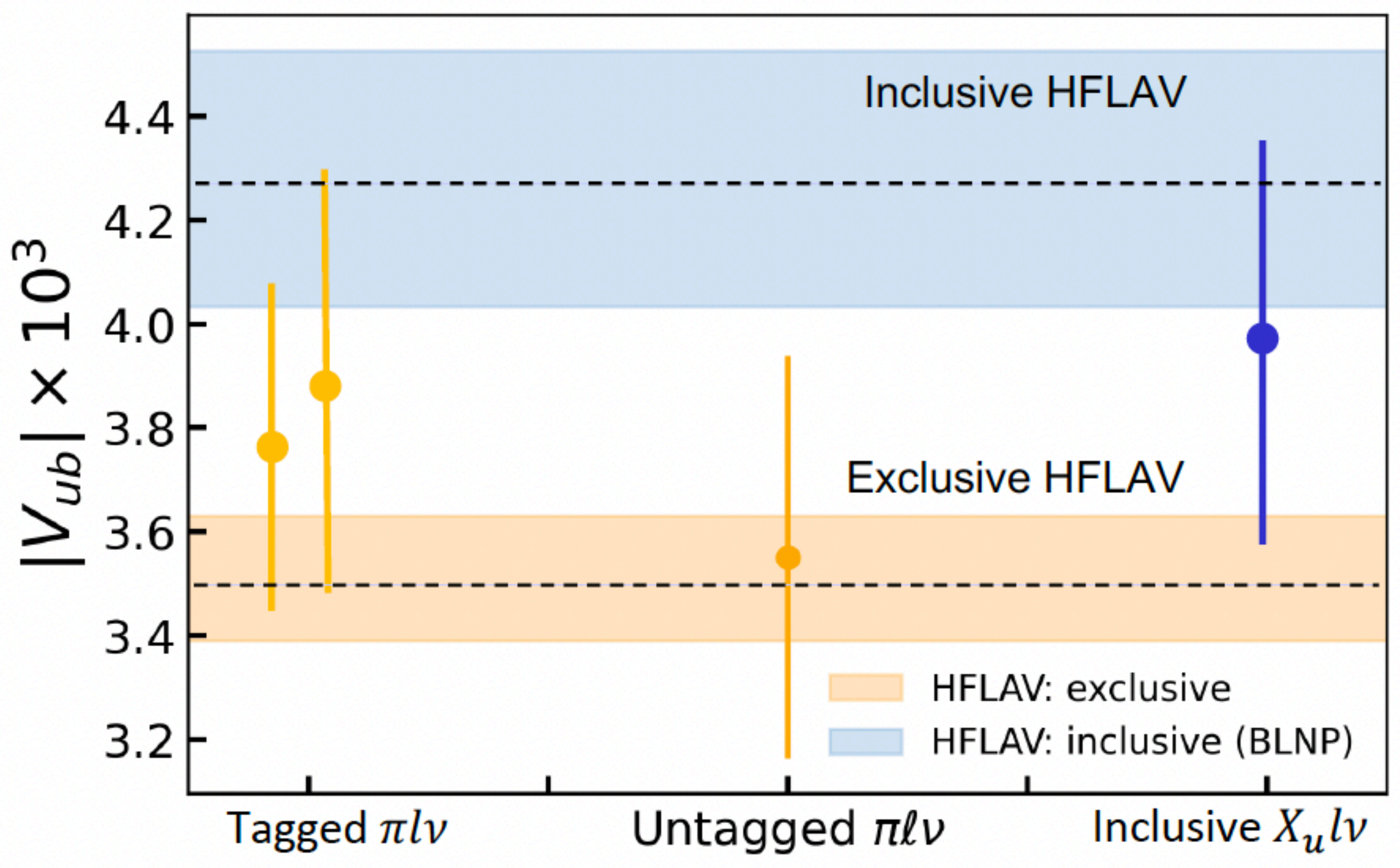
$$\mathcal{B}(B^+ \to \pi^0 e^+ \nu_e) = (8.33 \pm 1.67(\mathrm{stat}) \pm 0.55(\mathrm{syst})) \times 10^{-5}$$

$$|V_{\mathrm{ub}}| = (3.88 \pm 0.45) \times 10^{-3}$$

Results are statistically limited.
Consistent with world average

The developing picture





Belle, [arXiv: 2301.07529]

BelleII, [arXiv:2301.04716]

BelleII, [arXiv:2210.13143]

Belle II, PRD107,072002(2023) [arXiv: 2301.04716]
Moment measurement only, extraction of V_{cb} with Belle result
Belle PRD104,112011(2021) by F.Bernlocher, et.al, JHEP10(2022)068

BelleII, [arXiv:2206.08102]

Belle, [arXiv: 2303.17309]
Simultaneous determination of Exclusive and inclusive V_{ub}

BelleII, [arXiv:2210.04224]

Belle, [arXiv: 2303.17309]
Simultaneous determination of Exclusive and inclusive V_{ub}

Future Prospects at Belle II

- Reduce systematic uncertainties related to the reconstruction of the slow pion.
- Improve continuum modelling and related uncertainty with larger off-resonance sample.
- For tagged approach, also reduce systematic uncertainties related to tagging efficiency.
  - Clean up low purity hadronic B modes.
- Achieve higher accuracy with improved measurements for $N_{B\bar{B}}$ and f^{+0}
- Achieve higher precision in the measurements of the moments for inclusive $|V_{cb}|$ with larger dataset.
  - Valuable input for theory!
- Improved measurements of $B \to D^{**}\ell\nu$.

Belle II will restart data-taking in Fall 2023.

Conclusion

- Measurements in progress to resolve $|V_{ub}|$ and $|V_{cb}|$ puzzle.

- Exclusive $|V_{cb}|$:
  - Recent results at Belle II with untagged $B \to D^* \ell \nu$ and the form factors in the BGL and CLN parameterization.
    - Consistent with both inclusive and exclusive determinations.
    - Examine beyond zero recoil constraints from Fermilab/MILC.
  - Recent results with tagged $B \to D^* \ell \nu$ approach and CLN parametrisation.
    - Consistent with world average of exclusive determinations.
    - Work in progress for a full extraction of the form factors as in the untagged analysis.

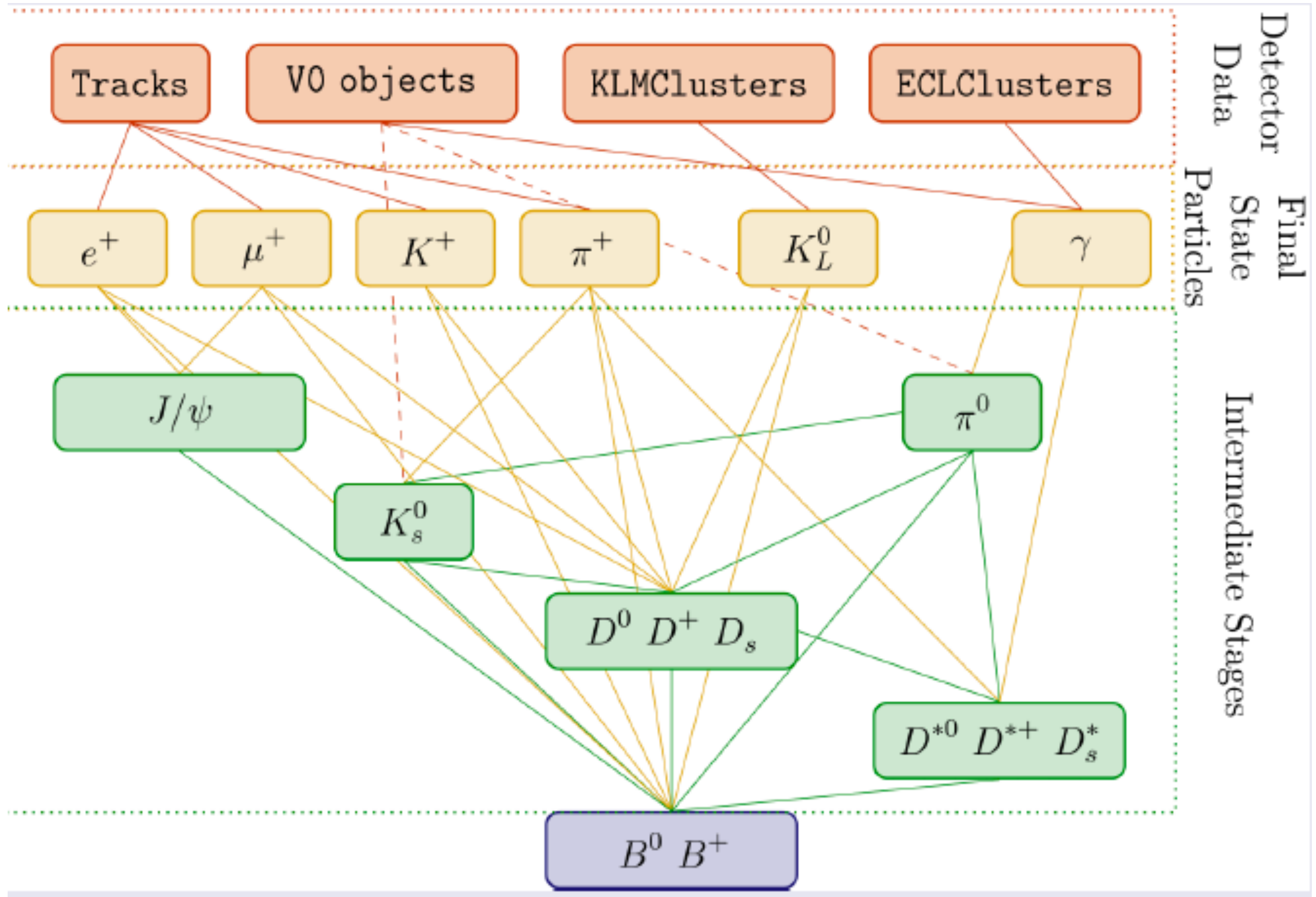
- Inclusive $|V_{cb}|$:
  - Novel $< q^{2n} >$ moments measurements at Belle II using tagged approach.
    - First determination of inclusive $|V_{cb}|$ using this information.

