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THE SCHOOL OF PHYSICS & ASTRONOMY

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Search for a heavy neutral lepton that mixes  
predominantly with the tau neutrino

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## Abstract

Heavy neutral leptons (HNLs) are highly motivated as a solution to the problems of neutrino masses, dark matter and the baryon asymmetry of the universe. In light of tight limits set by previous searches for HNLs that mix with electron and muon neutrinos, we search for an HNL that mixes predominantly with the tau neutrino. Such an HNL can be produced in tau decays, which are best studied in B-factory experiments. We search for the HNL in a sample of  $8.8 \cdot 10^8$   $e^+e^- \rightarrow \tau^+\tau^-$  events collected by the Belle experiment at center-of-mass energies around 10.58 GeV. The search focuses on long-lived HNLs with mass in the range 0.3-1.6 GeV exploiting the displaced-vertex signature for suppressing background. We set a new expected limit on the mixing between the HNL and  $\nu_\tau$ .

13 **CONTENTS**

14	<b>1. Introduction</b>	5
15	<b>2. Theoretical Overview</b>	6
16	2.1. The standard model	6
17	2.2. Neutrino flavor oscillations	7
18	2.3. Right handed neutrinos	8
19	2.3.1. Neutrino mass terms	8
20	2.3.2. The seesaw mechanism	10
21	2.4. $\tau$ physics	10
22	2.5. HNL model for this thesis	11
23	<b>3. Experimental Setup</b>	12
24	3.1. KEKB collider	12
25	3.2. Belle detector	12
26	3.3. Analysis software framework	17
27	<b>4. Analysis Method Overview</b>	18
28	<b>5. Data and Monte-Carlo samples</b>	20
29	5.1. Data samples	20
30	5.2. MC samples	21
31	<b>6. Event selection</b>	22
32	6.1. Online event selection	23
33	6.1.1. Primary online selection criteria	23
34	6.1.2. Vertex-fitting	23
35	6.1.3. Last online selection criteria	24
36	6.2. Offline signal region selection	24
37	6.3. Control region selection	34
38	6.4. Validation region selection	36
39	6.4.1. VRHeavySameSign and VRLightSameSign	36
40	6.4.2. VRHeavy $\pi\pi$ and VRLight $\pi\pi$	37
41	6.4.3. VR $K_S$	39

42	7. Signal efficiency and expected number of signal events	41
43	8. Systematic uncertainties	45
44	9. Unblinding plan	48
45	10. Statistical analysis and results	48
46	11. Conclusion	50
47	A. HNL mass calculation	52
48	B. The lifetime-reweighting efficiency-calculation method	53
49	C. Cross section, branching fractions, and lifetimes	54
50	D. Comparison of efficiency between basf2 vs b2bii for 1 GeV HNL of $c\tau = 30$ cm	54
51	References	59

## 52 1. INTRODUCTION

53 The Standard Model (SM) of particle physics, which was developed during the 20th  
54 century, represents the elementary particles in nature, and the fundamental interactions  
55 they experience with one another. The SM explains successfully diverse phenomena in  
56 physics, and all its major predictions were measured in experiments (one of their highlights  
57 was the discovery of Higgs boson in 2012). However, there are significant issues in today's  
58 physics, that cannot be explained by SM alone, and "new physics" should be formulate in  
59 order to do so.

60 One phenomenon that the SM is not able explain is the neutrino flavor oscillations which  
61 were observed firstly at the end of the last century. These oscillations necessitate neutrinos  
62 have mass states as well as their flavor states, while in SM the neutrinos are massless. A  
63 possible mechanism that incorporates these mass states to the SM can be derived from  
64 adding right handed neutrinos SM, in contrast to the known left handed in the SM (left  
65 and right handed refers to the neutrino's chirality). Sec. 2 elaborates about the nature  
66 neutrino oscillations and explain their relation to to right handed neutrinos. This Section  
67 also introduces the heavy neutrino lepton (HNL or  $N$ ), a superposition of the left and right  
68 handed neutrinos and a new mass states, and its detection is the main challenge this thesis  
69 faces. Quite amazingly, the existence of HNLs can solve more problem in physics than  
70 just the neutrino mass origins. It can explain the baryon asymmetry of the universe via  
71 leptogenesis [1-3]. It can also provide a dark matter candidate, because the HNL's lifetime  
72 is dictated by the its mass and the mixing parameters of the particles it can decay to. If  
73 one the HNLs has sufficiently low mass and small mixing parameters, its lifetime is greater  
74 than the age of universe. [4-8].

75 The efforts to discover new physics take place in various frontiers, and a main one is the  
76 experiments that involve particle accelerators and detectors. Colliding particles in GeV-TeV  
77 energy scales, allow us create and observe particles which are not present in our everyday  
78 nature. This thesis focuses on Belle experiment, a particle detector which ran and collected  
79 data in the years 1999-2010. It was located at the High Energy Accelerator Research Or-  
80 ganisation (KEK) in Tsukuba, Ibaraki Prefecture, Japan. The data was collected from the  
81 collisions produced in the KEKB particle accelerator. It should be mentioned that in the  
82 last years they superseded by their upgrades, Belle II and SuperKEKB respectively. Sec. 3  
83 deals more deeply with the technical details of the experiment. Additionally, in Sec. 3 we  
84 will see that this experiment is a fantastic environment for studying tau lepton physics.

85 In light of that, this research takes advantage of the relatively large number of  $\tau^+\tau^-$  events  
86 at belle, in order to search for an HNL that predominantly mixes with  $\nu_\tau$  in  $\tau$  decays. This  
87 type of mixing is highly motivated because various collider (and non-collider) experiments  
88 have already set tight bounds on mixing parameters of the HNL and  $\nu_e$  or  $\nu_\mu$  [9-19]. Until  
89 recently, the only published search that was directly sensitive to the  $N-\nu_\tau$  mixing parameter  
90  $V_{\tau N}$  in our mass range is the one by DELPHI experiment (in the HNL mass range this thesis  
91 focuses on) [13]. Recently, BABAR also put out a search based on an invisible HNL which  
92 mixes with  $\nu_\tau$ . It found no signal, but set a bound  $\nu_\tau$  on the mixing parameter in the range of  
93 100-1300 MeV [20].

94 This thesis explains this new method, and describes every steps of this execution, starting  
95 from the generating of the Monte-Carlo simulation samples, and ending with the final plot

## Standard Model of Elementary Particles

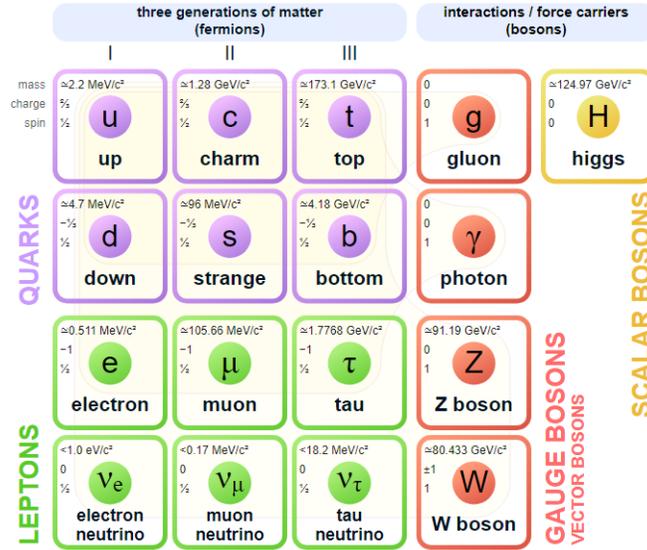


FIG. 1: The elementary particles of the Standard Model. The model contains 12 spin 1/2 fermions: 6 quarks and 6 leptons, divided into 3 flavor families (columns). 4 gauge bosons account for strong (gluon), electromagnetic (photon) and weak (Z, W) interactions. Finally, the Higgs scalar boson generates the masses of leptons and gauge bosons through the spontaneous symmetry breaking.

96 that presents the bound we managed to achieve on the  $N - \nu_\tau$  mixing.

## 97 2. THEORETICAL OVERVIEW

### 98 2.1. The standard model

99 The Standard Model (SM) of particle physics describes the elementary particles and the  
 100 fundamental interactions between them. Mathematically the Standard Model is a gauge  
 101 theory of the strong (SU(3)) and electroweak (SU(2) x U(1)) interactions. However, gravi-  
 102 tation is not included within the Standard Model. The elementary particles of the SM can be  
 103 categorised into fermions (leptons, quarks, and their anti-particles) and to bosons. Among  
 104 the elementary particles, the fermions are basically are the elementary units of matter (or  
 105 anti matter), and the bosons are the force carries, responsible for the interactions between  
 106 the particles. Fig. 1 presents all these particles with their chrage, mass and spin values [21].

107 Among the leptons, there are 3 flavors ( $e, \mu, \tau$ ) and their corresponding neutrinos ( $\nu_e, \nu_\mu, \nu_\tau$ ).  
 108 The neutrinos have only left-handed (LH) states, which are charged only under SU(2)<sub>L</sub>.  
 109 However, the observations of neutrino flavor oscillations [22] indicates (according to most  
 110 models) the existence of right-handed (RH) neutrino states that carry no SM gauge charges.  
 111 Sections 2.2.2 and 2.2.3 explains why, and what exactly is the nature of the RH neutrino.

112 **2.2. Neutrino flavor oscillations**

113 As a starting point of this discussion, we assume the neutrinos have masses, and therefore,  
 114 there is a spectrum of neutrino eigenstates  $\nu_i$ , that their eigenvalues are these masses, which  
 115 we denote by  $m_i$ . It should be mentioned that the  $\nu_i$  states are different from the flavor  
 116 states, which are denote here as  $\nu_\alpha$ . In fact, the two groups of states are two different bases  
 117 to describe the neutrino quantum states. Therefore, we can write a flavor state in term of  
 118 the mass basis (and vice versa of course) [23]:

$$|\nu_\alpha\rangle = \sum_i U_{i\alpha} |\nu_i\rangle, |\nu_i\rangle = \sum_\alpha U_{i\alpha}^\dagger |\nu_\alpha\rangle, \quad (1)$$

119 Where  $U$  is called the leptonic mixing matrix, and has a completely analogous role as the  
 120 CKM matrix, which is used for quark mixing in the SM. Obviously,  $U$  is unitary matrix,  
 121 and the corresponding mixing matrix that takes us back from the flavor basis to the mass  
 122 basis is  $U^\dagger$ .

123 In order to understand how neutrino oscillations indicate the existence of neutrinos mass  
 124 states, we discuss the nature of neutrino oscillations in vacuum. Of course, we are interested  
 125 of the probability of such process  $P(\nu_\alpha \rightarrow \nu_\beta)$ , because eventually it needs to be verified  
 126 by an experiment. For that, the amplitude is needed. By using the superposition property,  
 127 the amplitude can be divided into 3 contributions. If we look at a specific mass state  $\nu_i$ , we  
 128 need to consider the mixing matrix elements that connect this state to the initial and final  
 129 states,  $U_{\alpha i}^*$  and  $U_{\beta i}$ . The last contribution is the amplitude of  $\nu_i$  propagation the distance  
 130  $L$ , which we denote as  $Prop(\nu_i)$ . In conclusion, we need to consider all possible  $\nu_i$  states, so  
 131 the final amplitude for such process is:

$$Amp(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i} Prop(\nu_i) U_{\beta i} \quad (2)$$

132 In order to find  $Prop(\nu_i)$ , Schrödinger equation needs to be solved for a neutrino with  
 133 an energy  $E$  (considering that due to lightness of neutrinos, we have  $E \gg m_i$ ). So by  
 134 substituting the solution  $Prop(\nu_i) = e^{-im_i^2 L/2E}$  in Eq. [2], the squared absolute value of the  
 135 amplitude can be taken for getting the wanted probability:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} Re\{U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j}\} \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right) \\ + 2 \sum_{i>j} Im\{U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j}\} \sin \left( \frac{\Delta m_{ij}^2 L}{2E} \right) \quad (3)$$

136 If neutrinos are massless, then  $\Delta m_{ij}^2 = 0$  and  $P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta}$ . It means that neutrino  
 137 oscillations, namely a change in the neutrino's flavor across the distance  $L$ , indicate that the  
 138 neutrinos are massive. An important note is that this phenomenon has been observed as part  
 139 of experiments in which the neutrinos were passing through matter, and not only through  
 140 vacuum. In some cases the neutrinos-matter interactions are negligible, but sometimes  
 141 they are needed to be considered, and Eq. [3] has to be modified. Additional important  
 142 fact which rises from Eq. [3], is that neutrino oscillation experiment can only tell about the  
 143 neutrino squared mass splittings (the squared mass spectral pattern), as the masses' absolute

144 differences from zero remains unknown. However, there is a limit to this difference, as the  
 145 effective squared mass of the electron based neutrino mass, which is defined as  $m_{\nu_e}^{2(eff)} \equiv$   
 146  $\sum_i U_{ei}^2 m_{\nu_i}^2$ , has an upper bound of  $0.9 \text{ eV}^2$  (other flavors effective masses are relative to  
 147 this) [22].

148 The neutrino oscillation experiments can be categorized according to the neutrino sources  
 149 they deal with. In the first category there are the experiments which research solar neutrinos.  
 150 Solar neutrinos, as their name implies, originate directly from nuclear activity in the sun.  
 151 The main contribution comes from a proton-proton decay chain, namely:  $p+p \rightarrow d+e^++\nu_e$ .  
 152 There are also other decay chains, such as  ${}^8\text{B} \rightarrow {}^8\text{Be}^* + e^+ + \nu_e$ , which is the one that NSO  
 153 (Sudbury Neutrino Observatory) [24] used for solar neutrino detecting. The second category  
 154 is atmospheric neutrinos, which are generated through interaction of incoming cosmic rays  
 155 with air nuclei in Earth's atmosphere. [25] The production of these atmospheric neutrinos  
 156 is dominated by the decay of  $\pi \rightarrow \mu + \nu_\mu$  which is followed by  $\mu \rightarrow e + \nu_\mu + \nu_e$ . There are  
 157 also experiments that research neutrinos from artificial sources such as nuclear reactors and  
 158 particle accelerators. In conclusion, the experiments that were studying and validating the  
 159 neutrino oscillation are numerous and diverse.