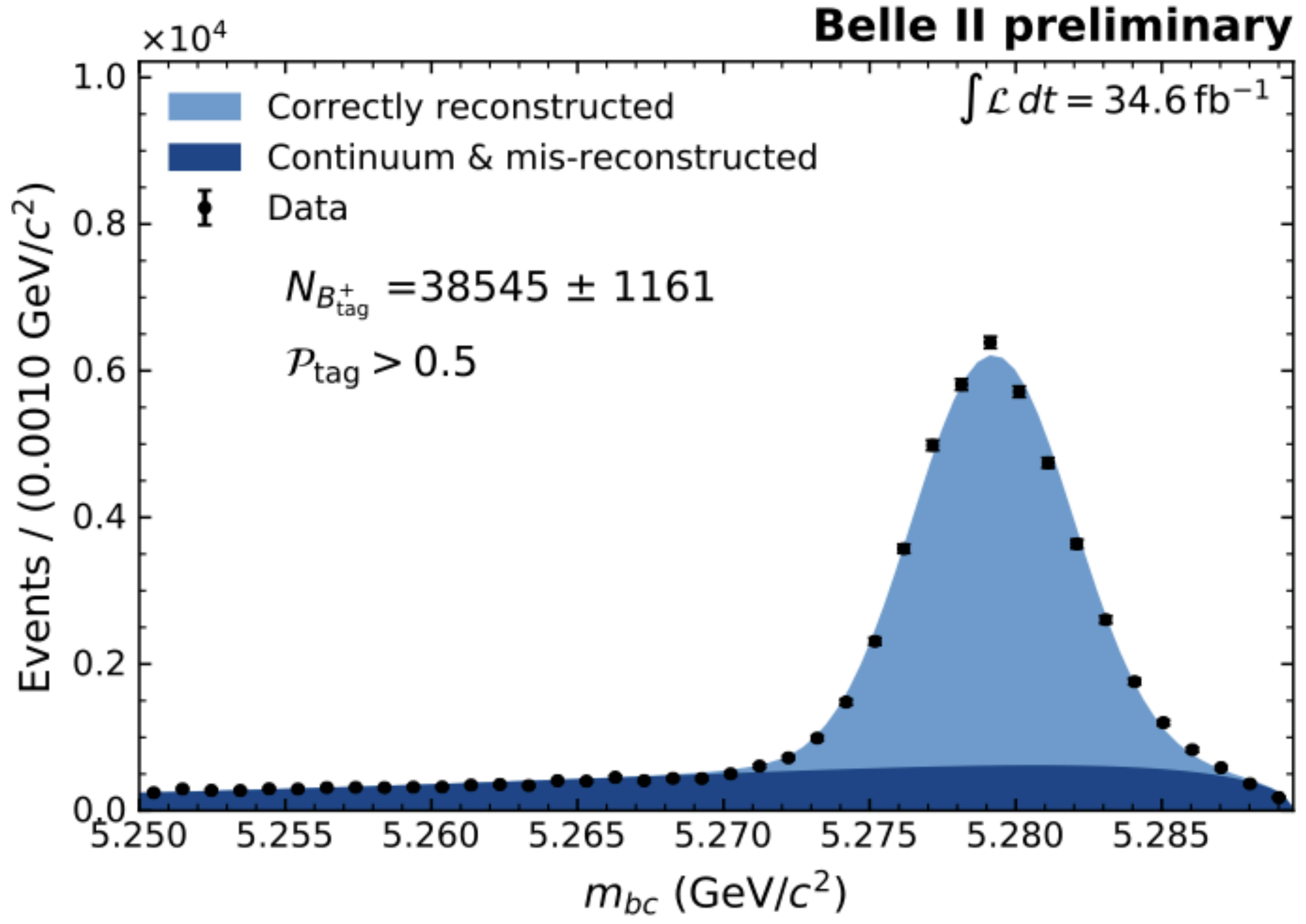
- Exclusive $|V_{ub}|$:
  - First results at Belle II with tagged and untagged $B \to \pi \ell \nu$.
    - Both results are consistent with the world average.
  - Work in progress for a precision measurement with current Belle II dataset (362 fb^{-1}) and the inclusion of $B \to \rho \ell \nu$.

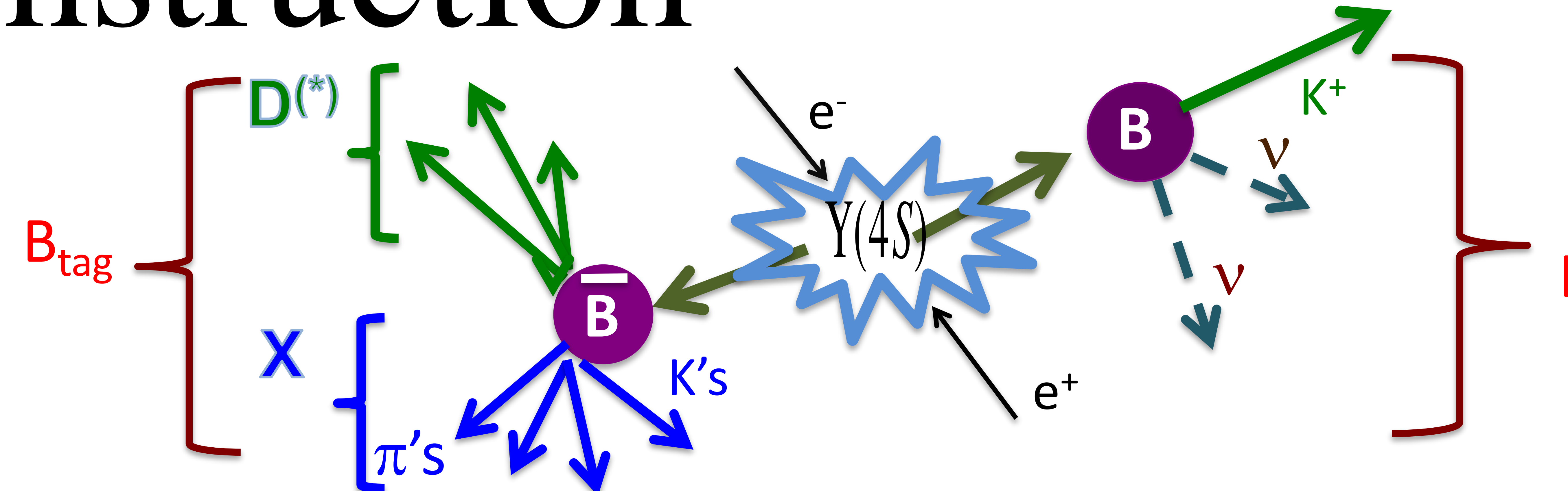
Back up

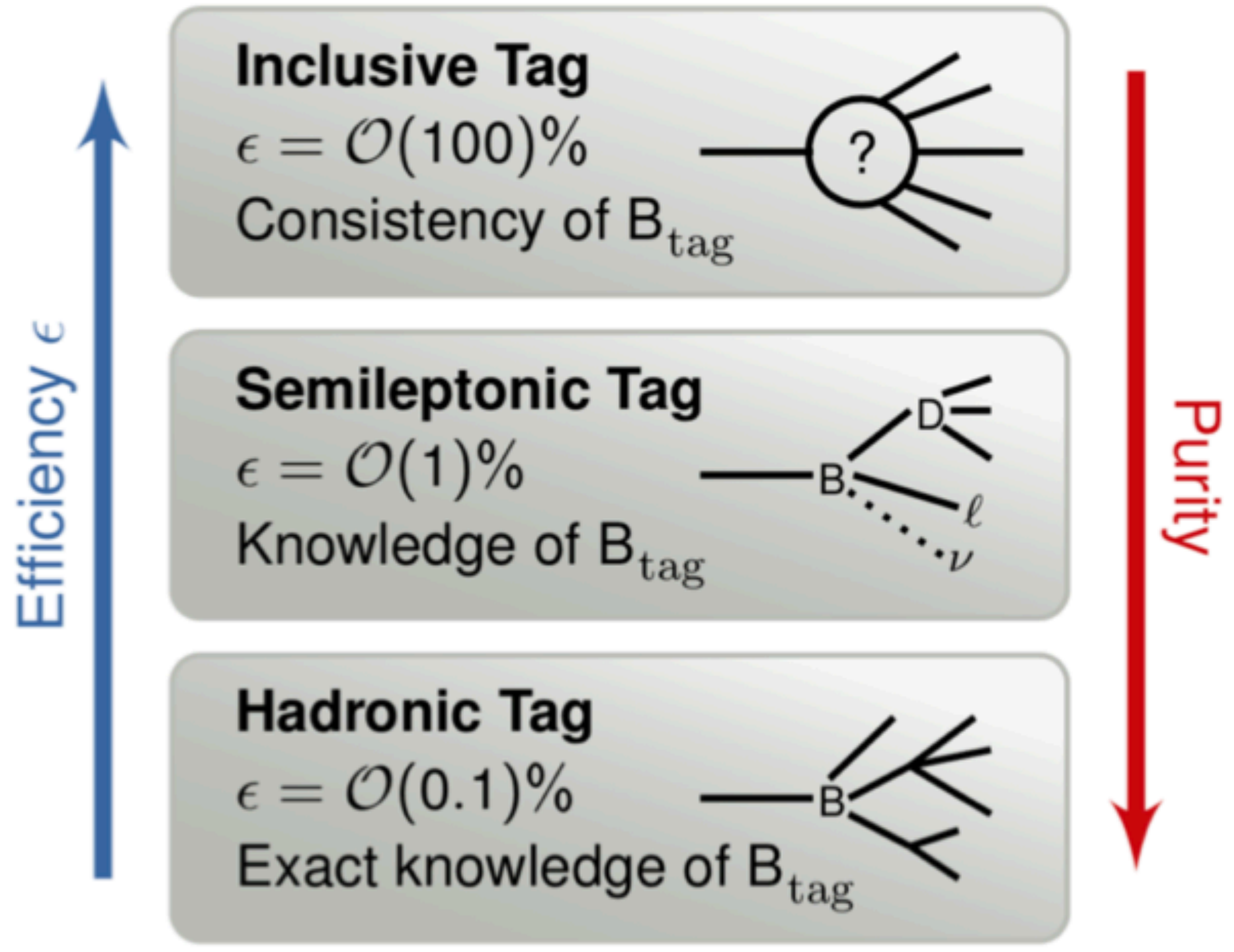
FEI reconstruction

- Exclusive reconstruction of hadronic B modes.
- Multivariate algorithm with hierarchal approach









Infer momentum and direction of signal B candidate:

$$p_{Bsig} \equiv (E_{Bsig}, \vec{p}_{Bsig}) = \left(\frac{m_{\Upsilon(4S)}}{2}, -\vec{p}_{B\mathrm{tag}}\right)$$

Ideal for decays with neutrinos, missing energy signatures!

Tagging Algorithm	Had B^+/B^0	SL B^+/B^0
Full Reconstruction Belle	0.28/0.18	0.67/0.63
FEI Belle	0.78/0.46	1.80/2.04

MC Tagging efficiency at 10% purity!

Inclusive $|V_{cb}|$

- HQE in powers of $1/m_b$
- Determine parameters of HQE using moments of the differential rate.

$$\langle E^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, E_\ell^n \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\rm cut}} dE_\ell \frac{d\Gamma}{dE_\ell}} \qquad \langle (M_X^2)^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, (M_X^2)^n \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\rm cut}} dM_X^2 \frac{d\Gamma}{dM_X^2}} \qquad R^*(E_{\rm cut}) = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \frac{d\Gamma}{dE_\ell}}{\int_0 dE_\ell \frac{d\Gamma}{dE_\ell}}$$

$$\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, m_b, (m_c)$$

- Using the branching fraction, determine $|V_{cb}|$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_0 + \Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} \right]$$

	Kinetic scheme	1S scheme
$O(1)$	m_b, m_c	m_b
$O(1/m_b^2)$	μ_π^2, μ_G^2	λ_1, λ_2
$O(1/m_b^3)$	ρ_D^3, ρ_{LS}^3	ρ_1, τ_{1-3}

JHEP 1109 055 (2011)
Phys Rev D 70, 094017 (2004)

Alternative Inclusive $|V_{cb}|$

NOVEL APPROACH!!!

- Achieve more precision by including higher order:

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\mu_\pi^2}{m_b^2}\left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right) + \frac{\mu_G^2}{m_b^2}\left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}\Gamma^{(D,0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right)\cdots\right)$$

- Number of parameters: 4 up to $1/m_b^3$, 13 up to $1/m_b^4$ and 31 up to $1/m_b^5$
- Use reparametrization invariance to link different orders of 1/mb and reduce the number of total parameters
- Requires RPI observables such as q^2

$$\langle (q^2)^n \rangle_{\rm cut} = \int_{q^2 > q^2_{\rm cut}} dq^2\, (q^2)^n \frac{d\Gamma}{dq^2} \Big/ \int_{q^2 > q^2_{\rm cut}} dq^2 \frac{d\Gamma}{dq^2}$$

$$R^*(q^2_{\rm cut}) = \int_{q^2 > q^2_{\rm cut}} dq^2 \frac{d\Gamma}{dq^2} \Big/ \int_0 dq^2 \frac{d\Gamma}{dq^2}$$

Fael, Mannel, Vos , JHEP 02 (2019) 177

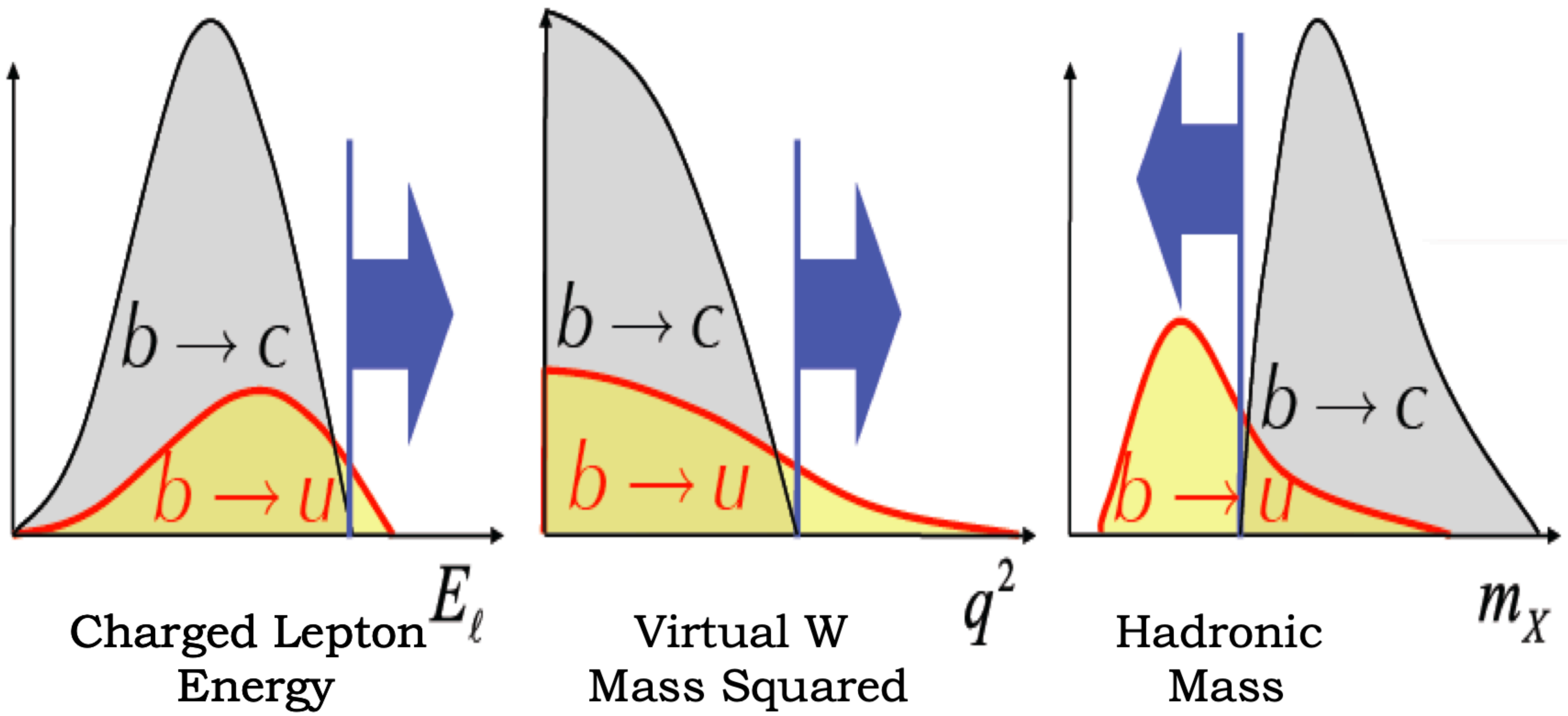
- $2M_B r_G^4 \equiv \frac{1}{2}\langle B|\bar{b}_v\,[iD_\mu, iD_\nu]\,[iD^\mu, iD^\nu]\, b_v|B\rangle \propto \langle \vec{E}^2 - \vec{B}^2\rangle$
- $2M_B r_E^4 \equiv \frac{1}{2}\langle B|\bar{b}_v\,[ivD, iD_\mu]\,[ivD, iD^\mu]\, b_v|B\rangle \propto \langle \vec{E}^2\rangle$
- $2M_B s_B^4 \equiv \frac{1}{2}\langle B|\bar{b}_v\,[iD_\mu, iD_\alpha]\,[iD^\mu, iD_\beta]\,(-i\sigma^{\alpha\beta}) b_v|B\rangle \propto \langle \vec{\sigma}\cdot\vec{B}\times\vec{B}\rangle$
- $2M_B s_E^4 \equiv \frac{1}{2}\langle B|\bar{b}_v\,[ivD, iD_\alpha]\,[ivD, iD_\beta]\,(-i\sigma^{\alpha\beta}) b_v|B\rangle \propto \langle \vec{\sigma}\cdot\vec{E}\times\vec{E}\rangle$
- $2M_B s_{qB}^4 \equiv \frac{1}{2}\langle B|\bar{b}_v\,[iD_\mu, [iD^\mu, [iD_\alpha, iD_\beta]]]\,(-i\sigma^{\alpha\beta}) b_v|B\rangle \propto \langle \Box\, \vec{\sigma}\cdot\vec{B}\rangle$.

8 parameters instead of 13 !

$|V_{ub}|$

- Experimentally challenging due to dominant $B \to X_c \ell \nu$ background.
- Only certain kinematic regions allow for clean separation: lepton momentum endpoint spectrum or low m_X.
- Inclusive via $B \to X_u \ell \nu$:
  - Precision of (~7%)
  - Operator Product Expansion (OPE) = Heavy Quark Expansion.
  - HQE breaks down and a non-perturbative shape function is required.

$$d\Gamma = d\Gamma_0 + d\Gamma_2 \left(\frac{\Lambda_{QCD}}{m_b}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{QCD}}{m_b}\right)^3 + d\Gamma_4 \left(\frac{\Lambda_{QCD}}{m_b}\right)^4$$



- Exclusive via $B \to \pi \ell \nu$
  - Most precise determination of $|V_{ub}|$ (~4%)
  - Form factor determined non-perturbative from lattice QCD (high q^2) or LCSR (q^2~0) .

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f_+(q^2)|^2$$

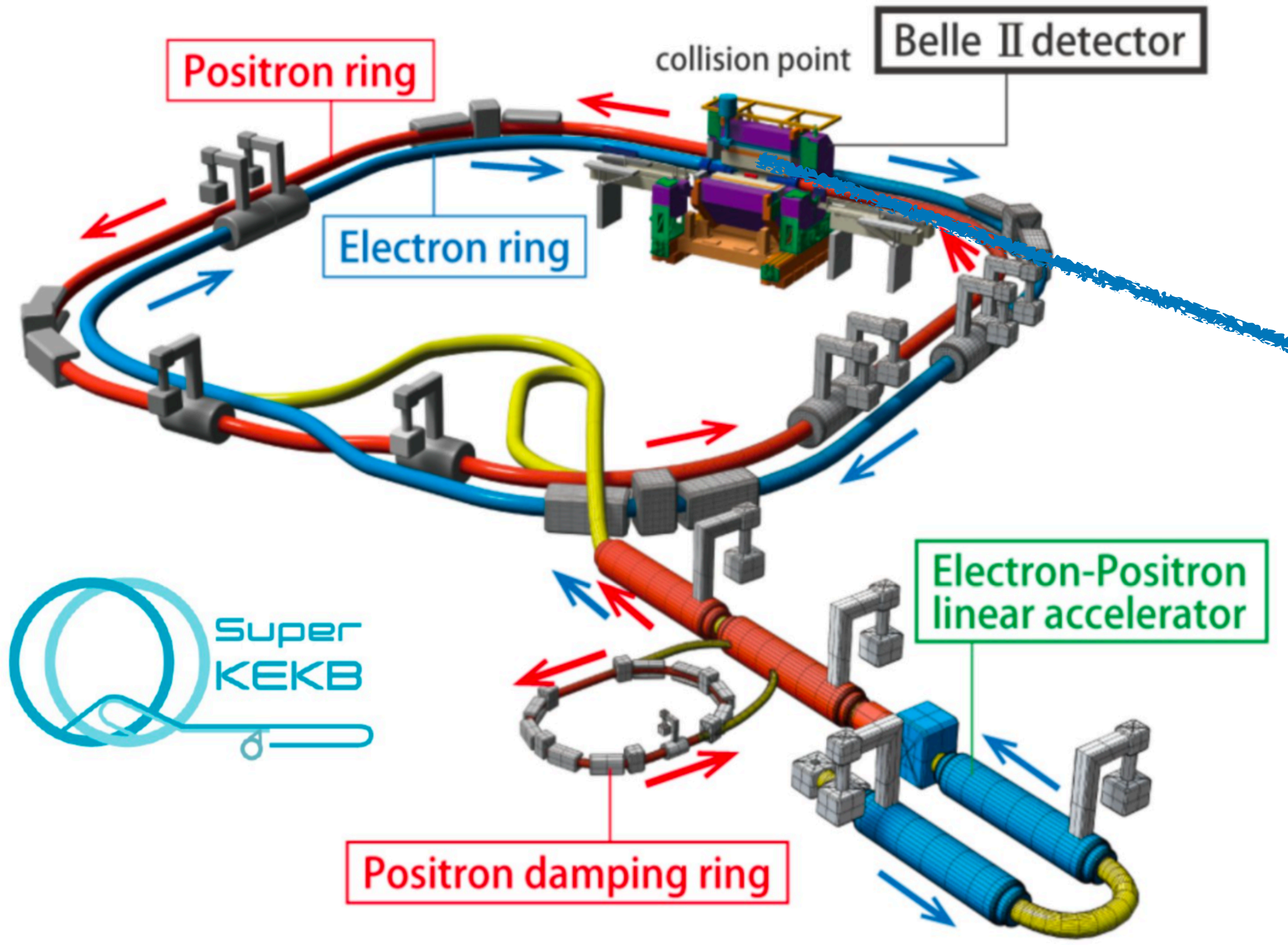
$|V_{ub}| = (4.25 \pm 0.12^{+0.15}_{-0.14} \pm 0.23) \times 10^{-3}$ **PDG inclusive**