160 Eventually, the first solid discoveries of the neutrino oscillations took place in the Super-  
 161 Kamiokande (atmospheric neutrinos experiment) and in NSO. The conductors of these stud-  
 162 ies were awarded the 2015 Nobel Prize for Physics for their achievements.

### 163 2.3. Right handed neutrinos

#### 164 2.3.1. Neutrino mass terms

165 After understanding why neutrino flavor oscillations indicate the neutrinos have mass, it  
 166 is now appropriate that we introduce the physical mechanism that lays the foundations for  
 167 the generation of these masses [21].

168 For simplicity, let us add a single right handed (RH) neutrino to the SM, which we mark  
 169 as  $\nu_R$  (later, we can consider a more complicated model with  $n$  flavors of RH neutrinos). RH  
 170 neutrinos are *sterile*, namely they do not interact via any of the SM interactions (strong,  
 171 weak, electromagnetic). In other words, they are singlets of the complete SM gauge group.  
 172 Moreover, the RH neutrinos couple to left handed (LH) leptons in the same way RH charged  
 173 leptons couple to LH charged leptons in the SM, i.e. via Yukawa interactions. Hence, The  
 174 most general renormalizable Lagrangian that is possible to write is:

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_R \not{\partial} \nu_R - \bar{l}_L Y^\nu \nu_R \tilde{\Phi} - \frac{1}{2} \bar{\nu}_R^c M_M \nu_R + h.c. \quad (4)$$

175 Where  $\mathcal{L}$  is the Lagrangian of the SM; the second term is the kinetic energy of the neutrino;  
 176  $Y^\nu$  and  $M_M$  are the matrices of Yukawa coupling and the Majorana mass term, corre-  
 177 spondingly (the RH neutrinos can have mass terms because such a term converts a parti-  
 178 cle to its anti-particle, which is allowed only for particles that have no conserved charge);  
 179  $l_L = (\nu_L, e_L)^T$  are the left handed lepton doublets;  $\Phi$  is Higgs doublet and  $\tilde{\Phi} = (\epsilon\Phi)^\dagger$ , where  
 180  $\epsilon$  is the  $SU(2)$  anti-symmetric tensor; h.c. is the hermitian conjugate of the corresponding

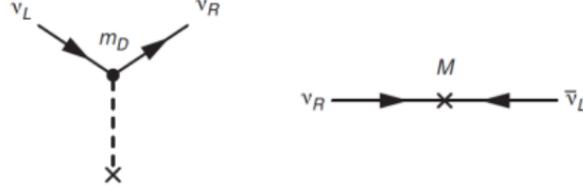


FIG. 2: Feynmann diagrams of the Dirac and Majorana mass terms, derived from Eqs. [6](#) and [7](#)

181 terms. Additionally, the CP conjugate of  $\nu$  is defined as:

$$\nu^c = CP\nu = i\gamma^2\gamma^0\nu^* \quad (5)$$

182 Where  $\gamma^i$  are the Dirac matrices.

183 This new Lagrangian implies on two mass terms for the RH handed neutrino. The first  
184 one is the Majorana term, which was mentioned earlier and appear explicitly in Eq. [4](#). The  
185 Majorana mass term involves a neutrino turning into an anti-neutrino. We can write it here:

$$\mathcal{L}_M = -\frac{1}{2}M(\bar{\nu}_R^c\nu_R + \bar{\nu}_R\nu_R^c), \quad (6)$$

186 The second term is the Dirac mass term. It is generated from a spontaneous electroweak  
187 symmetry breaking from the Yukawa interactions.

$$\mathcal{L}_D = -m_D(\bar{\nu}_R\nu_L + \bar{\nu}_L\nu_R), \quad m_D = Y^\nu \frac{v}{\sqrt{2}}. \quad (7)$$

188 Where  $v$  is the vacuum expectation value of the Higgs field. The Dirac term conserves total  
189 lepton number but it can break the lepton flavor number symmetries.

191 The sum of the two mass terms can be written in matrix form

$$\mathcal{L}_{DM} = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L^c & \nu_R \end{pmatrix} + h.c. \quad (8)$$

192 The masses of the physical neutrino states will be the eigenvalues of the matrix above,  
193 which are:

$$m_{\pm} = \frac{M \pm M\sqrt{1 + 4m_D^2/M^2}}{2} \quad (9)$$

194 The eigenstates are:

$$\nu = \cos\theta(\nu_L + \nu_L^c) - \sin\theta(\nu_R + \nu_R^c) \quad (10a)$$

195

$$N = \cos\theta(\nu_R + \nu_R^c) + \sin\theta(\nu_L + \nu_L^c) \quad (10b)$$

196 Where  $\tan\theta \approx m_D/M$ . As we can see, the effect of the Majorana mass term is reducing the  
197 weak charged-current of light neutrino states by a  $\cos\theta$  factor.

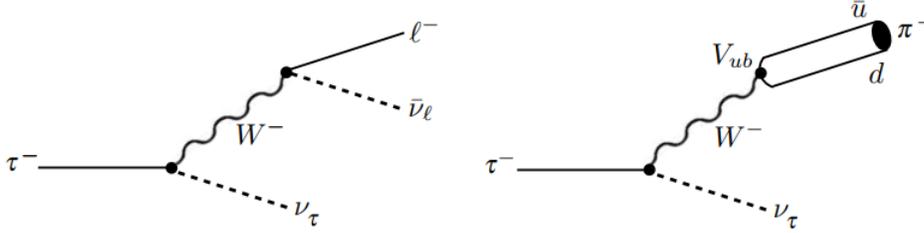


FIG. 3: Feynman diagrams to the tree level of leptonic (left) and hadronic (right)  $\tau$ -lepton decays.

198 2.3.2. *The seesaw mechanism*

199 This seesaw limit [26-32] is defined as the limit where  $M \gg m_D$ . In this case, the  
200 eigenvalues from Eq. 9 become:

$$m_\nu = \frac{m_D^2}{M}, \quad m_N = M, \quad (11)$$

201 corresponding to the light ( $\nu$ ) and heavy ( $N$ ) neutrino states. The seesaw mechanism actu-  
202 ally explains why the light neutrinos are so light, and predicts the existence of the "heavy  
203 neutral leptons" (HNLs, signed with  $N$ ) with mass of  $m_N = M$ . Under this limit, the final  
204 states from Eq. 10 becomes:

$$\nu \approx (\nu_L + \nu_L^c) - \frac{m_D}{M}(\nu_R + \nu_R^c) \quad (12a)$$

205

$$N \approx (\nu_R + \nu_R^c) + \frac{m_D}{M}(\nu_L + \nu_L^c) \quad (12b)$$

206 Hence, the light neutrino couples to the weak-charged current in the same way as the SM  
207 neutrinos. From the same reason,  $N$  is mostly RH (sterile), so it does not participate in any  
208 kind of weak interaction.

209 **2.4.  $\tau$  physics**

210 As our search focuses on HNL in  $\tau$  decays, some of the  $\tau$ -lepton properties should be dis-  
211 cussed. Like the other leptons, it interacts directly only via the weak interaction. However,  
212  $\tau$  is the only lepton that can decay into hadrons without violating mass-energy conservation,  
213 due to its relative high mass ( $m_\tau = 1.776\text{GeV}$ ). Hence, the  $\tau$ -lepton's decays can be divided  
214 into two types: leptonic or hadronic. Fig. 3 presents the simplest decay modes possible of  
215 these two types. Some of the most common decay modes and their branching fractions are  
216 presented in Table II. Note that "1-prong decay" means that the tau has only 1 charged  
217 daughter.

Decay mode of $\tau^-$	Branching fraction
$\mu^- \bar{\nu}_\mu \nu_\tau$	$17.39 \pm 0.04$
$e^- \bar{\nu}_e \nu_\tau$	$17.82 \pm 0.04$
$\pi^- \nu_\tau$	$10.82 \pm 0.05$
$\pi^- \pi^0 \nu_\tau$	$25.49 \pm 0.09$
1-prong decay	$85.24 \pm 0.06$

TABLE I: Common decay modes of  $\tau$ -lepton

## 2.5. HNL model for this thesis

As declared before, the main goal of this thesis is to present our search for the HNL. In the naive seesaw mechanism presented above, the mixing coefficient is  $V_{\ell N} \approx \sqrt{m_\nu/m_N}$ . We are interested in  $m_N \sim \text{GeV}$ , so with  $m_\nu < \text{eV}$ , this implies  $V_{\ell N} < 10^{-4}$ , making HNL production at colliders much too small to be observed. Therefore, all searches, including this one, rely on more complicated models, in which the value of  $V_{\ell N}$  is independent of that of the HNL mass  $m_N$ . In these models, the SM neutrino can be written as a superposition of the  $\nu$  and  $N$  mass states in the following way [33]:

$$\nu_L = \sum_{i=1}^3 U_{\ell i} \nu_i + V_{\ell N} N \quad (13)$$

where  $U_{\ell i}$  and  $V_{\ell N}$  are the mixing parameters. We took  $V_{\ell N} \ll 1$ . This small mixing between LH neutrino and HNL, enables HNL production and decay in SM processes.

As mentioned in the Sec. 1, most previous researches utilized the mixing parameters with electrons and muons  $V_{eN}$ ,  $V_{\mu N}$ . Our research focus on mixing with  $\tau$  lepton and the parameter  $V_{\tau N}$  under the assumption that  $|V_{\tau N}| \ll |V_{eN}|, |V_{\mu N}|$ , in which the  $N$  mixes mainly with the  $\tau$  neutrino, and its mixing with the electron or muon neutrinos can be neglected. Therefore, it is advantageous to search the HNL in  $\tau$ -decays, and the best place to that is at B-factories. B-factories are particle collider experiments designed to produce and detect a large number of B mesons via electron-positron collision, but they are also a significant source for  $e^+e^- \rightarrow \tau^+\tau^-$  processes.

The decay rate of the process  $\tau \rightarrow \pi N$  is obtained by replacing  $N \rightarrow \tau$  and  $\ell \rightarrow N$  in Eq.(3) of Ref. [34]:

$$\begin{aligned} Br(\tau^- \rightarrow N\pi^-) &= \frac{G_f^2}{16\pi} f_\pi^2 |V_{ud}|^2 |V_{\tau N}|^2 m_\tau^3 \lambda^{1/2} \left( 1, \frac{m_N^2}{m_\tau^2}, \frac{m_{\pi^-}^2}{m_\tau^2} \right) \\ &\times \left[ 1 + \frac{m_N^2}{m_\tau^2} - \frac{m_{\pi^-}^2}{m_\tau^2} \left( 1 + \frac{m_N^2}{m_\tau^2} \right) - 4 \frac{m_N^2}{m_\tau^2} \right] / \Gamma_\tau \end{aligned} \quad (14)$$

Where  $G_f$  is Fermi coupling constant,  $m_{\pi^-}$  and  $m_N$  denote the mass of the charged pion and sterile neutrino, respectively;  $V_{ud}$  is the CKM matrix,  $f_\pi$  is the pion decay constant and  $\Gamma_\tau$  is the  $\tau$  lepton's decay rate; the function  $\lambda(x, y, z)$  is defined as  $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$ .

242 The method that this thesis presents (and is discussed elaborately in the following sec-  
 243 tions), exploits the long lifetime of the low-mass  $N$ , which goes as [35]:

$$c\tau_N = 0.324 \text{ cm} \times \left(\frac{m_N}{1 \text{ GeV}}\right)^{-5.44} |V_{\tau N}|^{-2} \quad (15)$$

244 Particularly when produced at relativistic speeds, the  $N$  travels macroscopic distance inside  
 245 the detector before decaying. The resulting displaced-vertex signature is particularly useful  
 246 for suppressing background (see [6.6.2] [33]).

### 247 3. EXPERIMENTAL SETUP

#### 248 3.1. KEKB collider

249 KEKB [36] was an asymmetric electron-positron collider, which operated mostly at the  
 250 center of mass energy of 10.58GeV, corresponding to the center-of-mass energy of  $\Upsilon(4S)$   
 251 resonance. The CM frame is boosted due to the asymmetry in energies of the electron and  
 252 the positrons. Other resonance states were also produced  $\Upsilon(1S), \Upsilon(2S), \Upsilon(3S), \Upsilon(5S)$ , as  
 253 well as off-resonance states, which were set 60MeV bellow each of these on-resonance states.  
 254 The collected data of the states is listed in Table II [37]. KEKB accelerated the electrons and  
 255 the positrons in two different rings: the electrons in a High Energy Ring (HER) with 8GeV  
 256 and the positrons in a Low Energy Ring (LER) with 3.5GeV. Each ring has a circumference  
 257 of 3016m and is composed of four straight sections and four bends. The two rings are located  
 258 side by side in the accelerator tunnel. KEKB layout is depicted in Fig.4 KEKB in particular,  
 259 and  $e^+e^-$  colliders in general, are suitable for studying tau physics.  $\tau$  leptons are produced  
 260 in pairs through the following electroweak process:  $e^+e^- \rightarrow \gamma/Z \rightarrow \tau^+\tau^-$ . The cross section  
 261 of this process for center of mass energy of the  $\Upsilon(4S)$  is  $0.919 \pm 0.003 \text{ nb}$  [38], what allowed  
 262 the Belle detector to collect high number of  $8.8 \times 10^8$  of  $e^+e^- \rightarrow \tau^+\tau^-$  events [33].

Resonance state	On-resonance lumi. [fb <sup>-1</sup> ]	Off-resonance lumi. [fb <sup>-1</sup> ]
$\Upsilon(1S)$	5.7	1.8
$\Upsilon(2S)$	24.9	1.7
$\Upsilon(3S)$	2.9	0.25
$\Upsilon(4S)$	711	89.4
$\Upsilon(5S)$	121.4	1.7

TABLE II: Summary of luminosity integrated by Belle

#### 263 3.2. Belle detector

264 The Belle detector was installed in the Tsukuba hall, where the accelerated particles  
 265 collide. The position at which the particles' beams cross is called interaction point (IP).  
 266 The detector's purpose is to detect the particles that are produced in this collision. The  
 267 Belle detector was about  $3.6\pi$  solid angle composite detector with rotational symmetry

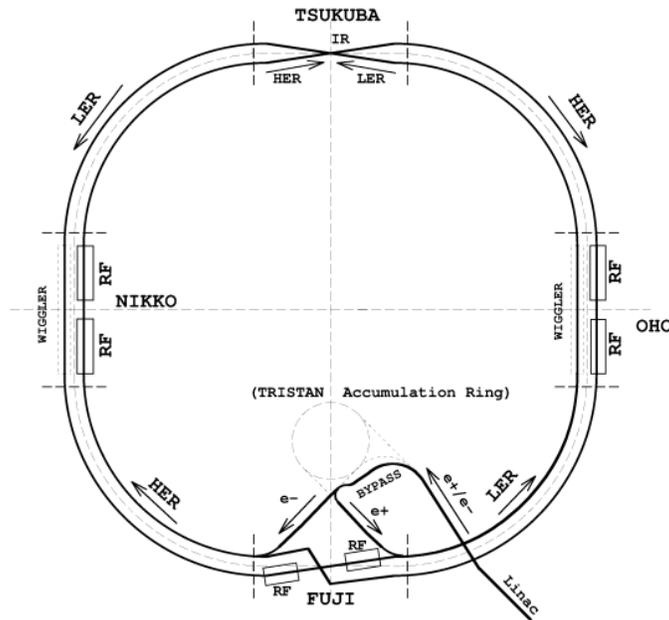


FIG. 4: Schematic layout of KEKB accelerator complex

268 around the beam axis. It was, however, asymmetrical in the forward-backward direction  
 269 due to the asymmetry of energy between the electron and positron beams.

270 The Belle detector [39] was designed with multiple layers, going inside-out, similarly to  
 271 other particle detectors. Belle had a different types of a sub-detectors, and all together gave  
 272 detailed information about the collision event. Some of the information is used for particle  
 273 identification, which is provided to us by the reconstruction software (see Secion 33.3 for  
 274 more details about the software framework) as the parameter **particle ID (PID)**. PID  
 275 is a likelihood function that gives the degree of confidence that the particle is really the  
 276 particle related to the PID (eID is the function for electrons and muID for muons). This  
 277 distribution is calculated based on the information achieved by different components of the  
 278 detector, which are described in the following paragraphs. Fig. 5(6) presents the eID (muID)  
 279 distribution of an electron (muon) and a non-electron (non-muon) particle, in this case a  
 280 pion.

281 The sub-detectors are shown in Fig. 7, and, and their roles are described as follows, going  
 282 from the inner part of Belle outside:

- 284 • **SVD – Silicon Vertex Detector:** The SVD is located outside the cylindrical beryl-  
 285 lium beam pipe, which holds the vacuum needed for the beams. Its purpose is to  
 286 measure the  $z$ -axis vertex position of the  $\tau$ ,  $B$  and  $D$  mesons, with the best possible  
 287 resolution. Information from the SVD is used for PID estimation. Two different SVDs  
 288 were used during the experiment. The first one, named SVD1, had 3 layers (30, 45.5,  
 289 60.5 mm radii) in a barrel-only design and covered an angle of  $23^\circ < \theta < 139^\circ$ , corre-  
 290 sponding to  $86^\circ$  of the full solid angle, where  $\theta$  is the angle from the beam axis (polar  
 291 angle). Afterwards, SVD1 was replaced by SVD2 due to radiation damage. SVD2  
 292 consisted 4 layers (20, 43.5, 70 and 88 mm radii) and covered  $17^\circ < \theta < 150^\circ$ . This is  
 293 the closest the SVD can be installed to the beam pipe, since a double-wall beryllium

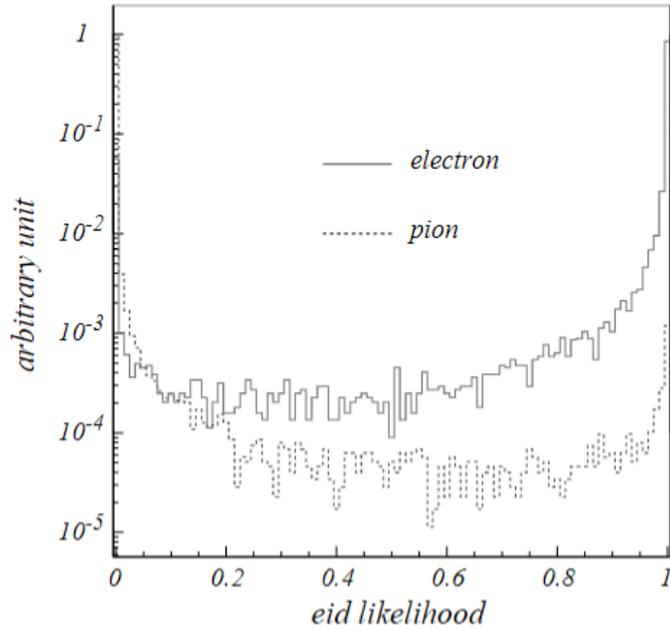


FIG. 5: eID likelihood distributions for electrons (solid line) and for pions (dashed line). [40]

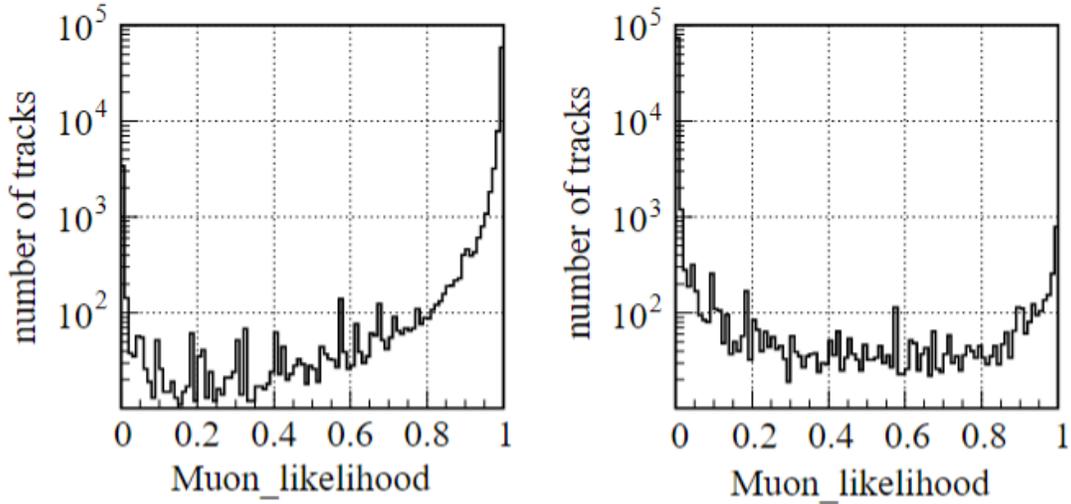


FIG. 6: muID likelihood distributions for muons (left) and for pions (right). [40]

294  
295

cylinder with an inner diameter of 40 mm is built around the pipe, used as a shielding mechanism for the SVD.

296  
297  
298  
299  
300

- **CDC – Central Drift Chamber:** The CDC is a component of great importance for charged particles reconstruction, which includes tracking, momentum measurement and particle identification via energy loss ( $dE/dX$ ) measurement. The momentum measurement exploits the magnetic field of a super-conducting solenoid of 1.5T, which resides between the ECL and the KLM. The inner and outer radii of the CDC are 103.5

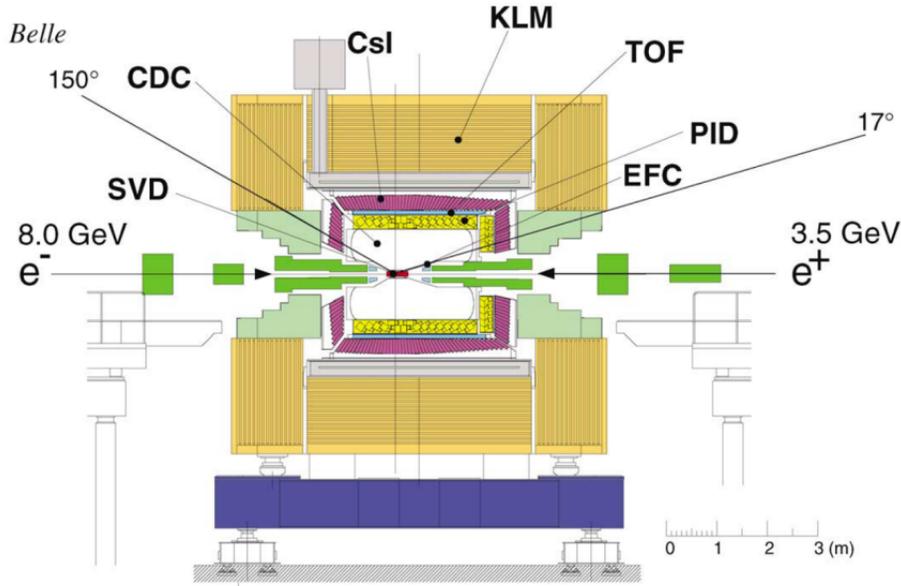


FIG. 7: Schematic layout of Belle detector [39]

301 mm and 874 mm, respectively. The structure of the CDC is of great importance for  
 302 the understanding of the selection cuts we apply in Sec 6, so its overview is presented  
 303 in Fig. 8.

304 The CDC has 8400 drift cells in 50 cylindrical layers. Most of the drift cells are almost  
 305 square and have a drift distance between 8 and 10 mm. Each cell is formed by a grid  
 306 of field wires and a sense wire. As a charged particle moves through the drift cell, it  
 307 ionizes the gas. The field wires are negative and the sense wire is positive, so that the  
 308 electrons from the ionization drift under the electric field toward the sense wire. When  
 309 they get very close to the sense wire, the field (which goes like  $1/r$ ) is so large that  
 310 the electrons gain enough kinetic energy during their mean free path that they ionize  
 311 more electrons, which ionize more electrons etc., until all the ionized electrons reach  
 312 the sense wire and are collected there. This avalanche yields a signal amplification of  
 313 order  $10^4$  and reduces the need for strong electronic amplification of the signal.

314 The transverse momentum resolution for charged particles with  $p_t \geq 100 MeV/c^2$  is  
 315  $\frac{\sigma_{p_t}}{p_t} \sim \%0.3\sqrt{1 + p_t^2}$  ( $p_t$  in GeV/c) in the polar angle region of  $17^\circ < \theta < 150^\circ$  (the  
 316 CDC is asymmetric in the z-axis). This high resolution above is achieved by low-Z gas  
 317 (50% helium, 50% ethane mixture), in order to reduce multiple scattering.

- 318 • **ACC – Aerogel Cherenkov Counter system:** The main functions of the ACC  
 319 are distinguishing  $\pi^\pm$  and  $K^\pm$  mesons, extending the momentum coverage for particle  
 320 identification beyond the reach of  $dE/dx$  measurements in the CDC and time-of-  
 321 flight measurements in the TOF. When charged particles travel with velocity  $v$  higher  
 322 than the speed of light in a dielectric medium of refractive index  $n$  ( $v > c/n$ ) in the  
 323 material, they emit radiation called "Cherenkov light". This radiation is emitted at  
 324 an angle given by  $\cos \theta = 1/(\beta n)$ , where  $\beta \equiv v/c$ . For beta too small,  $\cos \theta > 1$  which  
 325 means that there is no radiation. The value of  $n$  is chosen such that for most of the

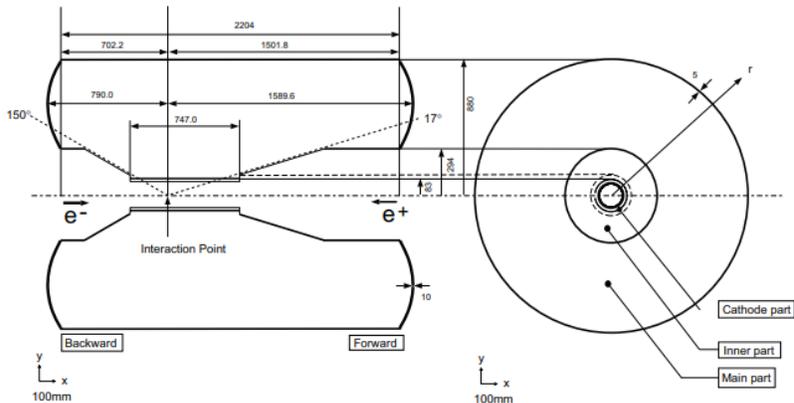


FIG. 8: Schematic layout of CDC

326 momentum range of interest, pions emit Cherenkov light while kaons don't, since their  
 327  $\beta$  is too slow. The silica aerogels that is used as the radiators in the ACC have a  
 328 refractive indices between 1.01 to 1.03. The ACC consists of 960 counter modules for  
 329 the barrel part and 228 modules for the forward end-cap part of the detector. One  
 330 or two fine mesh-tubes photo multiplier tubes (FM-PMT) are used per module, for  
 331 detection of the Chenekov light. The FM-PMT are attached directly to the aerogel  
 332 for that purpose.

333 • **TOF – Time of Flight detection system:** The TOF system is composed of plastic  
 334 scintillation counters which are used as an additional tool for particles identification.  
 335 The TOF has a 100ps time resolution and is situated such that hard particles have  
 336 a flight path of 1.2m, which makes this system effective for particle momenta below  
 337 about 1.2 GeV/c. The TOF is also utilized for providing fast timing signals for the  
 338 trigger system. The TOF system includes 128 counters and 64 trigger scintillation  
 339 counters (TSC), which are divided to 65 modules located at a radius of 1.2m from the  
 340 IP. The system polar angle coverage is  $34^\circ < \theta < 120^\circ$ . The counters of both types  
 341 use FM-PMTs for their scintillation counting operation.