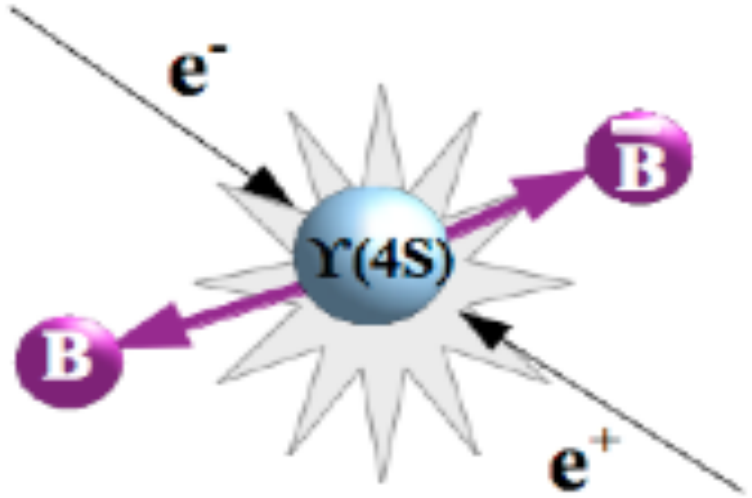
$|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$ **PDG exclusive**

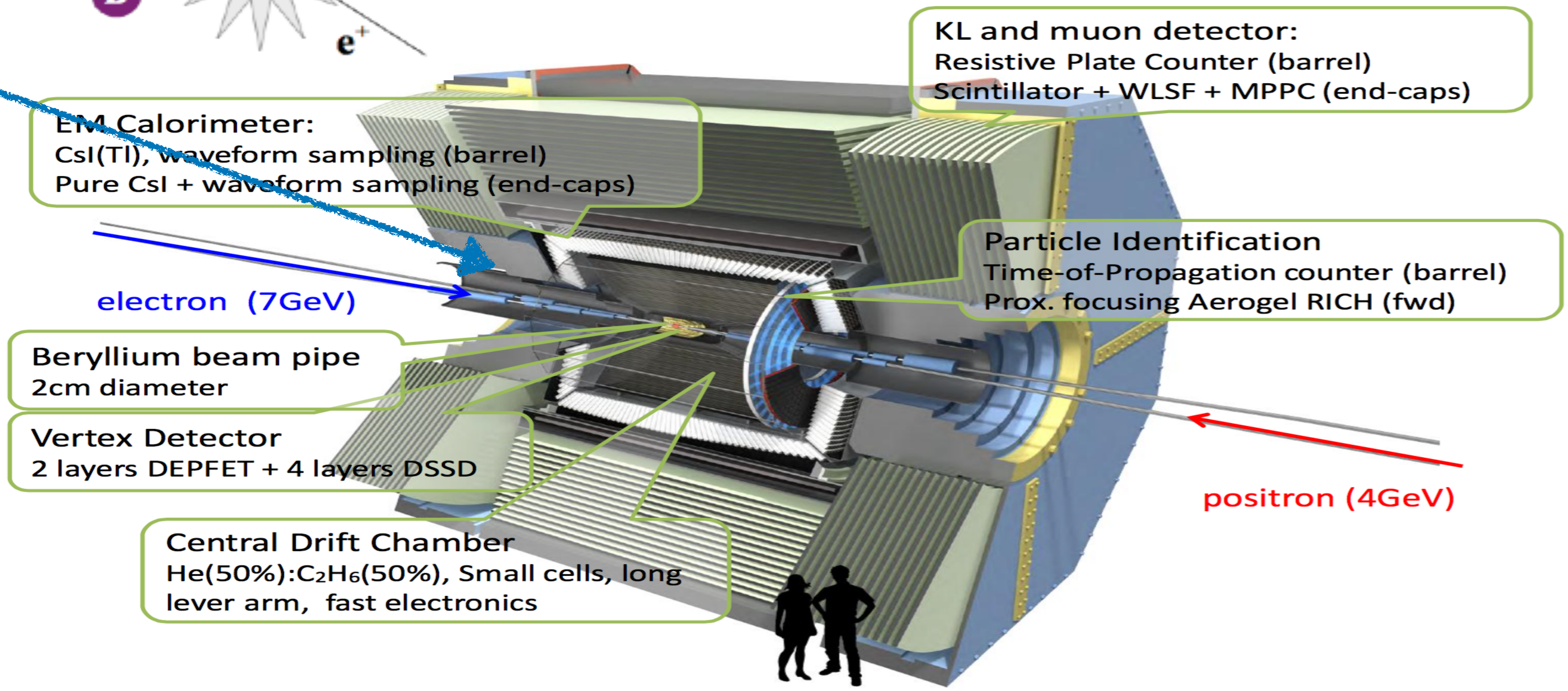
Current ~3σ tension between inclusive and exclusive determinations

Belle II experiment

- A *B* meson factory in Tsukuba, Japan based on the SuperKEKB accelerator complex.
- Upgrade of its predecessor Belle at KEKB.