342 • **EFC – Extreme Forward Calorimeter:** The EFC is installed in order to extend a  
 343 polar angle area, which otherwise wouldn't be covered by the electromagnetic calorime-  
 344 ter (ECL):  $6.4^\circ < \theta < 11.5^\circ$  in the forward direction, and  $163.4^\circ < \theta < 171.2^\circ$  in the  
 345 backward direction. Additional role the EFC plays is being used as a beam mask,  
 346 reducing the background in the central drift chamber (CDC). It is also used for mon-  
 347 itoring the beam and the luminosity of Belle. For the sake of this goals, the EFC is  
 348 installed in the front faces of the cryostats of the compensation solenoid magnets of the  
 349 KEKB accelerator, surrounding the beam pipe. To withstand the high radiation near  
 350 the interaction point, the EFC is made of Bismuth Germanate (BGO,  $Bi_4Ge_3O_{12}$ ).  
 351 Its energy resolution is:

$$\frac{\sigma_E}{E} = \frac{(0.3 - 1)\%}{\sqrt{E[GeV]}} \quad (16)$$

352 • **ECL – Electromagnetic Calorimeter:** The ECL is the main tool in the detector for  
 353 detection of photons (which are charge-less particles, hence undetectable in the CDC)

and for measuring their energies. The photons that the ECL detects are high-energy photons that come mostly from  $\pi^0$  decays, but also other sources. Such a photon creates an EM shower in the ECL. The energy of most photons in Belle is below 500 MeV, what makes the performances of the ECL in this energy range very important. The ECL in Belle consists of 8736 CsI(Tl) crystals with a silicon photodiode readout. This type of crystal was chosen due to its important features e.g. high photon yield, weak hygroscopicity and mechanical stability.

The ECL has a barrel section of with a length of 3.0 m and an inner radius of 1.25 m, and annular end-caps at  $z = +2.0m$  and  $z = -1.0$  from the IP. The polar angular coverage of this system is  $17^\circ < \theta < 150^\circ$ .

- **KLM –  $K_L^0$  and  $\mu$  detector:** This sub-detector is responsible for identification of muons and of  $K_L^0$  mesons. The muon identification is done only for candidates with a momentum greater than 600 MeV/c. Otherwise, they either don't reach or do not penetrate the KLM deep enough to leave a clear muon signal. For  $K_L$  there is no such momentum limit.

The polar angular coverage of this system is  $20^\circ < \theta < 155^\circ$  (including barrel part and endcap parts). The KLM is divided to 15 layers of charged-particle detectors and to 14 iron layers in each the octagonal barrel region. Moreover, there are 14 detector layers in each of the forward and backward end-caps. Between each layer, there is an RPC (resistive plate chamber) detectors to detect the muons that pass through from the absorber layers.

The  $\mu$  ( $K_L^0$ ) detection is done by observing clusters in the KLM which are (not) associated with charged tracks in the CDC. The multiple layers of charged particle detectors and iron allow the discrimination between muons and charged hadrons, as muons travel much farther with smaller deflections on average than strongly interacting hadrons. As for the  $K_L^0$ , they interact hadronically and produce a shower of ionizing particles in the ECL or the iron. The location of this shower determines the direction of the  $K_L^0$  (fluctuations in the size of the shower prevent a useful measurement of the energy).

- **Detector solenoid and iron structure:** The superconducting solenoid induces a magnetic field of 1.5 T. The solenoid has a diameter of 3.4 m and a length of 4.4 m. The iron yoke surrounding the magnet has several functions. First, it is used as a return path of the magnetic flux. Second, it serves as the absorber material for the KLM. The iron yoke's components masses are 608 tons for the barrel yoke and 524(2x262) for the end-cap yokes.

### 3.3. Analysis software framework

Fig. 9 displays the workflow for both real data and Monte-Carlo (MC) simulation processes from the starting point (which is different for real data and MC) and up to the final samples that will be used for offline analysis. The data was collected from the Belle detector's measurements. The specific data samples that we use are elaborated in Sec. 5. The MC is generated with the number of appropriate generators. Each generator knows how to generate the physics for particular processes. The generators and the corresponding

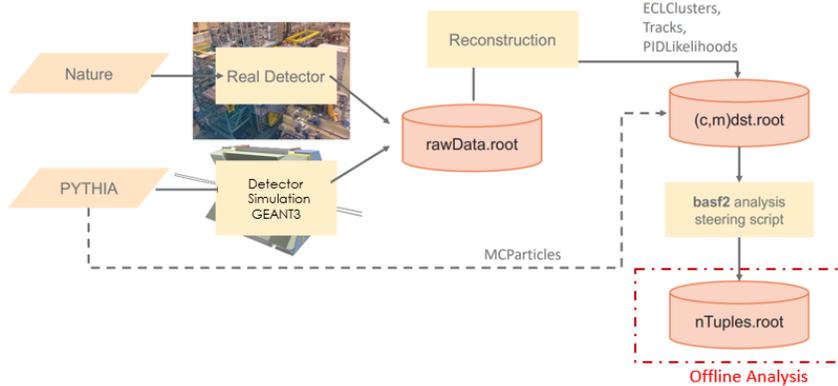


FIG. 9: Data and MC schematic process tree

396 processes are elaborated in Sec. 5.5.2. The outcome of the generators' work is a list of par-  
 397 ticles with mother-daughter relations, 4-momenta, and production positions. This is then  
 398 fed into a GEANT3 simulation of the particle's interactions with the detector material and  
 399 the electronic response of the detector. The electronic response output has the same format  
 400 (channel for channel) as the real data that's obtained from the detector.

401 Afterwards, both data and MC samples need to be processed from raw-data configuration  
 402 to a data-format which will eventually includes the reconstructed tracks, 4-momenta of the  
 403 each detectable particles and additional essential parameters (e.g. PID and decay vertices).  
 404 This format is called an "ntuple". All these steps demand an efficient and reliable software.

405 Such software is Belle Analysis Framework (`basf`) [41], which is the software framework  
 406 intended for generation, reconstruction and analysis of Belle events. Eventually, we preferred  
 407 to use the software for Belle II (`basf2`), because it is more updated and includes more useful  
 408 features. In Sec. 5.5.2 we explain where we use each one. When we used `basf` for MC  
 409 generation processes, we then converted the output files of `basf` to `basf2` output format file,  
 410 with a package in `basf2` called `b2bii` [42].

411 The work of `basf2` is implemented with processing blocks called *modules*, each executing a  
 412 defined task. A sequence of modules creates a *path*. When a path is processed, the modules  
 413 it includes are executed in order.

414 For our final limit plots of  $V_{\tau N}$ , we use `pyhf` [43, 44], which a python implementation of the  
 415 `HistFactory` framework [45]. `HistFactory` is a tool to build parametrized probability density  
 416 functions (p.d.fs) based on simple ROOT histograms organized in an XML file. Although  
 417 the p.d.f has a restricted form, it is able to describe various analyses based on template  
 418 histograms. The tool takes a modular approach to build complex p.d.fs from more primitive  
 419 conceptual building blocks. See Sec. 10 for more explanation about the p.d.f and the way  
 420 we use it.

#### 421 4. ANALYSIS METHOD OVERVIEW

422 This analysis probes directly  $V_{\tau N}$ , the coupling of the HNL and  $\tau$  lepton. It is done by  
 423 searching for HNL production via  $\tau^- \rightarrow N\pi^-$  following the method of Ref. [33].

424 This production mechanism implies  $m_N < m_\tau - m_\pi$ . Further restricting ourselves to the  
 425 scenario in which the HNL mixings with the  $\nu_e$  and  $\nu_\mu$  are negligible, we see that the HNL  
 426 can decay only via the weak neutral current to  $Z^*\nu_\tau$ . In this analysis we consider only the  
 427  $\mu^+\mu^-$  final state of the  $Z^*$ . The branching fractions  $Br(\tau^- \rightarrow N\pi^-)$ ,  $Br(N \rightarrow \mu^+\mu^-\nu_\tau)$ ,  
 428 and the HNL lifetime are taken from Ref. [46], and they are tabulated in Appendix C

429 We reconstruct  $e^+e^- \rightarrow \tau_{tag}^+\tau_{sig}^-$  in which the  $\tau_{tag}^+$  undergoes a 1-prong decay. The signal  
 430 decay is  $\tau_{sig}^- \rightarrow \pi^-N$  followed by  $N \rightarrow \mu^+\mu^-\nu_\tau$ . The Feynman diagram corresponding to  
 431 the signal  $\tau$  decay is shown in Fig. 10.

432 Due to Eq. 14 and 15,  $V_{\tau N}$  impacts both the number of signal events produced and the  
 433 lifetime. Therefore, the range of  $V_{\tau N}$  that the analysis is sensitive to is governed by both of  
 434 these properties. It so happens that this range is such that the HNL is long lived, as will be  
 435 shown later. Because HNL lifetime, the  $\mu^+\mu^-$  form a displaced vertex (DV). To suppress  
 436 background from promptly produced tracks,  $K_S$  and  $\Lambda$  decays, and particle interactions in  
 437 dense material, the radial position  $r_{DV}$  of the DV is required to satisfy  $r_{DV} > 15cm$ . This  
 438 tight cut implies that the analysis focuses on small values of the squared mixing parameter  
 439  $|V_{\tau N}|^2$  between the HNL and the SM neutrino.

440 The decay chain cannot be fully reconstructed, due to the unobservable neutrino in the fi-  
 441 nal state. There are 12 unknowns, namely, the 4-momenta  $p_{\nu_\tau}^\mu, p_N^\mu, p_\tau^\mu$  of the unreconstructed  
 442  $\nu_\tau$ ,  $N$  and  $\tau$  respectively. However, the decay chain has 12 constraints: 4-momentum con-  
 443 servation in the  $\tau$  and  $N$  decays (8 constraints), the known masses of the  $\tau$  and the  $\nu_\tau$  (2  
 444 constraints), and the unit vector from the production point of the  $\pi$  system to that of the  
 445  $\mu^+\mu^-$  system, which is the direction of  $\vec{p}_N$  (2 constraints). Solving the constraint equations,  
 446 one determines the 4-momenta of all the particles up to a two-fold ambiguity arising from a  
 447 quadratic equation. [33]. The calculation is shown in detail in Appendix A.

448 The resulting two HNL-mass solutions are referred to as  $m_+$  and  $m_-$ , depending on the  
 449 sign in front of the term  $\sqrt{A_{sq}}$ , which arises from a quadratic equation. Due to the very low  
 450 background, a detailed fit of the  $m_+$  vs.  $m_-$  distribution is not needed. Rather, we use only  
 451 a course cut on these variables in one of the signal regions (see Sec. 6). Ref. [33] also suggests  
 452 exploiting the signal- $\tau$  CM-energy solutions  $E_\pm$  for additional background suppression. As  
 453 the plots presented below demonstrate, given the low background there is no particular  
 454 advantage in using  $E_\pm$ .

455 After event selection, the final statistical analysis is performed with `pyhf` in 2 signal  
 456 regions, SRHeavy and SRLight which target light and heavy HNLs in low-background large-  
 457 radius region of the detector. The fit background model is obtained from MC, and data  
 458 control regions (CRs) are used in the fit to determine the background level in the SRs.

459 A comparison between the  $V_{\tau N}$  bound set by DELPHI, to the expected  $V_{\tau N}$  bound from  
 460 Belle (and other future experiments), using the method offered by Ref. [33], is presented in  
 461 Fig. 11.

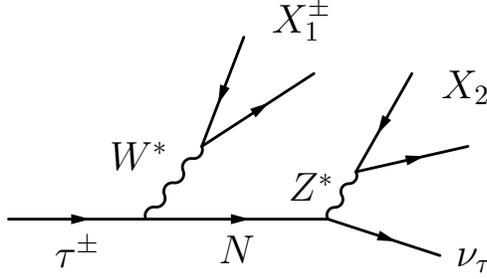


FIG. 10: The decay chain searched for in this analysis, with  $X_1 = \pi^\pm$  and  $X_2 = \mu^+\mu^-$ .

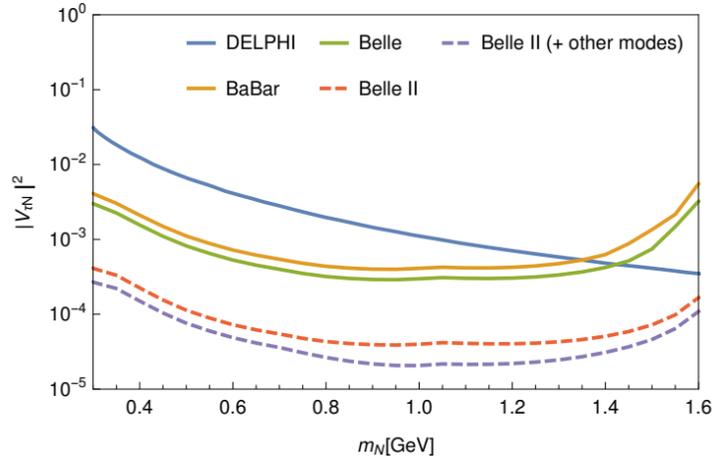


FIG. 11: Expected 95% confidence-level limits on the coupling  $V_{\tau N}$  vs  $m_N$ , obtainable from the decay chain described in Fig. 10 with  $X_1 = \pi^\pm(\pi^0)$  and  $X_2 = \ell^+\ell^-$ , between the different experiments. Also shown is the potential impact of adding the modes  $X_1 = \ell\nu$  and  $X_2 = \pi^+\pi^-$

## 462 5. DATA AND MONTE-CARLO SAMPLES

### 463 5.1. Data samples

464 The analysis uses data taken from the period of time during the Belle experiment opera-  
 465 tion. The events of the above data-set passes tau\_skinB, which is a collection of high-level  
 466 analysis scripts that reduce the data set to a manageable size by applying a simple selection.  
 467 The exact conditions are presented in in Table III. The data samples have an integrated  
 468 luminosity of 702.623/89.454 fb<sup>-1</sup> for events on/off-resonance  $\Upsilon(4S)$ , and 121.061/1.73 fb<sup>-1</sup>  
 469 for events on/off-resonance  $\Upsilon(5S)$ , respectively. Currently, the data is still blinded, so the  
 470 data analysis has not been done yet, but only the MC. An internal belle review committee  
 471 has been formed for examining the analysis and giving an approval for unblinding of the  
 472 data.