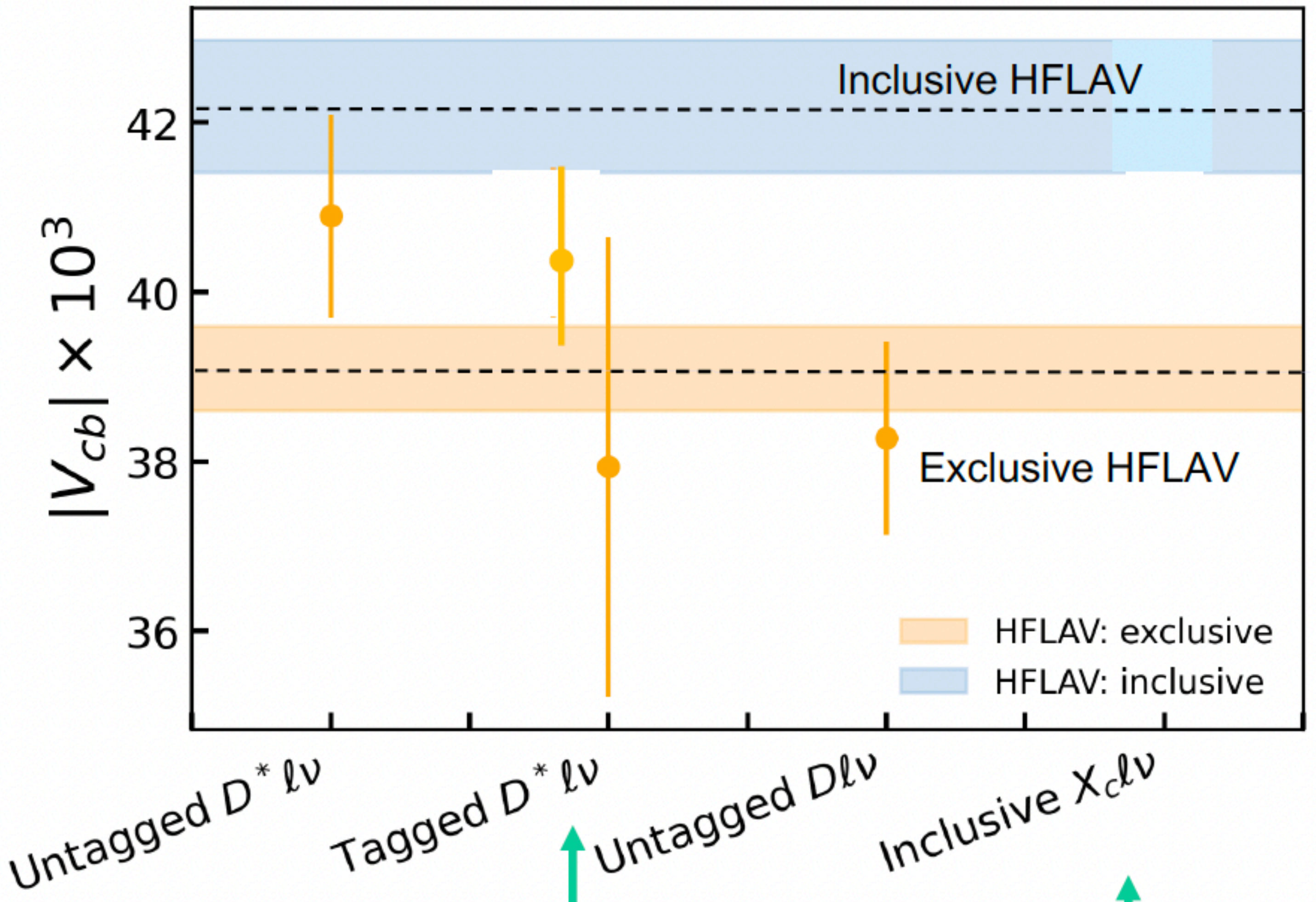


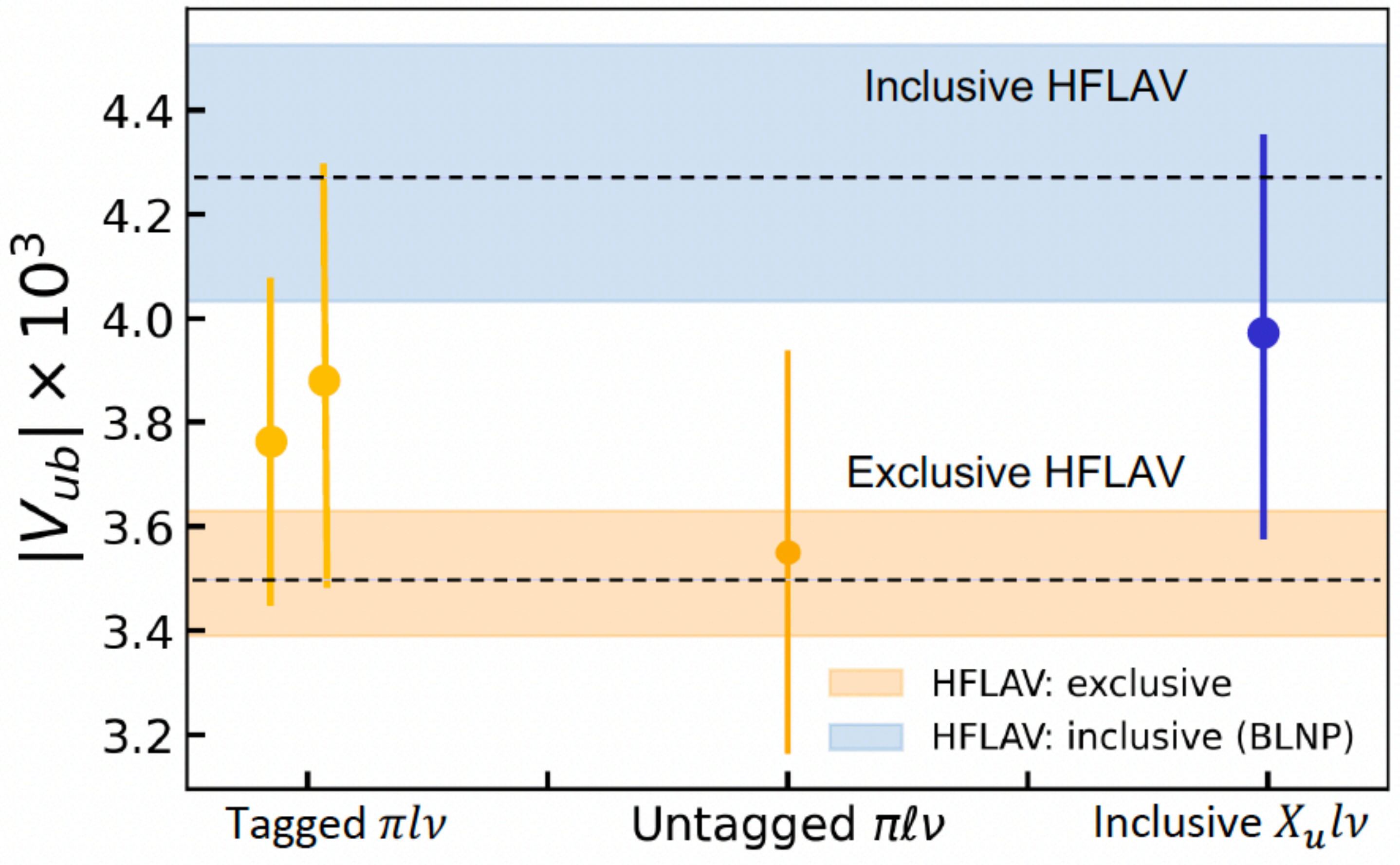




- Luminosity projected to be 30 x larger than that of Belle.

The big picture





Belle, [arXiv: 2301.07529]

BelleII, [arXiv:2301.04716]

BelleII, [arXiv:2210.13143]

Belle II, PRD107,072002(2023) [arXiv: 2301.04716]
Moment measurement only, extraction of V_{cb} with Belle result
Belle PRD104,112011(2021) by F.Bernlocher, et.al,
JHEP10(2022)068

BelleII, [arXiv:2206.08102]

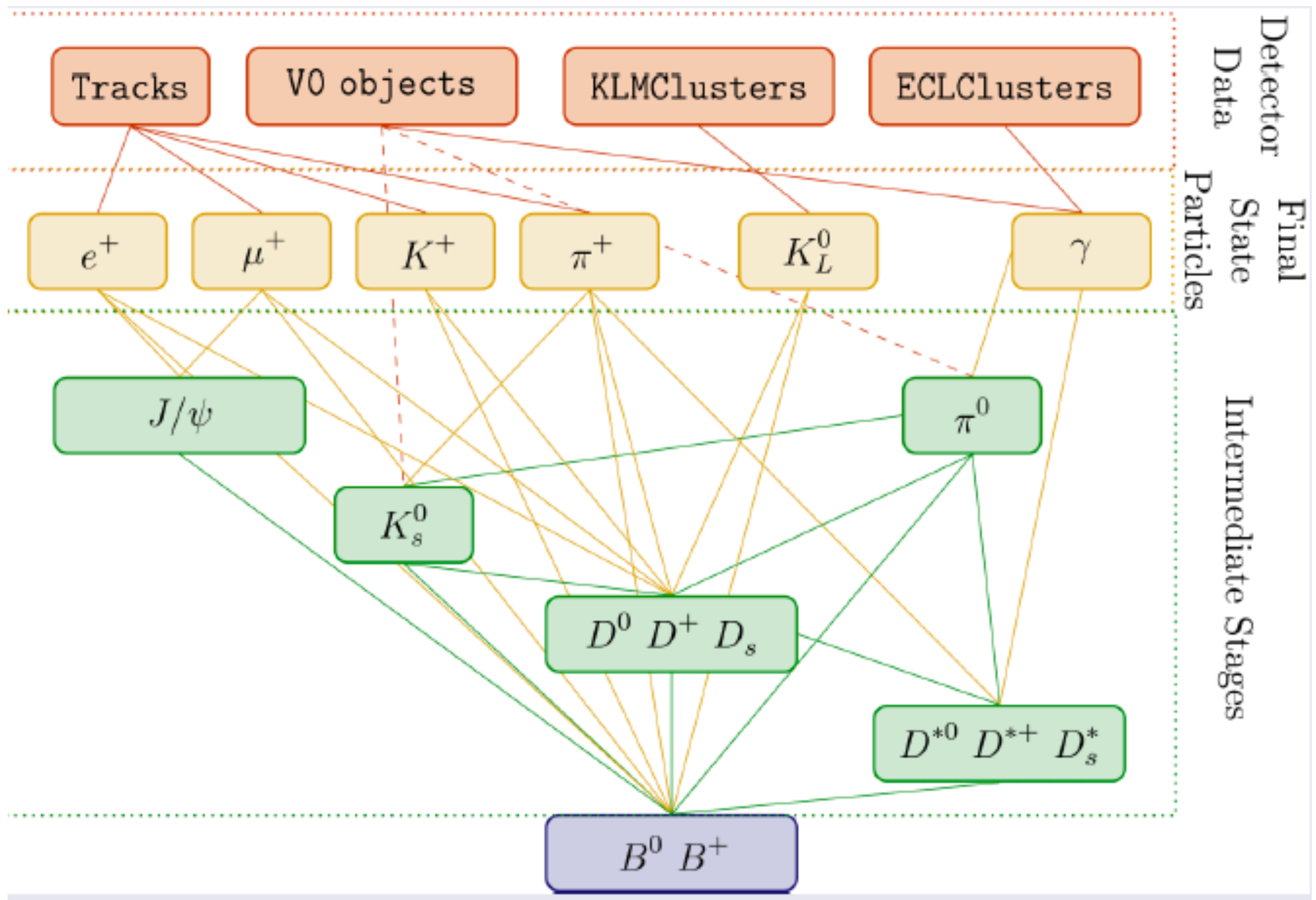
Belle, [arXiv: 2303.17309]
Simultaneous determination of
Exclusive and inclusive V_{ub}

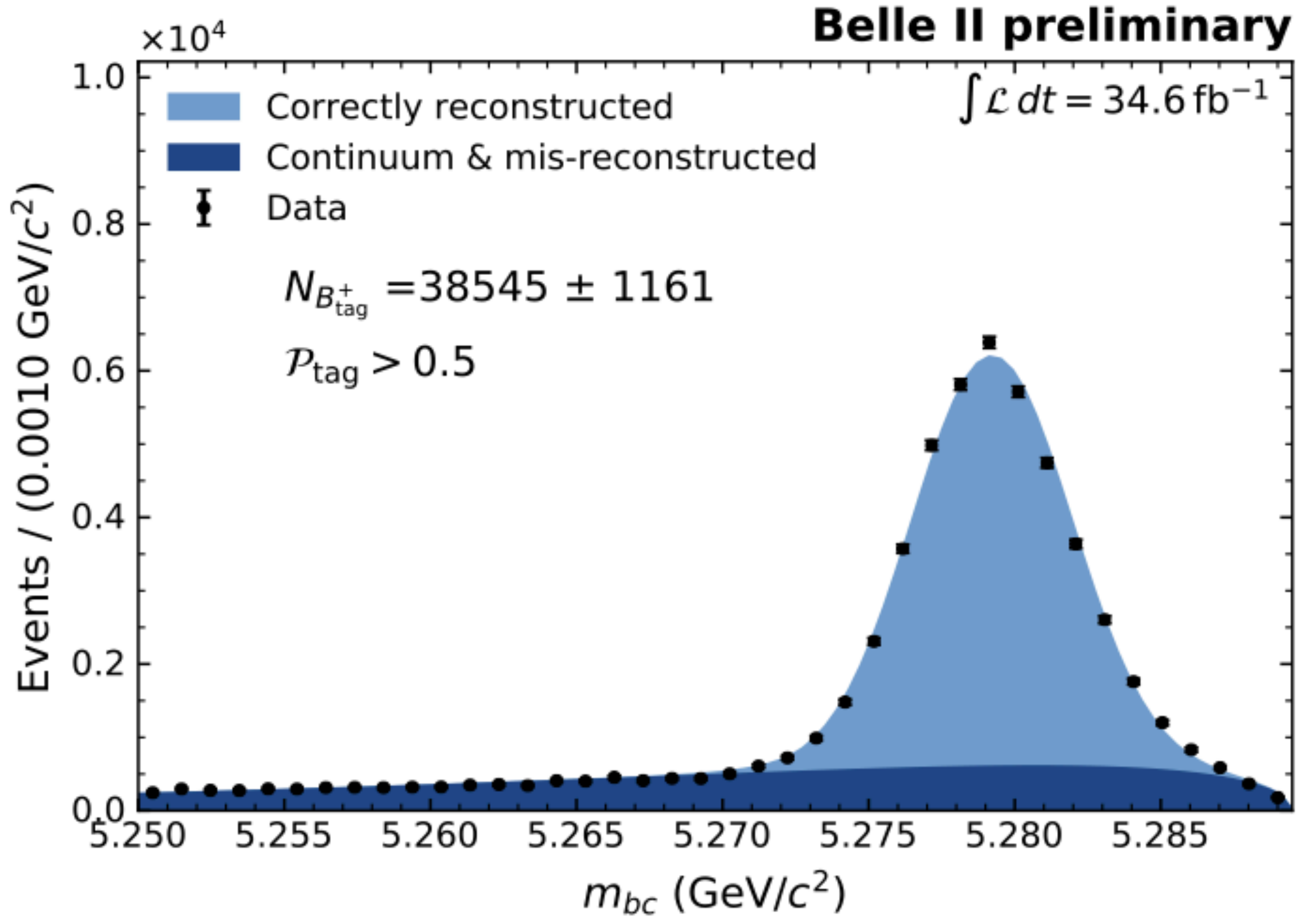
BelleII, [arXiv:2210.04224]