Index	Tauskim criteria
1	$2 < \text{Number of charged tracks} < 8$
2	Sum of charge: $ q_{\text{sum}}  \leq 2$
3	Maximum $p_T$ of charged track, $p_T^{\text{max}} > 0.5 \text{ GeV}$
4	Event vertex: $ dr  < 1.0 \text{ cm}$ and $ dz  < 3.0 \text{ cm}$
5	For two-track events, 5-a and 5-b must be satisfied.
5-a	$E_{\text{ECL}} < 11 \text{ GeV}$
5-b	$5^\circ < \theta_{\text{miss}} < 175^\circ$
6	$E_{\text{rec}} > 3 \text{ GeV}$ or $p_T^{\text{max}} > 1 \text{ GeV}$
7	For 2-4 track events, 7-a and 7-b must be satisfied.
7-a	$E_{\text{tot}} < 9 \text{ GeV}$ or $\theta^{\text{max}} < 175^\circ$ or $2 < E_{\text{ECL}} < 10 \text{ GeV}$
7-b	$N_{\text{barrel}} \geq 2$ or $E_{\text{ECL}}^{\text{trk}} < 5.3 \text{ GeV}$

TABLE III: Selection criteria included in tauskim package;  $dr$  and  $dz$  are defined in Sec. 6.6.1,  $E_{\text{ECL}}$  is the total energy measured in the ECL,  $\theta_{\text{miss}}$  is the polar angle of the missing momentum,  $E_{\text{rec}}$  is the sum of the total momenta of good charged tracks and gamma energy in CM frame,  $E_{\text{tot}}$  is the sum of  $E_{\text{rec}}$  and the missing momentum in CM frame;  $N_{\text{barrel}}$  is the number of tracks in the barrel region.

## 473 5.2. MC samples

474 To study the effect of the selection criteria and the performance of the fits, Monte Carlo  
475 (MC) samples are used. These MC simulate the production and decay of variety of particles  
476 and their interactions with the matter inside the detector. These MC samples are divided  
477 into two types: generic MC and signal MC. The goal of the generic MC is to simulate the  
478 background process as best as we can, and it is used to study the background. We use  
479 run-dependent generic Monte-Carlo samples which pass the tau skim criteria and trigger  
480 simulation. Belle's MC is divided into streams, where each stream of simulated events  
481 corresponds to the number of events in the recorded data sample for a given decay type.  
482 The details of the samples used are summarized in Table IV

483  
484 In signal simulates the signal we are searching for. We generate experiment-dependent  
485 signal  $\tau_{\text{sig}}^- \rightarrow \pi^- N (\rightarrow \mu^+ \mu^- \nu_\tau)$  MC using KKMC [48] and PYTHIA [49] event generators. KKMC  
486 is used for generating the process  $e^+ e^- \rightarrow \tau_{\text{tag}}^+ \tau_{\text{sig}}^-$  (including initial/final stater radiation).  
487 PYTHIA is used for the generating the  $\tau$  and HNL decays. The samples are generated in  
488 the BASF2 framework and the detector simulation is performed in the BASF framework.  
489 Table V lists the values of the  $N$  mass  $m_N$  and lifetime  $c\tau$  and the number of events  
490 generated for each sample. The lifetime values are chosen so as to yield a reasonably large  
491 number of events in the fiducial volume of the analysis (see Sec. 6) to enable high-statistics  
492 determination of the signal efficiency (see Sec. 7).

Process (production of $e^+e^-$ collision)	$\Upsilon(4S)$ Lumi ( $\text{fb}^{-1}$ ) On/Off	$\Upsilon(5S)$ Lumi ( $\text{fb}^{-1}$ ) On/Off	Streams
$\tau^+\tau^-$	702.623/89.454	121.061/1.73	5
$\mu^+\mu^-$	702.623/89.454	121.061/1.73	5
Bhabha	51.924/5.085	0/0	5
$e^+e^-e^+e^-$	455.294/58.121	0/0	3
$e^+e^-\mu^+\mu^-$	455.294/58.121	0/0	3
$eeqq(q = u, s, c)$	62.615/72.806	0/0	5
uds	702.623/89.454	121.061/1.73	5
charm	702.623/89.454	121.061/1.73	5
$B^+B^-$	702.623/00.000	121.061/00.00	5
$B^0\bar{B}^0$	702.623/00.000	121.061/00.00	5

TABLE IV: Generic MC samples for different processes and their corresponding luminosities per stream used in this analysis. The numbers of streams used for different processes are summarized as well. [\[47\]](#)

$m_N$ (GeV)	Generated events	$c\tau$ (cm)
0.3	1479720	15.0
0.4	1447884	15.0
0.5	1398591	15.0
0.6	1457528	22.5
0.7	1470659	22.5
0.8	1468197	22.5
0.9	1464336	22.5
1.0	1460280	22.5
1.1	1453751	30.0
1.2	1447480	30.0
1.3	1437136	30.0
1.4	1422052	30.0
1.5	1398034	30.0
1.6	1369152	30.0

TABLE V: Signal MC samples generated for different masses and lifetimes of  $N$ . The differences between the numbers of events are due to some crashed jobs.

## 493 6. EVENT SELECTION

494 We have online selection used to produce ntuples for offline processing, where events are  
495 further selected using the "offline" selection.

## 496 6.1. Online event selection

497 The application of the cuts is divided into 3 different steps: primary cuts, vertex-fitting  
498 and final cuts.

### 499 6.1.1. Primary online selection criteria

500 The selection of primary tracks and neutral particles through the online selection criteria  
501 are designed for rejecting most of the non- $\tau\tau$  backgrounds. We look for a topology in which  
502 one track is roughly back-to-back with 3 additional tracks, as was discussed in Sec. 4. The  
503 former is the "tag-side" (related to  $\tau_{tag}$ ) and the last is the "signal-side" (related to  $\tau_{sig}$ ).

504 Our primary selection criteria are described as follows:

- 505 • Track-quality selection: All tracks must have at least 20 CDC hits. In addition, we  
506 want to assure that the signal-side pion and the tag-side track really originated from  
507 the two taus, i.e. the tracks are close enough to the IP. For this goal, we use the  
508 variables  $|dr|$  and  $|dz|$ , which are defined as the distance between the IP and point of  
509 closest approach (POCA) to the IP of the tracks, in the  $r$  and  $z$  axis correspondingly.  
510 We select only events that satisfy  $|dr| < 0.5$  cm,  $|dz| < 2.0$  cm.
- 511 • Signal side muon selection: the muon ID has to be  $muID > 0.5$ .
- 512 • Signal side pion selection: no particle-ID cut.
- 513 • Tag side 1-prong: no particle-ID cut.

### 514 6.1.2. Vertex-fitting

515 The  $m^+$  and  $m^-$  daughters of signal HNL candidates that satisfy the above cuts, are  
516 vertex-fit with the `treeFitter` algorithm [50-52] to produce the DV. Vertex fitting is a  
517 technique in which one uses prior knowledge on the nature of a decay, namely, that the two  
518 muons are supposed to originate from the same point. This, in order to find the DV, so we  
519 can use it to select signal.

520 `treeFitter` is the standard `basf2` module for fitting a full decay chain simultaneously.  
521 It performs a progressive fit, using a Kalman filter algorithm [53]. This algorithm input  
522 is the measured 4-momenta of final states particles, and their masses which is given as a  
523 constraint. The output of `treeFitter` gives an optimised fit with a  $\chi^2$  minimisation for the  
524 4-momentum and the position of the vertex. [41]

525 The tracks are combined to reconstruct long-lived particle ( $N$ ) with various final states as  
526 mentioned above. The  $N$  is combined with one pion to reconstruct the signal-side  $\tau$  lepton.  
527 The tag-side  $\tau$  lepton is reconstructed from a single charged lepton without applying any  
528 particle identification criteria.

530 We define several selection regions with different purposes:

- 531 • The event selection for signal regions SRHeavy and SRLight is intended to suppress  
532 the reducible backgrounds while maintaining a high signal efficiency. The difference  
533 between the two regions is that SRHeavy targets heavy HNLs and SRLight targets light  
534 HNLs. The control regions CRHeavy (target heavy HNL) and CRLight (targets light  
535 HNL) are used to determine the background in SRHeavy and SRLight, respectively.  
536 The main feature of the CRs is that one of the tracks in the DV is required to be a  
537 muon and the other is required to not be a lepton (electron or muon), so that it is  
538 almost always a pion. This choice is motivated by the presence of backgrounds from  
539  $\tau \rightarrow K_L \pi \nu$ ,  $\tau \rightarrow K_L \pi \nu$  and  $\tau \rightarrow 3 \pi \nu$ , with 1 or 2 pions undergoing decay in flight  
540 to a muon or a hard scatter and mis-identification as a muon, as well as pions from  
541  $e^+ e^- \rightarrow q \bar{q}$  events. This is in contrast to the 2-muon DV selection in the SRs.
- 542 • We have 5 validation regions (VRs) which are used for data-MC comparison:
  - 543 – The selection for the same-charge validation regions VRHeavySameSign and  
544 VRLightSameSign is identical to those of SRHeavy and SRLight, except that  
545 the two DV tracks are required to have the same electrical charge. (2 VRs)
  - 546 – The VRHeavy $\pi\pi$  and VRLight $\pi\pi$  in which we require both DV daughters to  
547 be non-leptons (so that they are almost always pions), but outside the  $K_S$  mass  
548 region. (2 VRs)
  - 549 – The  $K_S$  validation region VR $K_S$  contains  $\tau \rightarrow \pi K_S$ ,  $K_S \rightarrow \pi^+ \pi^-$  decays. (1 VR)

550 After the online selections, we have the reconstructed vertices of the decay chain, so  
551 additional cuts can be applied on their parameters. This cuts are the following:

- 552 • The radial position of the DV must satisfy  $r_{DV} > 5$  cm. This reduces the sample size  
553 with a cut that is much looser than the one applied offline.
- 554 • The invariant mass  $m(\pi + DV)$  of the prompt pion plus the DV must be smaller  
555 than the  $\tau$  mass  $m_\tau = 1.776$  GeV, to quickly reject obviously irrelevant candidates (we  
556 search in  $\tau$  decays).
- 557 • The event is divided into two hemispheres centered on the event thrust axis [54],  
558 calculated with the observed tracks and photons. The three signal-side tracks are  
559 required to be in one hemisphere, and the tag-side single track is required to be in the  
560 other hemisphere. This is a standard 1-3 prong selection cut.

## 561 6.2. Offline signal region selection

562 Events that satisfy the online cuts are required to satisfy also the following offline cuts:

- 563 0.  $N_{\text{tracks}}$ : The number of tracks in the event must be  $N_{\text{tracks}} = 4$ . This strongly  
564 suppresses hadronic ( $q\bar{q}$  and  $B\bar{B}$ ) background.

- 565 1.  **$r_{\text{DV}}$  cut:** The decay position of the HNL in the xy plane needs to satisfy  $r_{\text{DV}} > 15$  cm.  
566 This suppresses background from prompt particles (particularly those that undergo  
567 large-angle multiple scattering in dense material or decay in flight), as well as  $K_S$  and  
568  $\Lambda$  decays.
- 569 2.  **$P(\chi^2)$  cut:** The  $\chi^2$  probability of the DV fit is required to be  $P(\chi^2) > 0.00001$ . This  
570 ensures consistency of the DV fit.
- 571 3.  **$\cos \theta_{\mu^+\mu^-}$  cut:** Cosine of the angle between the 2 muons must be  $\cos \theta_{\mu^+\mu^-} > 0.5$ .  
572 This selects tracks consistent with originating from a boosted parent.
- 573 4. **Prompt  $\pi$   $L_{\text{ID}}$  cut:** The muon ID and electron ID of the prompt  $\pi$  are both required  
574 to be less than 0.01. This ensures pion selection and rejects background from QED  
575 events, particularly 4-lepton final states.
- 576 5. **CDC Hits min cut:** Each HNL-daughter muon needs to satisfy  $r_{\text{hit}} - r_{\text{DV}} > -2$  cm,  
577 where  $r_{\text{hit}}$  is the radial position of the lowest-radius CDC hit on the track. This very  
578 loose cut rejects prompt tracks, for which there are hits at a smaller radius than  $r_{\text{DV}}$ .
- 579 6.  **$\mu_{\text{ID}}$  cut 1:** The muon ID of at least one HNL-daughter muon must be greater than  
580 0.9.
- 581 7.  **$K_S^0$  veto:** The DV mass calculated with the pion mass hypothesis for the two DV  
582 daughters, is not in the range of  $0.42 < m_{\pi\pi}^{\text{DV}} < 0.52$  GeV. This suppresses  $K_S^0$  back-  
583 ground, which is also a long-lived neutral particle, which can produce a DV similarly  
584 to the HNL.
- 585 8.  **$\sum E_\gamma$  cut:** The total lab-frame energy of photons in the event must be less than 1  
586 GeV, in order to reduce  $q\bar{q}$  background ( $\tau$  decays usually do not include high energy  
587 photons).
- 588 9.  **$E_{\pi^0}$  cut:** The lab-frame energy of any  $\pi^0$  in the signal hemisphere must be less than  
589 0.1 GeV. This suppresses background from, e.g.,  $\tau \rightarrow \pi\pi^0 K_S \nu$ , given that we are not  
590 searching for  $\tau \rightarrow \pi\pi^0 N$  signal.
- 591 10.  **$A_{\text{sq}}$  cut:** The argument  $A_{\text{sq}}$  of the square root in the HNL mass calculation Eq. A16  
592 is required to be  $A_{\text{sq}} < 0.4 \text{ GeV}$ . This takes advantage of the fact that background  
593 tends to have larger values of  $A_{\text{sq}}$ .
- 594 11.  **$\mu_{\text{ID}}$  (N) cut 2:** Muon ID of both muons (the two HNL daughters) must be greater  
595 than 0.9.
- 596 12. **Final  $m_{\pi\pi}^{\text{DV}}$  and  $m_\pm$  cuts:** these cuts define two signal regions that target heavy  
597 and light HNLs:
- 598 • Signal region **SRHeavy**:  $m_{\pi\pi}^{\text{DV}} > 0.52$  GeV.  
599 This cut is efficient for heavy HNLs and rejects light HNLs. E.g., it is obvious  
600 that an HNL with  $m_N < 0.52$  GeV cannot satisfy this cut.
  - 601 • Signal region **SRLight**:  
602 –  $m_{\pi\pi}^{\text{DV}} < 0.42$  GeV.  
603 This cut is efficient for light HNLs.

604                   –  $\mathbf{m}_+, \mathbf{m}_-$  cut: either  $m_+ < 0.9$  GeV or  $m_- < 0.6$  GeV. This takes advantage  
605                   of the fact that background events that satisfy  $m_{\pi\pi}^{DV} < 0.42$  GeV tend to  
606                   have high values of  $m_{\pm}$ , in contrast with signal.

607       The distributions of the cut variables for signal and for the  $\tau\tau$  and  $q\bar{q}$  backgrounds (which  
608       are by far the dominant background sources) in MC are shown in Figs. [12](#), [13](#), and [14](#). Each  
609       variable is presented before the associated cut is applied on the samples. All the distributions  
610       are normalized to the same area in order to compare them properly.

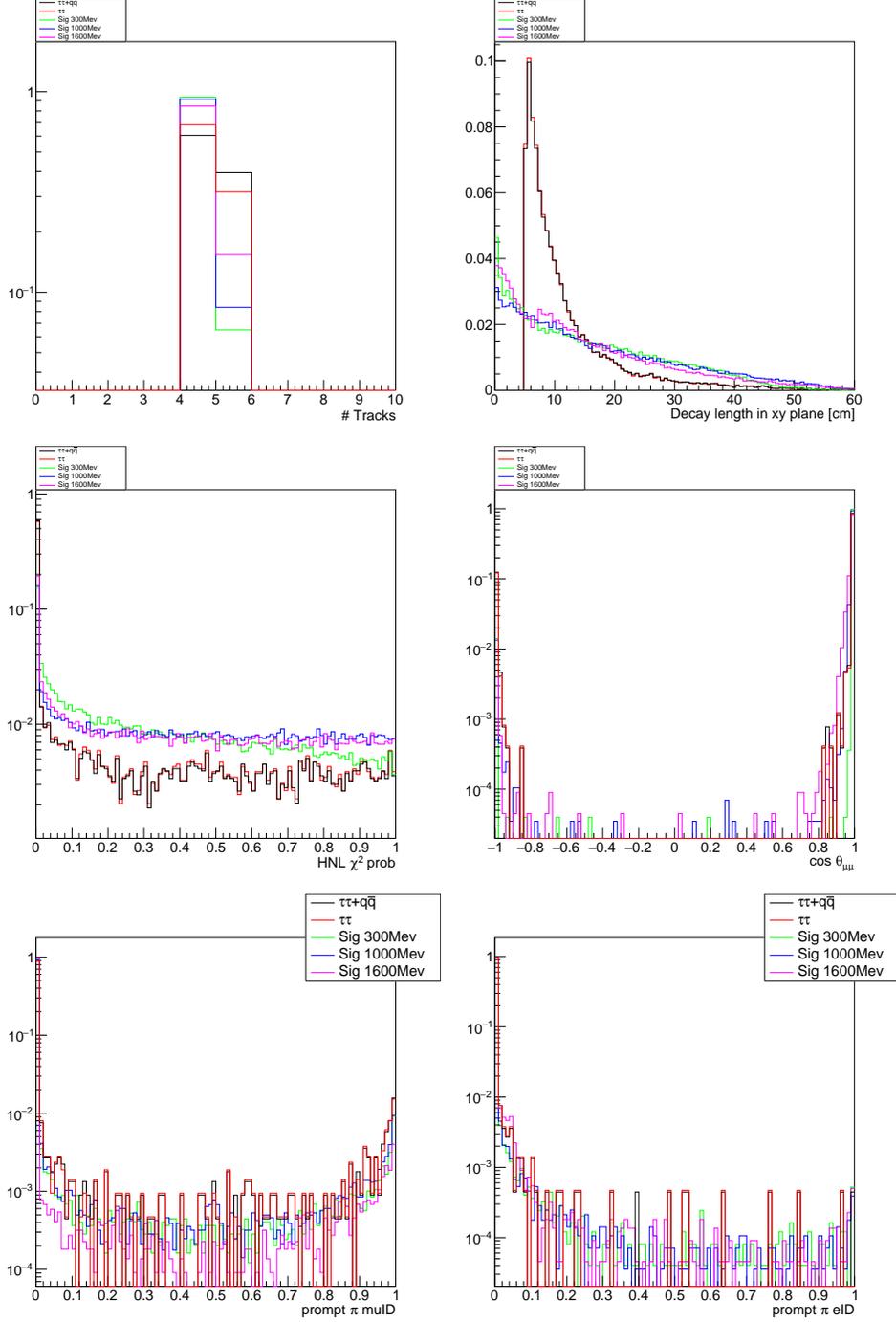


FIG. 12: Signal- and background-MC distributions for the number of tracks for 5 different samples (top left),  $r_{DV}$  after the  $N_{tracks}$  cut (top right), the  $\chi^2$  probability of the DV after the  $r_{DV}$  cut (middle left), cosine of the angle between the 2 muons after the  $P(\chi^2)$  cut (middle right),  $\mu$ -ID of prompt pion (bottom left),  $e$ -ID of prompt pion (bottom right); all histograms are normalized to the same area.

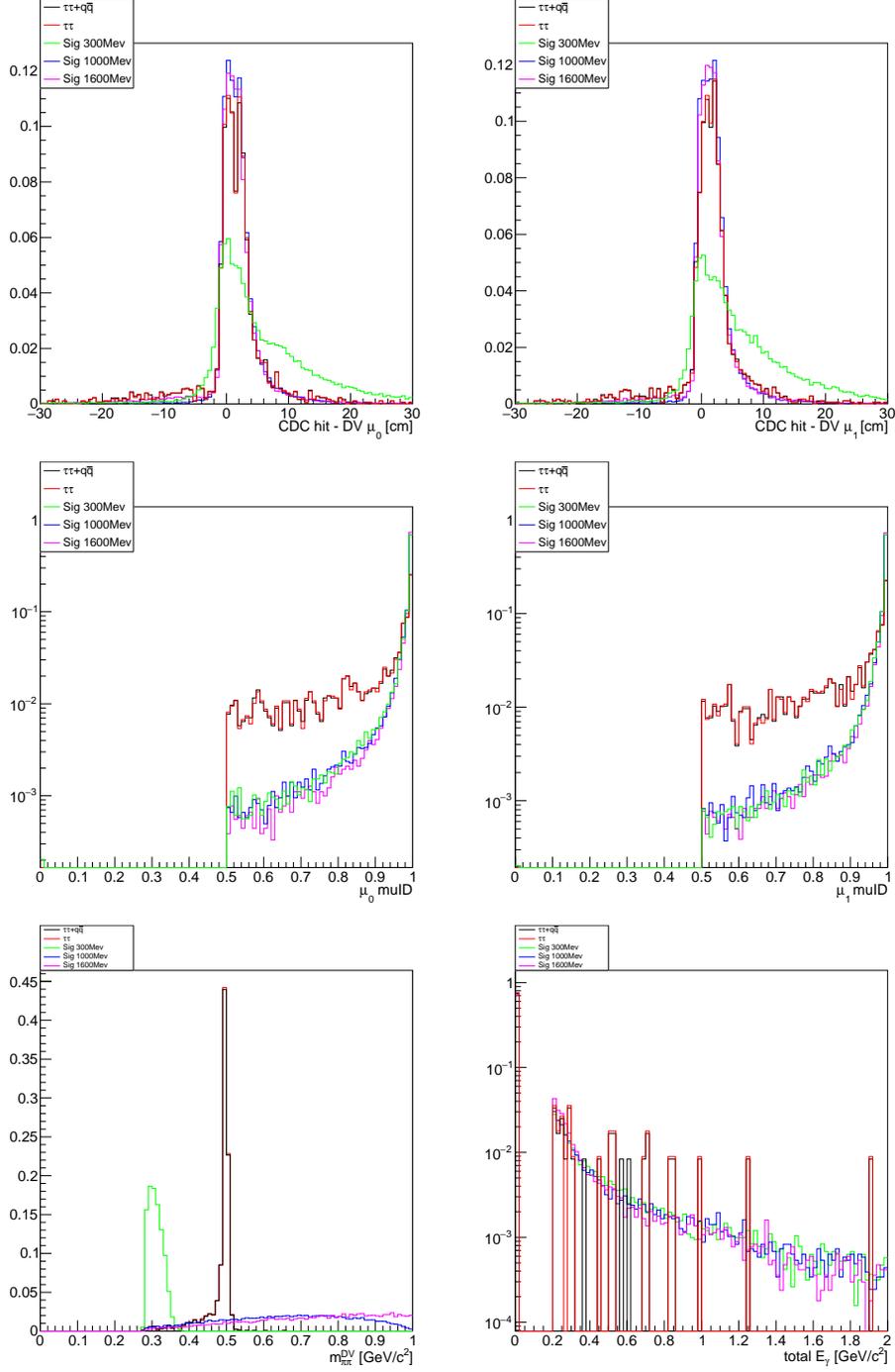


FIG. 13: Signal- and background-MC distributions for the difference between CDC hit position of  $\mu_0$  and HNL DV (top left), Difference between CDC hit position of  $\mu_1$  and HNL DV (top right),  $\mu$ -ID of  $\mu_0$  (middle left),  $\mu$ -ID of  $\mu_1$  (middle right), invariant mass of HNL daughters with pion mass hypothesis applied (bottom left),  $\sum E_\gamma$  distribution (bottom right); all histograms are normalized to the same area.

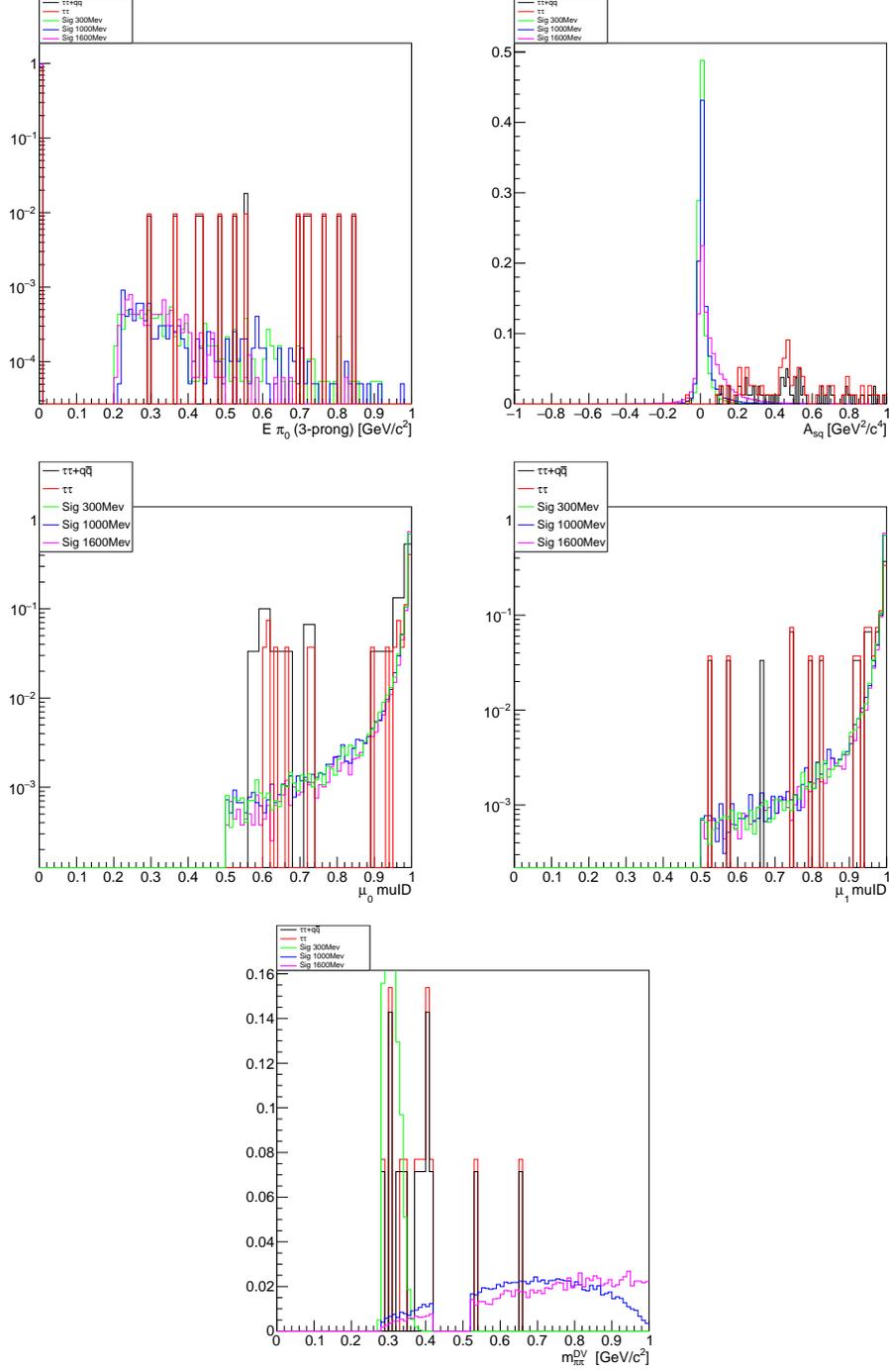


FIG. 14: Signal- and background-MC distributions for the  $E_{\pi^0}$  distribution (top left),  $A_{sq}$  distribution (top right),  $\mu$ -ID of  $\mu_0$  (middle left),  $\mu$ -ID of  $\mu_1$  (middle right), invariant mass of HNL daughters with pion mass hypothesis applied (bottom left); all histograms are normalized to the same area.

611 The number of MC events that pass each cut are shown in the cut-flow Table [VI](#) for  
 612 the generic backgrounds and for three representative signal samples, generated with HNL  
 613 masses of 300, 1000, and 1600 GeV - see Sec [5.5.2](#). For each cut, Table [VI](#) also shows the  
 614 total expected number of background events in the data and the MC-statistical uncertainty  
 615 on this number. These values constitute the background model for the pyhf fit.