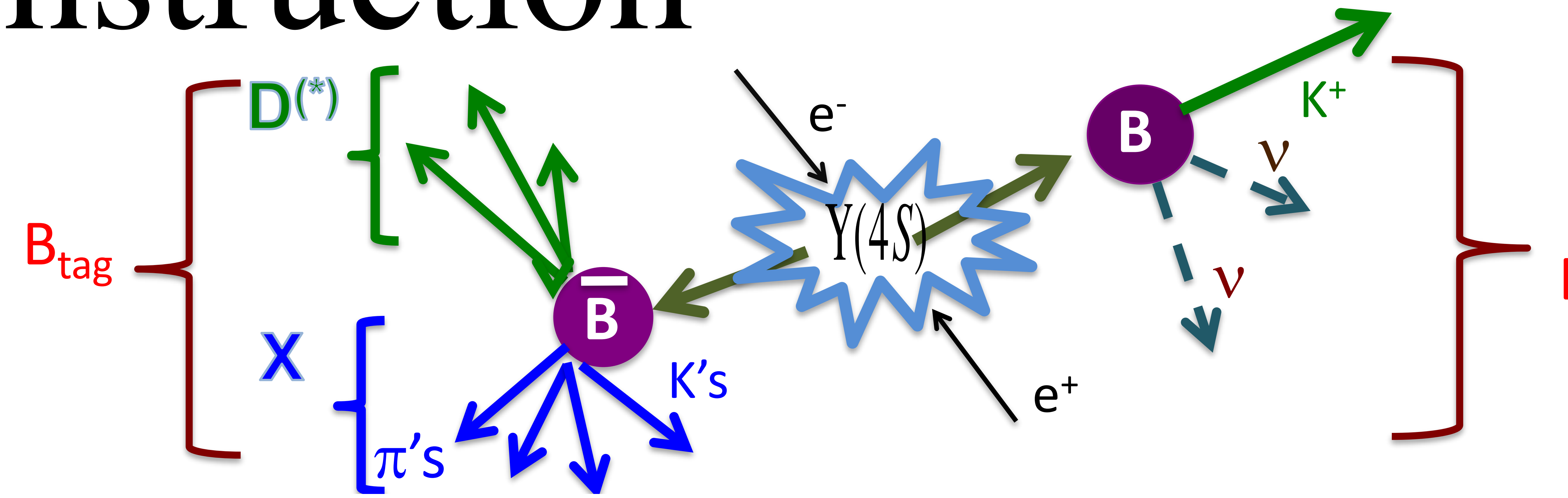
Belle, [arXiv: 2303.17309]
Simultaneous determination of
Exclusive and inclusive V_{ub}

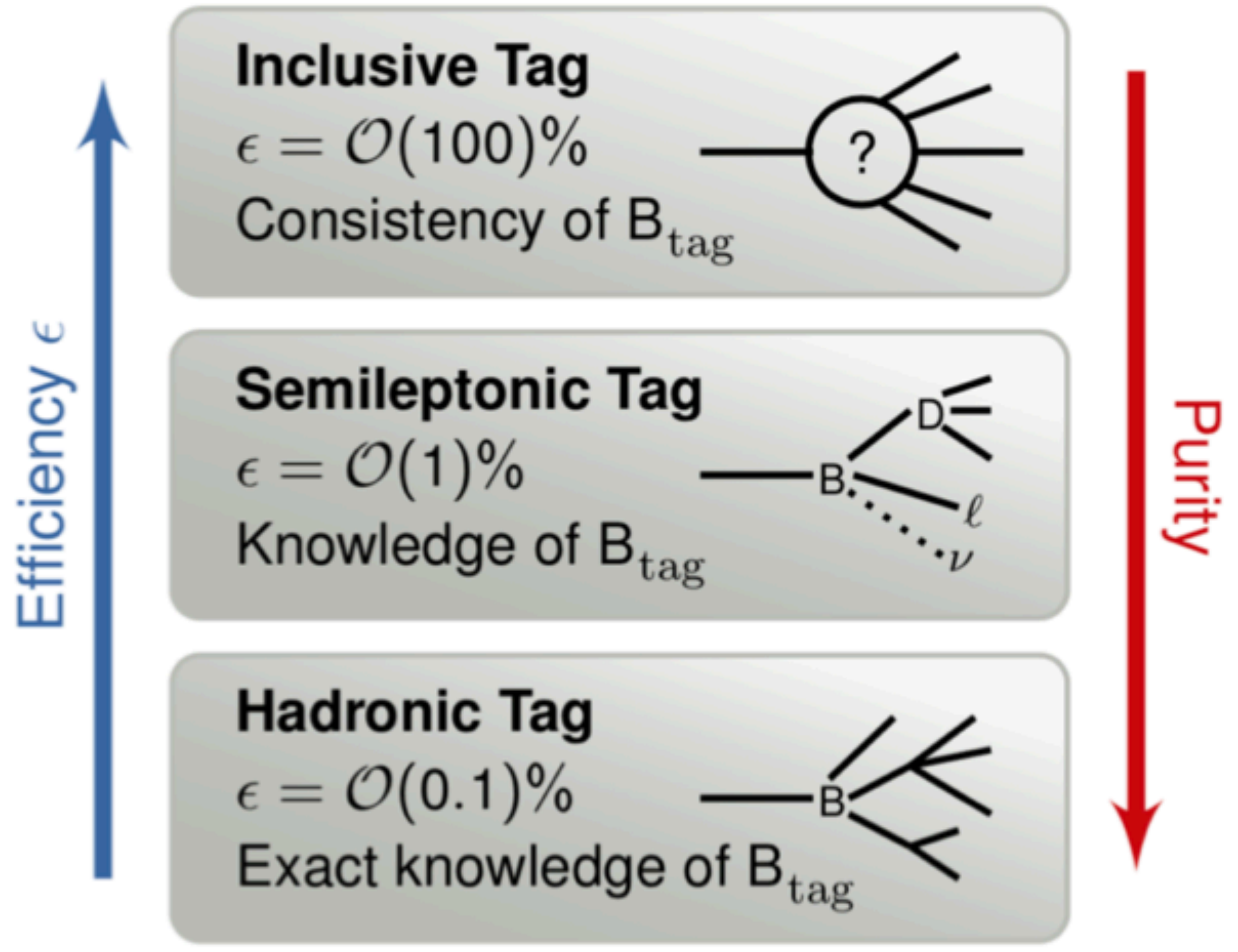
FEI reconstruction

- Exclusive reconstruction of hadronic B modes.
- Multivariate algorithm with hierarchal approach









Infer momentum and direction of signal B candidate:

$$p_{Bsig} \equiv (E_{Bsig}, \vec{p}_{Bsig}) = \left(\frac{m_{\Upsilon(4S)}}{2}, -\vec{p}_{B\mathrm{tag}}\right)$$

Ideal for decays with neutrinos, missing energy signatures!

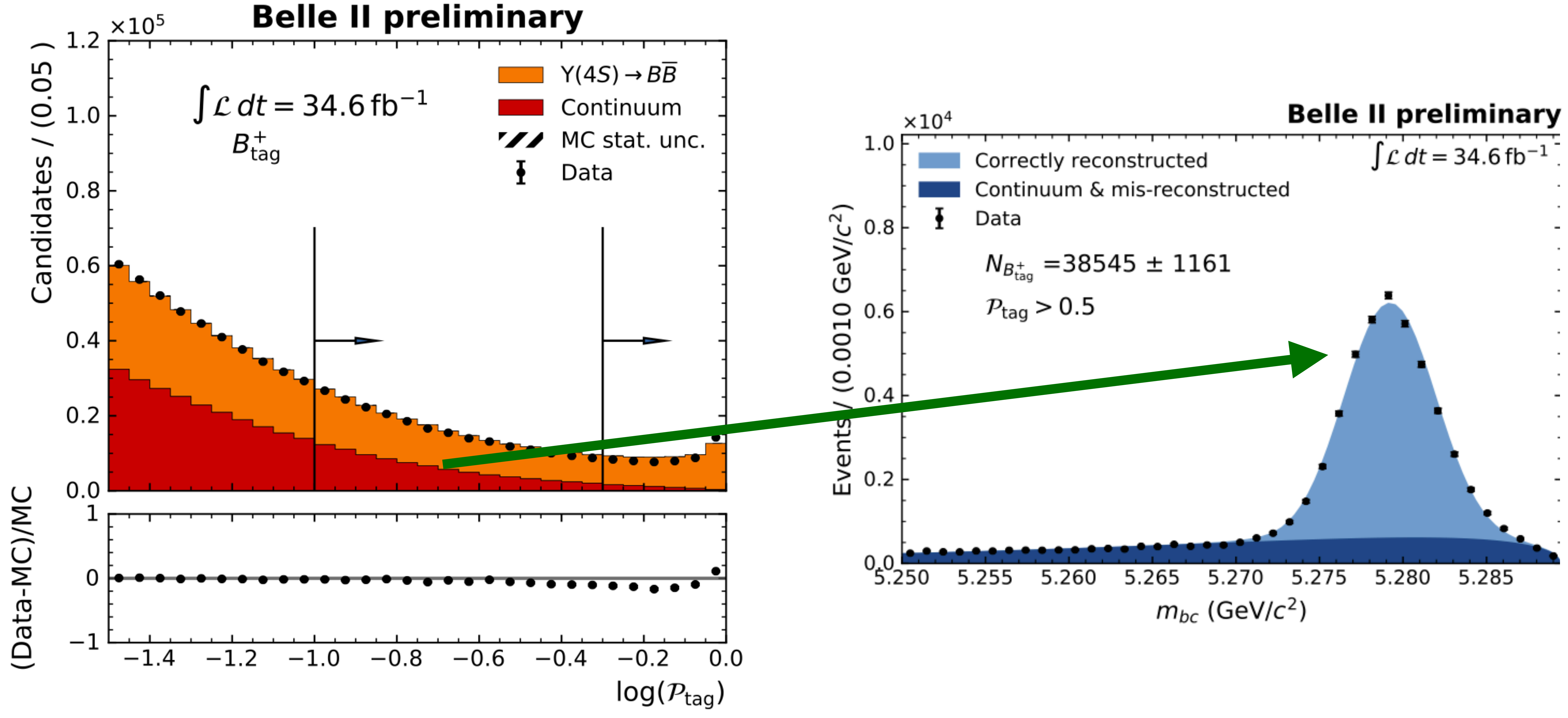
Tagging Algorithm	Had B+/B0	SL B+/B0
Full Reconstruction Belle	0.28/0.18	0.67/0.63
FEI Belle	0.78/0.46	1.80/2.04

MC Tagging efficiency at 10% purity!

arXiv:2008.06096

B-tagging at Belle II

- Outputs a signal probability which separates correctly reconstructed B mesons.