616 The third-last (last) row in Table [VI](#) corresponds to the final cuts of SRHeavy and  
 617 SRLight. From these rows, we see that the expected numbers of background events in these  
 618 SRs are

$$\begin{aligned} N_{SRHeavy}^{MC} &= 0.40 \pm 0.28, \\ N_{SRLight}^{MC} &= 0.80 \pm 0.40. \end{aligned} \tag{17}$$

619 These expectations arise from the MC yields, as follows.

620 In SRHeavy we find seven  $\tau\tau$  events, and nothing else. One of these events contains the  
 621 decays

$$\tau^+ \rightarrow \mu^+ \bar{\nu}_\tau \nu_\mu, \quad \tau^- \rightarrow \nu_\tau \pi^- K^+ K^-. \tag{18}$$

622 Both the  $\pi^-$  and the  $K^+$  undergo decay in flight to muons, and the two muons form the  
 623 DV. The second event contains

$$\tau^+ \rightarrow \mu^+ \bar{\nu}_\tau \nu_\mu, \quad \tau^- \rightarrow \nu_\tau \pi^- K_S. \tag{19}$$

624 Both pion daughters of the  $K_S$  undergo decay to muons, and the two muons form the DV.  
 625 In SRLight we find 2  $\tau\tau$  events and 2  $q\bar{q}$  events. The  $\tau\tau$  events have the decays

$$\begin{aligned} \tau^+ &\rightarrow \bar{\nu}_\tau K_L \pi^+, \quad \tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau, \\ \tau^+ &\rightarrow \bar{\nu}_\tau \pi^+ K_L, \quad \tau^- \rightarrow \nu_\tau \rho^-. \end{aligned} \tag{20}$$

626 In both events the  $K_L$  decays to  $\mu^\pm \pi^\mp \nu_\mu$ , and the pion and muon form the DV.

627 The first  $\bar{q}q$  event is a  $s\bar{s}$  event with the final state  $K_L, K_S, \pi^+ \pi^- \pi^0$ . The DV is formed  
 628 from a pion produced in the  $K_S$  decay and from a muon produced in a pion decay.

629 The second  $\bar{q}q$  event is a  $u\bar{u}$  event with the final state  $K_L, K^-, \pi^+ \pi^0$ . The DV is formed  
 630 from two muons, one produced in the  $K_L$  decay and the other produced in a pion decay.

Sample	$ee$	$\mu\mu$	$\tau\tau$	$4\ell$	$eeq\bar{q}$	$q\bar{q}$	$B\bar{B}$	MC	$\sigma$ MC	Sig300	Sig1000	Sig1600
<b>Online</b>	1	341	29554	431	194	6793	866	7665	40	69546	73171	72619
$N_{\text{tracks}}$	1	42	20202	101	122	1524	75	4433	30	64803	66760	61105
$DL > 15cm$	0	17	4884	31	33	531	24	1108	15	28959	32125	25448
$\chi^2_{prob} > 10^{-5}$	0	12	2425	8	9	186	5	530	10	27550	28795	22359
$\cos\theta_{\mu+\mu^-}$	0	12	2118	7	8	158	1	462	10	27171	28378	22124
$L_{ID}$ prompt $\pi$	0	0	1817	6	2	135	1	392	9	24864	25560	20551
CDC Hits min	0	0	1483	4	0	97	1	317	8	21673	24359	18178
$\mu_{ID}$ (N) cut 1	0	0	1208	3	0	81	0	258	7	21575	24258	18117
$K_S^0$ exclusion	0	0	116	0	0	13	0	25.8	2.3	21575	21050	17252
$\sum E_\gamma$	0	0	110	0	0	10	0	24.0	2.2	20282	19860	16409
$E_{\pi^0}$	0	0	91	0	0	8	0	19.8	2.0	20038	19622	16240
$A_{sq}$	0	0	27	0	0	4	0	6.2	1.1	20037	19548	16028
$\mu_{ID}$ (N) cut 2	0	0	13	0	0	2	0	3.00	0.77	17695	17239	14455
$m_{\pi\pi}^{DV} > 0.52GeV$	0	0	2	0	0	0	0	0.40	0.28	0	15234	13948
$m_{\pi\pi}^{DV} < 0.42GeV$	0	0	11	0	0	2	0	2.60	0.72	17695	2005	507
$m_+, m_-$	0	0	2	0	0	2	0	0.80	0.40	17681	251	0

TABLE VI: Cut flow table for the SRHeavy and SRLight signal regions, showing the event yield at each stage of the selection for the generic MC samples and selected signal samples with the HNL mass indicat. MC is the total number of events expected in the data sample given the MC yields, and  $\sigma$ MC is its uncertainty, arising from MC statistics only.

631 Fig. 15 shows the distributions of  $m_-$  vs.  $m_+$ ,  $E_-$  vs.  $E_+$ , and  $A_{sq}$  for  $\tau^+\tau^-$  background  
632 MC events in the SRs (after the final cuts). While the variables  $E_-$  and  $E_+$  are not used in  
633 our selection, we show them here since they are suggested in Ref. 33. In practice, after the  
634 other cuts, cutting on them is not worthwhile.

635 Fig. 16 shows the same variables for the  $q\bar{q}$  background. Figs. 17, 18, and 19 show these  
636 distributions for the 300 MeV, 1000 MeV, and 1600 MeV signal samples. Based on these  
637 distributions, we chose to apply the cuts on  $A_{sq}$  and on  $m_\pm$  but not on  $E_\pm$ .

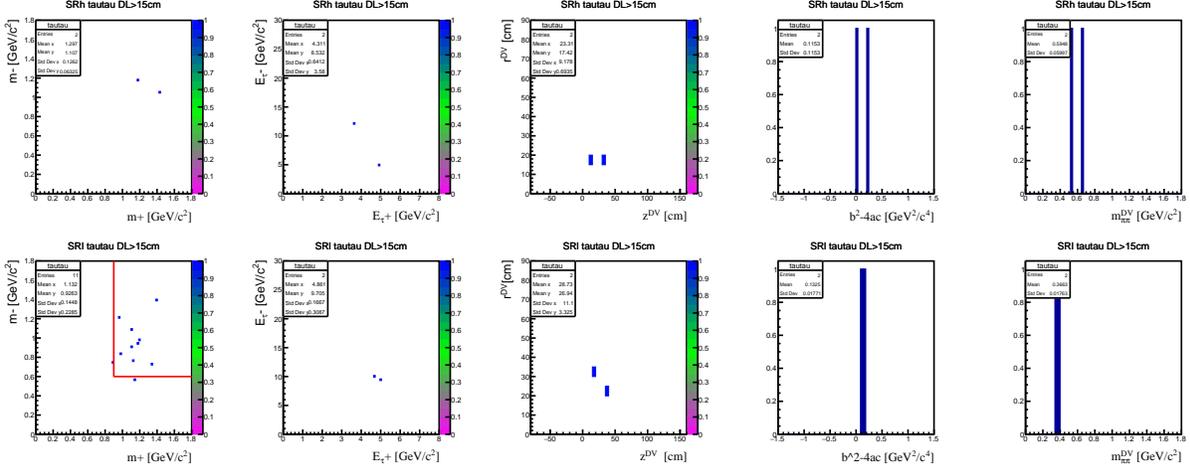


FIG. 15: Distributions of  $m_-$  vs.  $m_+$ ,  $E_{\tau^-}$  vs.  $E_{\tau^+}$ ,  $A_{sq}$ ,  $r_{DV}$  vs.  $z_{DV}$  and  $m_{\pi\pi}^{DV}$  for  $\tau^+\tau^-$  background MC events in (top plots) SRHeavy and (bottom plots) SRLight. The red square on the  $m_-$  vs.  $m_+$  plot shows the cut that either  $m_+ < 0.9$  GeV or  $m_- < 0.6$  GeV. This takes advantage of the fact that background events that satisfy  $m_{\pi\pi}^{DV} < 0.42$  GeV tend to have high values of  $m_{\pm}$ , in contrast with signal.

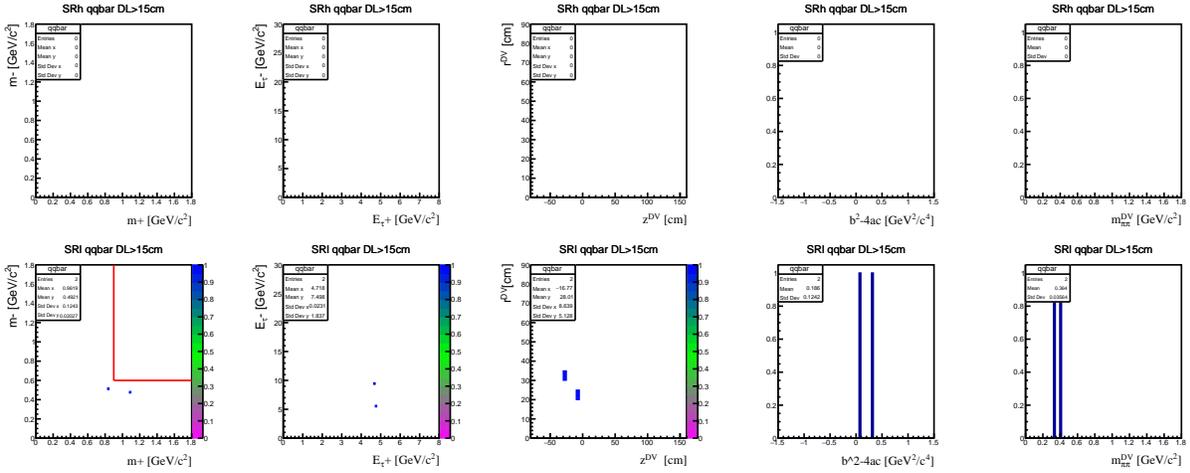


FIG. 16: Same as Fig. 15 for the  $q\bar{q}$  background MC.

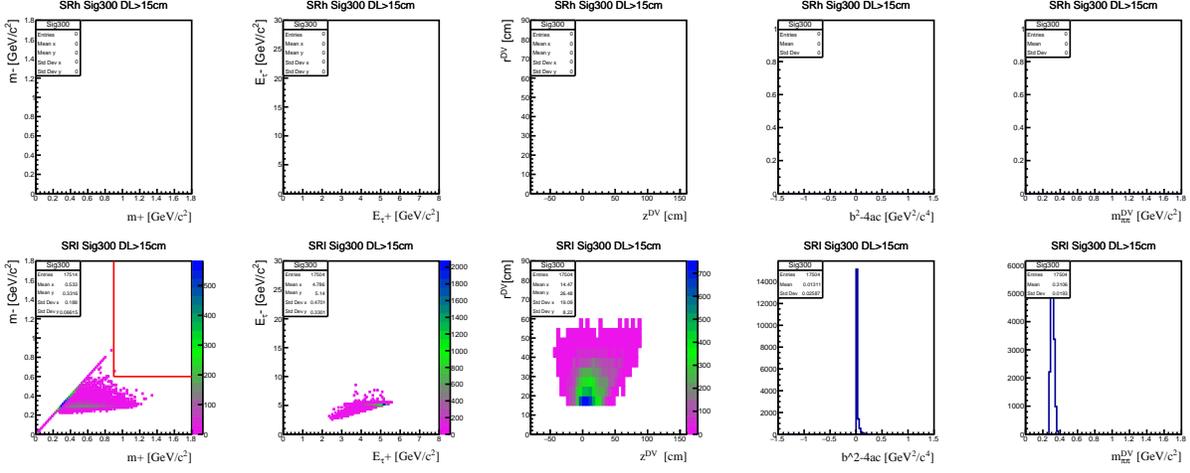


FIG. 17: Same as Fig. 15 for the  $m_N = 300$  MeV signal MC events.

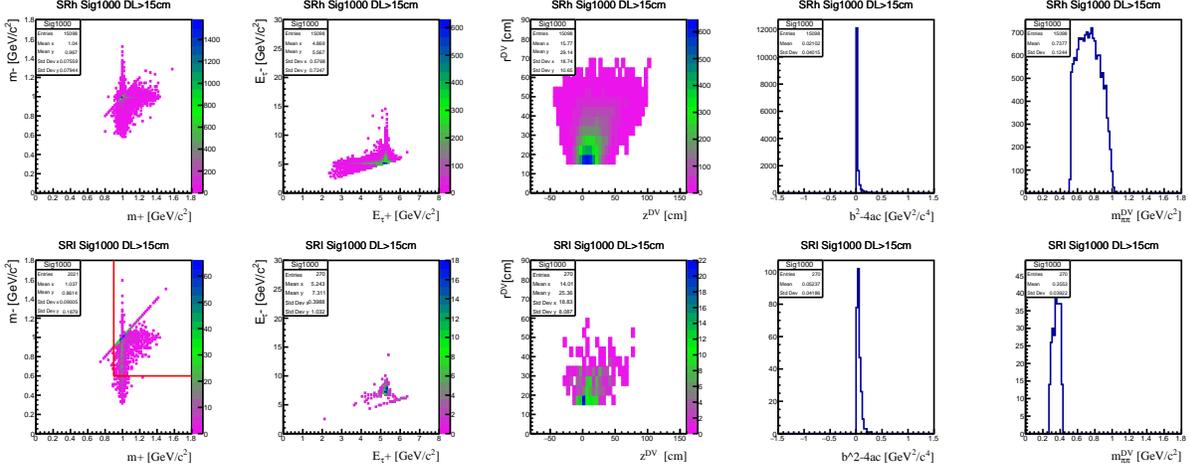


FIG. 18: Same as Fig. 15 for the  $m_N = 1000$  MeV signal MC events.