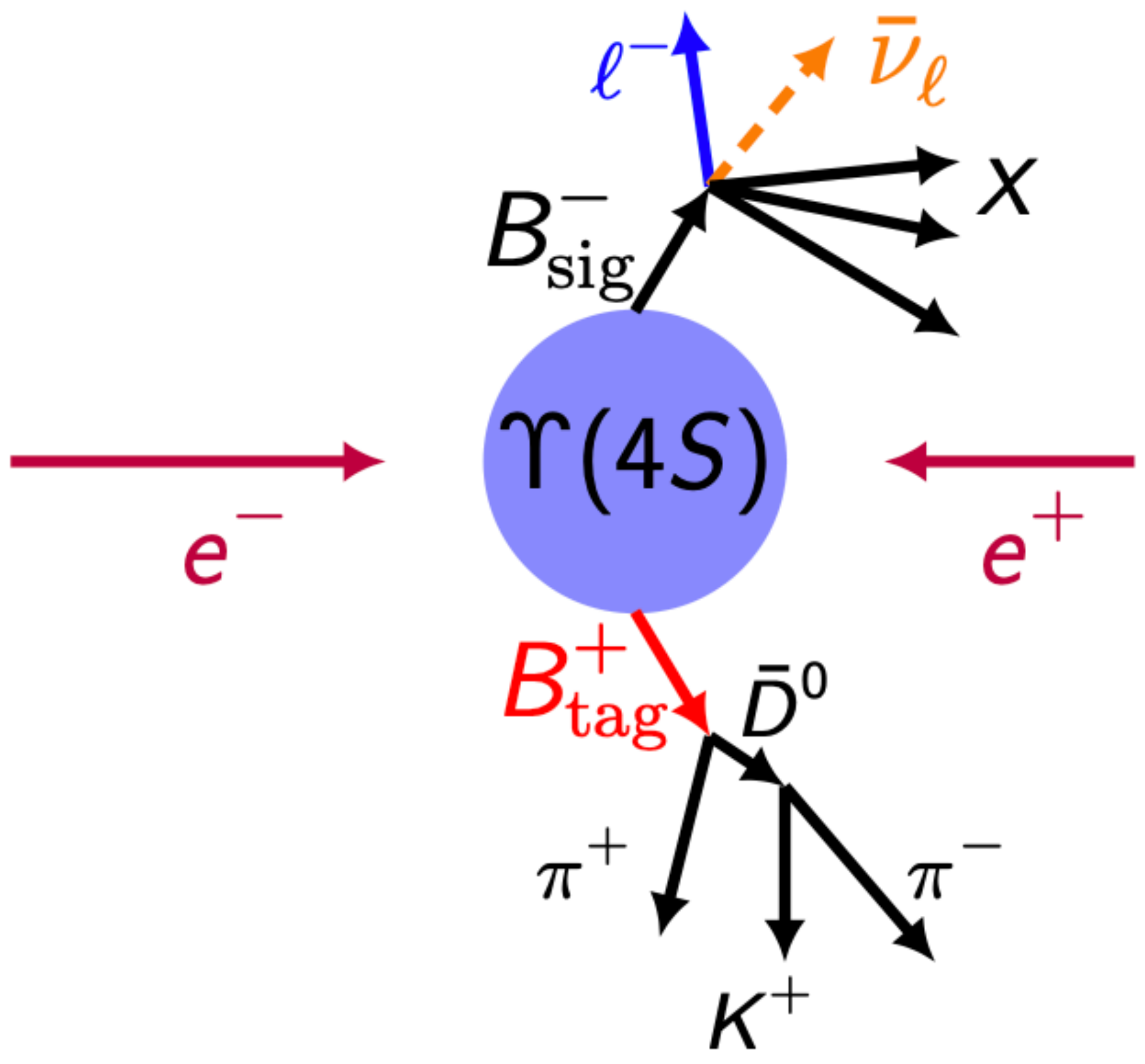
- 30-50% improvement in efficiency compared to Full Reconstruction at Belle.

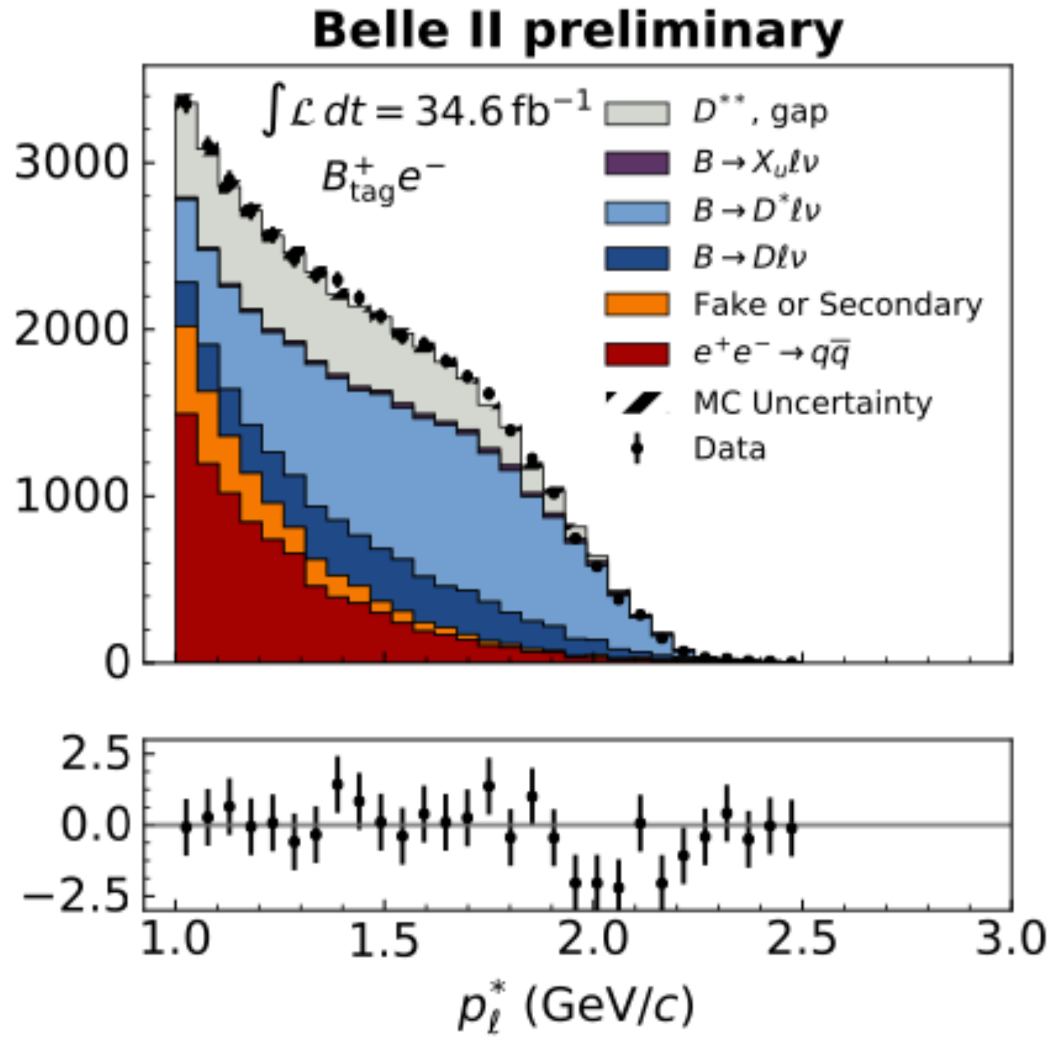
	$B^\pm$	B^0
Hadronic		
FEI with FR channels	0.53 %	0.33 %
FEI	0.76 %	0.46 %
FR	0.28 %	0.18 %
SER	0.4 %	0.2 %

	$B^\pm$	B^0
Semileptonic		
FEI	1.80 %	2.04 %
FR	0.31 %	0.34 %
SER	0.3 %	0.6 %

Comp. Softw. Big. Sci. 3 (2019)

- $B \to X\ell\nu$ channel employed to calibrate the hadronic FEI and account for data-MC differences.
- Calibration determined as $N^{X\ell\nu}_{Data}/N^{X\ell\nu}_{MC}$ after selecting signal side B



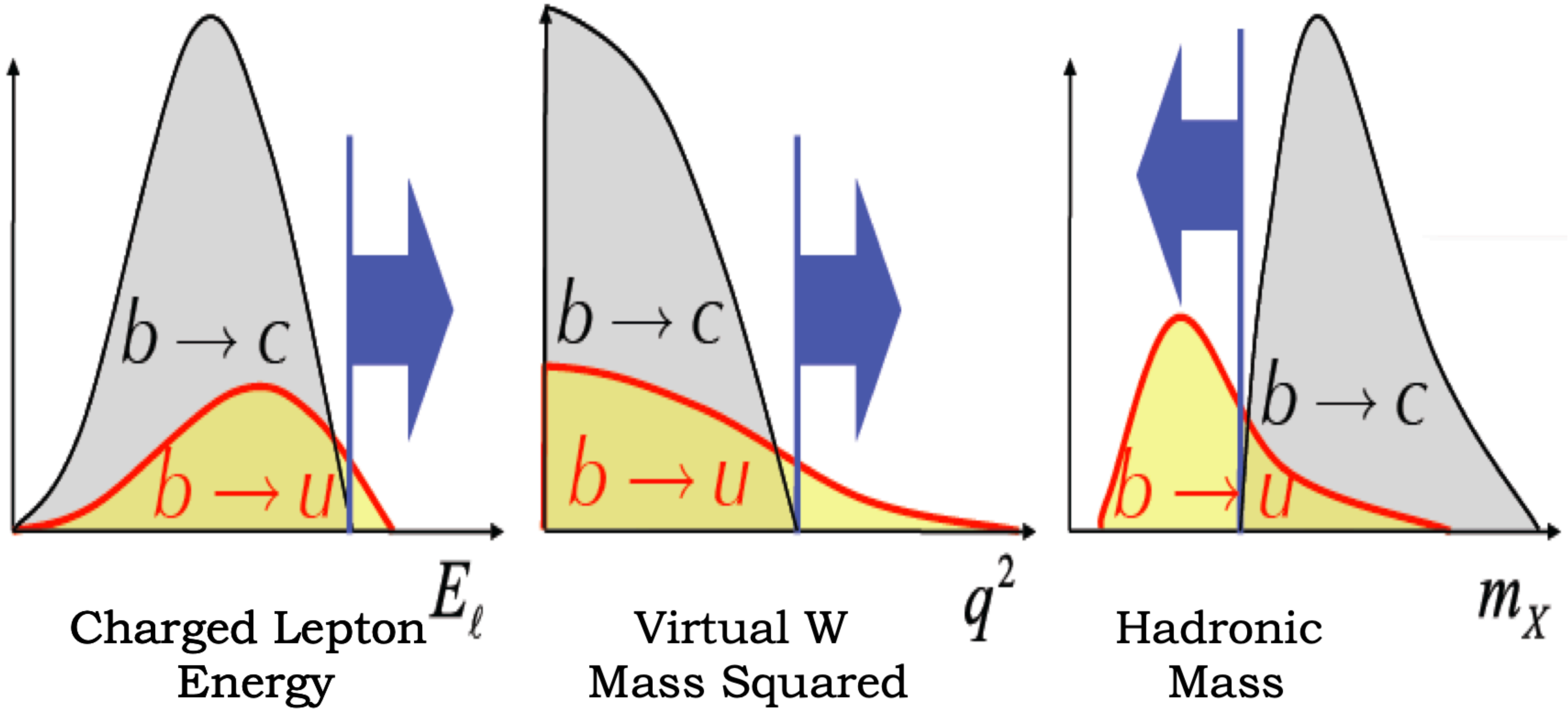


- Hadronic FEI calibration strategy has been established.
- Semi-leptonic FEI calibration and performance studies in progress for Summer 2022.

$|V_{ub}|$

- Experimentally challenging due to dominant $B \to X_c \ell \nu$ background.
- Only certain kinematic regions allow for clean separation: lepton momentum endpoint spectrum or low m_X.
- Inclusive via $B \to X_u \ell \nu$:
  - Precision of (~7%)
  - Operator Product Expansion (OPE) = Heavy Quark Expansion.
  - HQE breaks down and a non-perturbative shape function is required.

$$d\Gamma = d\Gamma_0 + d\Gamma_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 + d\Gamma_4 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4$$



- Exclusive via $B \to \pi \ell \nu$
  - Most precise determination of $|V_{ub}|$ (~4%)
  - Form factor determined non-perturbative from lattice QCD (high q^2) or LCSR (q^2~0) .

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f_+(q^2)|^2$$

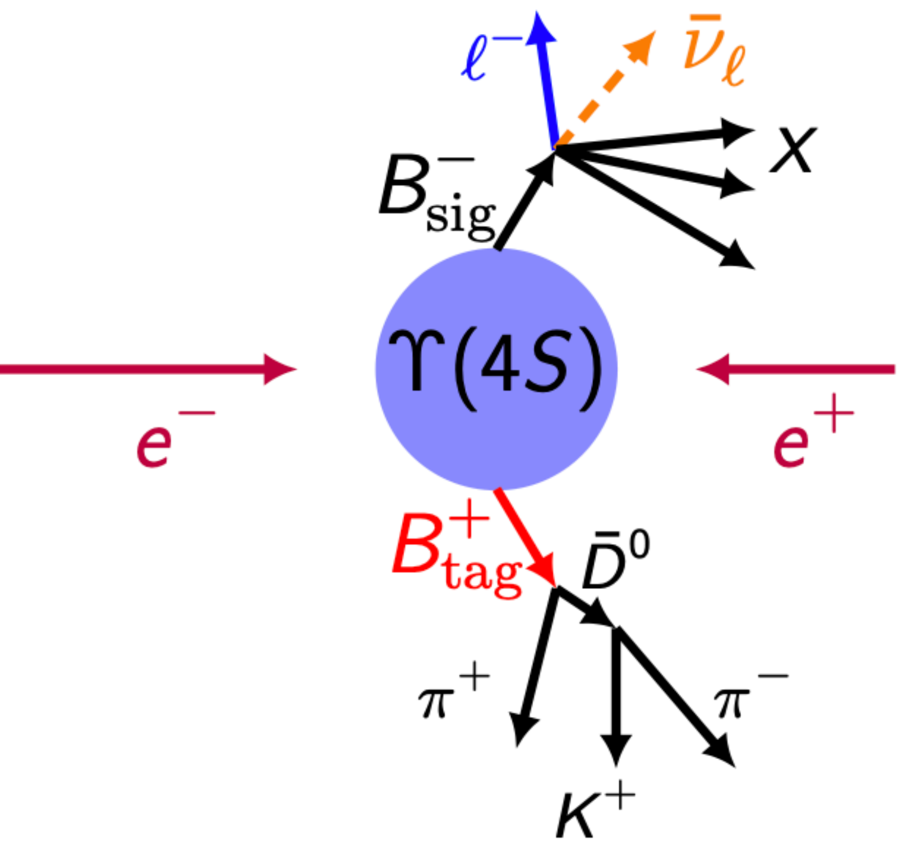
$$|V_{ub}| = (4.25 \pm 0.12^{+0.15}_{-0.14} \pm 0.23) \times 10^{-3}$$ **PDG inclusive**

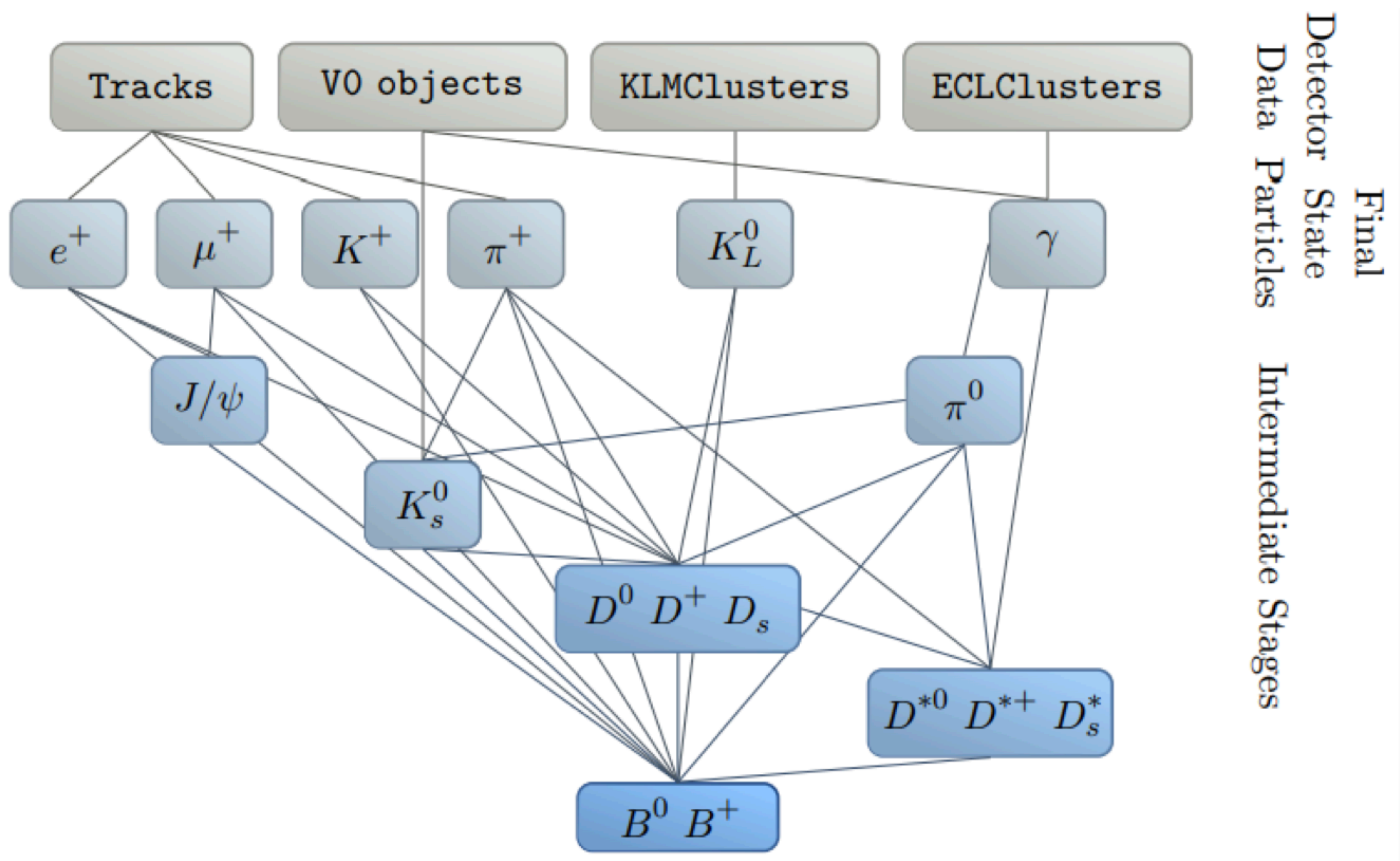
$$|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$$ **PDG exclusive**

Current ~3σ tension between inclusive and exclusive determinations

B-tagging at Belle II

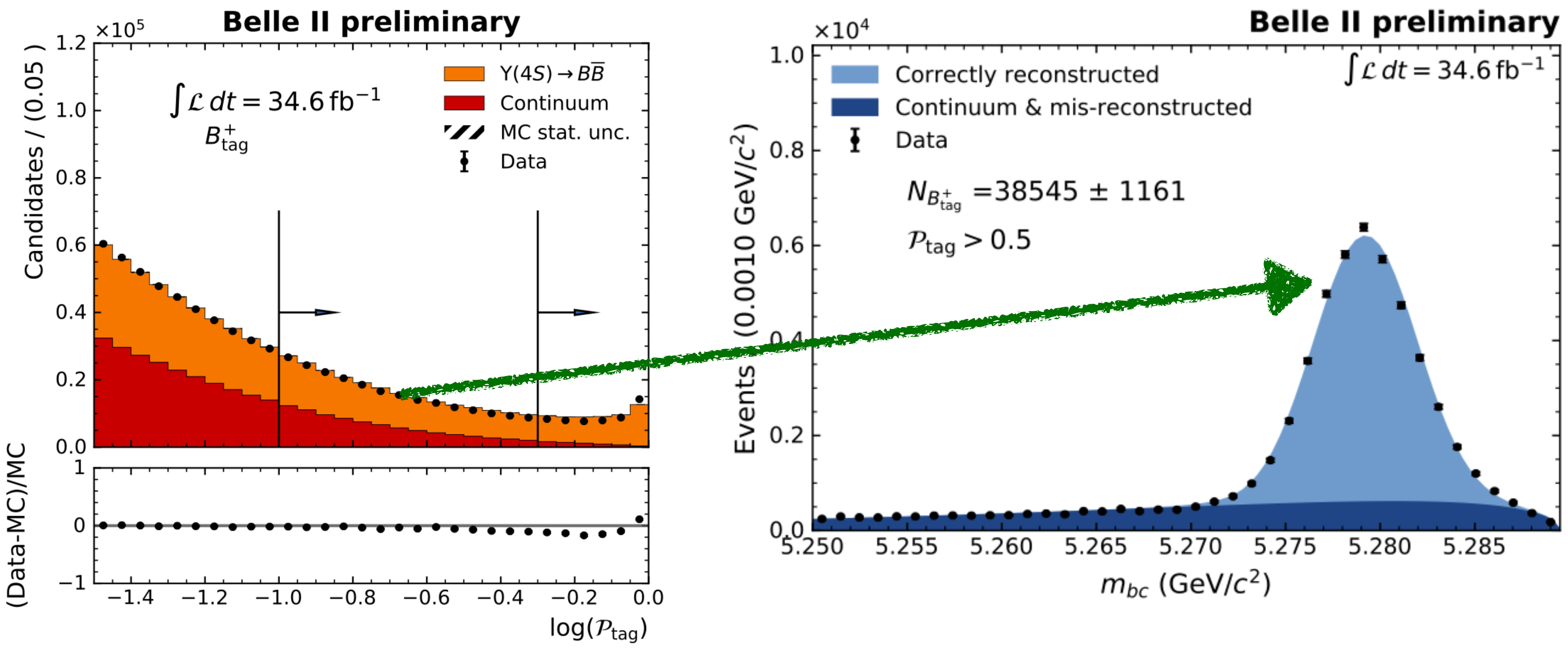
- Exclusive reconstruction of *B* mesons using hadronic and semi-leptonic modes.
- Achieved using the **Full Event Interpretation (FEI),** a multivariate algorithm based on a hierarchal approach.





- Employs over 200 Boosted Decision Trees to reconstruct ~10000 *B* decay chains.

- Outputs a signal probability which separates correctly reconstructed *B* mesons.



- **30-50% improvement in efficiency** compared to Full Reconstruction at Belle.

	$B^\pm$	B^0
Hadronic		
FEI with FR channels	0.53 %	0.33 %
FEI	0.76 %	0.46 %
FR	0.28 %	0.18 %
SER	0.4 %	0.2 %

	$B^\pm$	B^0
Semileptonic		
FEI	1.80 %	2.04 %
FR	0.31 %	0.34 %
SER	0.3 %	0.6 %

Comp. Softw. Big. Sci. 3 (2019)