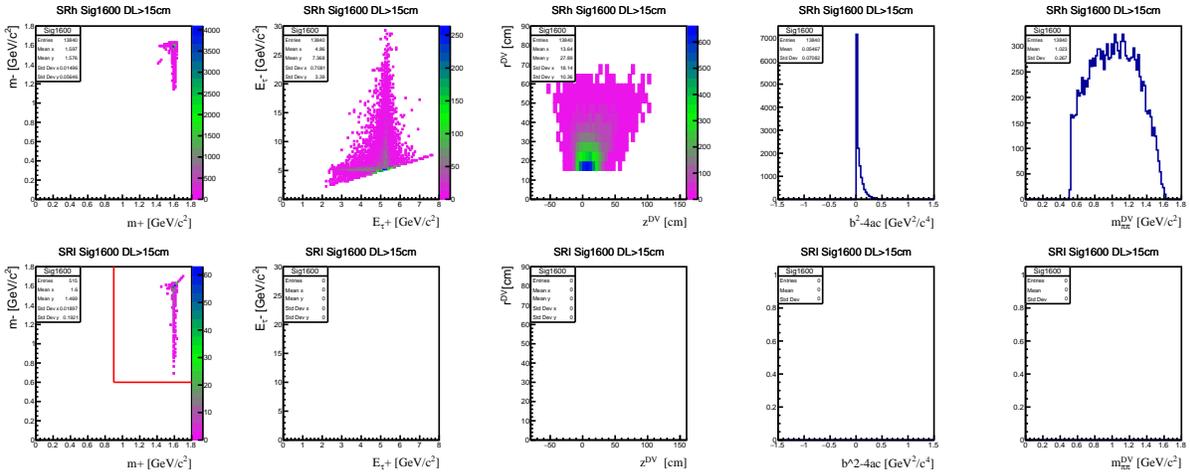


FIG. 19: Same as Fig. 15 for the  $m_N = 1600$  MeV signal MC events.

638 **6.3. Control region selection**

639 Two control regions (CRs) are used to estimate the background in the fit described in  
 640 Sec [10](#). The two control regions, CRHeavy and CRLight are defined identically to the  
 641 signal regions SRHeavy and SRLight except that one of the DV tracks is required to be  
 642 inconsistent with a lepton. This is enforced with the requirement:

- 643
  - one of the DV daughter tracks satisfies  $\text{MuonID} < 0.01$  &&  $\text{ElectronID} < 0.01$

644 Table [VII](#) compares the cut flow for data and generic MC events where the DV daugh-  
 645 ters are a pion and a muon, culminating in the two CRs. The table gives the expected  
 646 (luminosity-scaled MC) and observed (data) CR yields

$$\begin{aligned}
 N_{CRHeavy}^{MC} &= 73.6 \pm 3.8, \\
 N_{CRHeavy}^{data} &= 95 \pm 10, \\
 N_{CRLight}^{MC} &= 37.2 \pm 2.7, \\
 N_{CRLight}^{data} &= 43 \pm 7.
 \end{aligned}
 \tag{21}$$

647 The table shows that the  $N_{\text{data}}/N_{\text{MC}}$  ratio in CRHeavy is 1.29, and the statistical consistency  
 648 of this ratio with unity is  $(N_{\text{data}} - N_{\text{MC}})/\sigma = 2.0$ , where  $\sigma$  is the total statistical uncertainty  
 649 on  $N_{\text{data}} - N_{\text{MC}}$ . For CRLight this ratio is 1.16, and the consistency is  $0.8\sigma$ .

650 We note that the  $\tau\tau$  events involve decays that are well understood, including those of  
 651 the well known, long-lived  $K_L$  and  $K_S$ . Therefore, we expect the MC to provide a robust  
 652 prediction of the  $\tau\tau$  event yields. The same is not necessarily true of the  $q\bar{q}$  simulation,  
 653 since PYTHIA is rarely tested with low-multiplicity events at this energy scale. Therefore,  
 654 discrepancy between data and MC is no surprise. However, we can compare the MC events  
 655 produced in the SRs to those in the CRs, and thus determine whether data-MC comparison  
 656 in the CR can be used to give a reasonable estimate of the background in the SR. This  
 657 comparison is carried out in what follows.

658 In the MC, CRHeavy contains 761  $\tau\tau$  events and 26  $q\bar{q}$  events. The daughters of the DV  
 659 in the  $\tau\tau$  events originate from the following processes:

- 660
  - 312 events (41%):  $K_S \rightarrow \pi^+\pi^-$  decays. This includes 70 events (9%) in which the the  
 661 DV is formed from only one of the  $K_S$  daughters.
  
 662   - 193 events: prompt kaons or pions
  
 663   - 183 events: decay in flight of prompt kaons or pions
  
 664   - 62 events:  $K_S \rightarrow \pi^+\pi^-$  with pion decay in flight
  
 665   - 11 events:  $K_L$  decays

666 The CRLight contains 101  $\tau\tau$  events and 81  $q\bar{q}$  events, consistent with the two  $\tau\tau$  and  
 667 two  $q\bar{q}$  events in SRLight . Among the 101  $\tau\tau$  events, the DV is formed from the following  
 668 processes:

- 669 • 54 events:  $K_L \rightarrow \pi\mu\bar{\nu}_\mu$
- 670 • 26 events: photon conversion
- 671 • 15 events: prompt particles
- 672 • 4 events:  $K_L \rightarrow \pi e\bar{\nu}_e$  and  $K_L \rightarrow \pi\pi\pi^0$
- 673 • 2 events:  $K_S \rightarrow \pi^+\pi^-$  with one decay in flight

674 Among the 81  $q\bar{q}$  events, the DV arises from the following processes:

- 675 • 48 events:  $\Lambda \rightarrow p\pi$ , with 6 events involving pion decay in flight
- 676 • 32 events:  $K_L \rightarrow \pi\mu\bar{\nu}_\mu$
- 677 • 1 event:  $K_S \rightarrow \pi^+\pi^-$

678 We see that in each CR, the processes that give rise to the DV are statistically consistent  
679 with those in the SR. This validates the use of the CRs for estimating the background level  
680 in the SRs.

cut	$ee$	$\mu\mu$	$\tau\tau$	$4\ell$	$eeq\bar{q}$	$q\bar{q}$	$B\bar{B}$	MC	$\sigma$ MC	data	$\sigma$ Data	$\frac{data}{MC}$	cons.
<b>Online</b>	22	15071	2097913	66660	18925	462799	51032	545074	333	403387	635	0.74	-198
$N_{\text{tracks}}$	18	4649	1138959	13690	7231	85891	3373	251780	226	199944	447	0.79	-103
<b><math>DL &gt; 15cm</math></b>	2	856	316299	4934	3039	31557	1333	72015	121	58574	242	0.81	-50
$\chi^2_{\text{prob}} > 10^{-5}$	2	259	137726	984	441	9246	193	29835	78	23059	152	0.77	-40
<b><math>L_{ID}</math> prompt <math>\pi</math></b>	2	37	121715	877	252	8170	143	26279	73	20637	144	0.79	-35
<b><math>\mu\pi_{ID}</math> (N) cut 1</b>	0	19	63926	496	178	4471	95	13861	53	11771	108	0.85	-17
$\sum E_\gamma$	0	15	59646	466	165	3808	89	12860	51	10921	105	0.85	-17
$E_{\pi^0}$	0	15	49865	366	164	2498	50	10613	46	9269	96	0.87	-13
$\cos\theta_{\mu\pi}$	0	5	41619	247	71	1920	16	8785	42	7649	87	0.87	-12
<b>CDC Hits min</b>	0	0	33688	123	0	1418	5	7047	38	5941	77	0.84	-13
<b><math>K_S^0</math> exclusion</b>	0	0	2993	25	0	278	1	659	11	670	26	1.02	0.39
$A_{\text{sq}}$	0	0	1267	10	0	172	1	290	8	290	17	1.00	0
<b><math>m_{\pi\pi}^{DV} &gt; 0.52GeV</math></b>	0	0	337	5	0	26	0	73.6	3.8	95	10	1.29	2.00
<b><math>m_{\pi\pi}^{DV} &lt; 0.42GeV</math></b>	0	0	930	5	0	146	1	216	6	195	14	0.90	-1.4
$m_+, m_-$	0	0	101	3	0	81	1	37.2	2.7	43	7	1.16	0.77

TABLE VII: Cut flow for the control regions CRHeavy and CRLight in data and generic MC. MC and  $\sigma$ MC are the total luminosity-weighted MC yield and its MC-statistical uncertainty. data and  $\sigma$ Data are the data yield and its statistical uncertainty.  $\frac{data}{MC}$  is the ratio between the data and total-MC yields, and cons. is the statistical consistency, the difference between the data and luminosity-scaled MC divided by the total statistical uncertainty. See Table VI caption for additional details

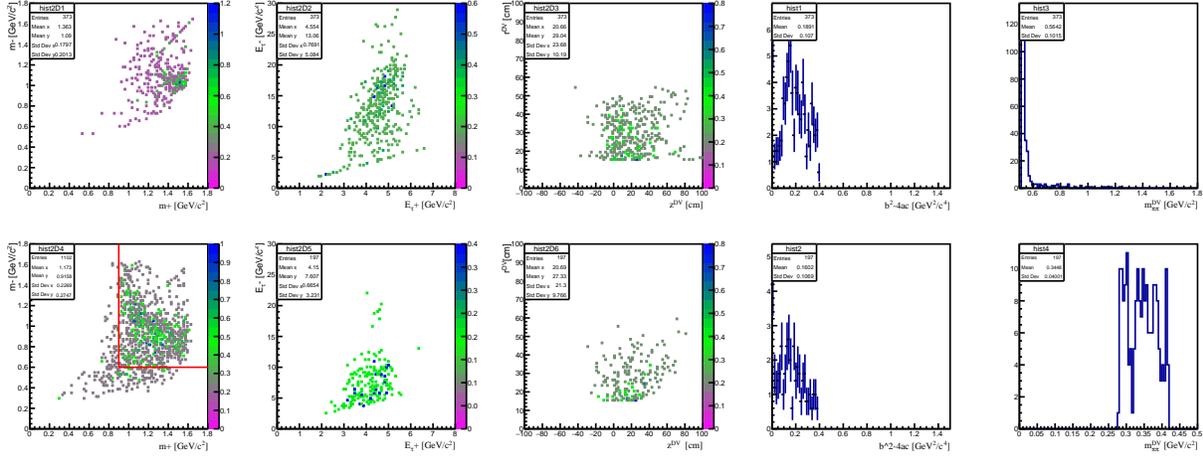


FIG. 20: Distributions of  $m_-$  vs.  $m_+$ ,  $E_{\tau^-}$  vs.  $E_{\tau^+}$ ,  $r_{DV}$  vs.  $z_{DV}$ ,  $A_{sq}$  and  $m_{\pi\pi}^{DV}$  for  $\tau^+\tau^-$  background MC events in (top plots) CRHeavy and (bottom plots) CRLight.

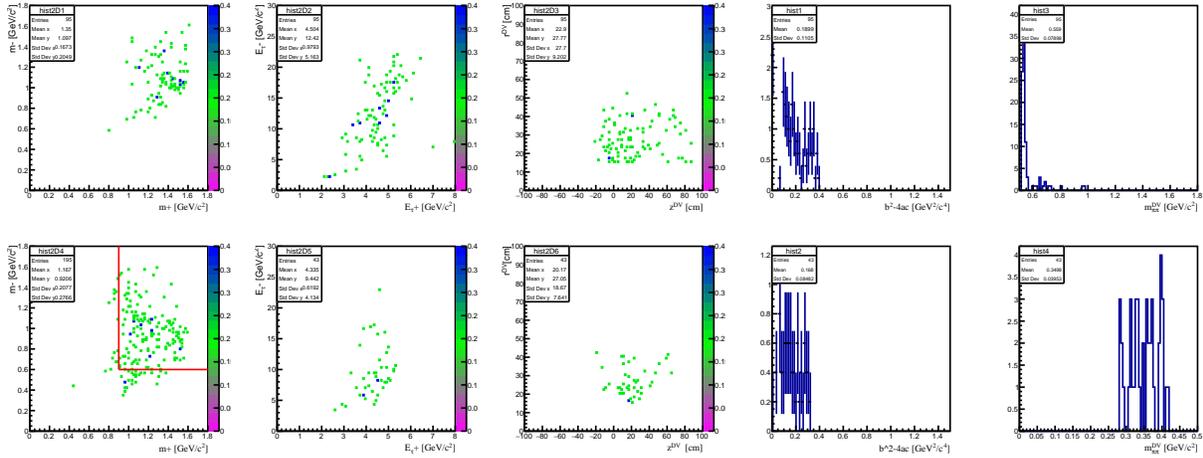


FIG. 21: Same as Fig. 20 for the background data in the CR.

#### 681 6.4. Validation region selection

682 MC-data agreement is further tested in the following validation regions (VRs), which are  
 683 not used in the final fit. The VRs come in pairs, depending on whether they satisfy the  
 684  $m_{\pi\pi}^{DV}$  and  $m_{\pm}$  cuts that correspond to SRHeavy or SRLight. For each VR, we list below the  
 685 event selection and show the cut-flow table comparing MC and data. We also compare the  
 686 data and MC distributions of several key variables.

##### 687 6.4.1. VRHeavySameSign and VRLightSameSign

688 The events for this validation region are selected in the same way as SR events, except  
 689 that

- The two muons emanating from the DV have the same electrical charge.

This targets random combinations of tracks that accidentally form a DV regardless of their charges. Due to the low multiplicity, the yields in the same-sign validation regions are expected to be somewhat smaller than those in the SRs. No same-sign events are observed in the the data. In MC, we see one  $\tau\tau$  event and one  $q\bar{q}$  event in VRHeavySameSign (corresponding to an expectation of 2/5 event in the data). The data and MC cut flows for these regions are shown in Table VIII. Plots of data and MC same-sign events are shown in Fig. 22.

cut	$ee$	$\mu\mu$	$\tau\tau$	$4\ell$	$ee\bar{q}q$	$q\bar{q}$	$B\bar{B}$	MC	$\sigma$ MC	data	$\sigma$ Data	$\frac{data}{MC}$	cons.
<b>Online</b>	4	3678	99920	1831	2250	9916	757	23983	71	18682	137	0.78	-34
$N_{tracks}$	3	825	39285	580	1646	1287	61	8966	44	8343	91	0.93	-6
$DL > 15cm$	0	345	15843	488	1342	747	23	3937	29	4199	65	1.07	3.7
$\chi_{prob}^2 > 10^{-5}$	0	104	3368	146	595	154	6	954	15	1116	33	1.17	4.5
$cos\theta_{\mu\mu}$	0	52	1725	44	157	64	3	430	10	460	21	1.07	1.3
$L_{ID}$ prompt $\pi$	0	5	1544	39	36	53	2	341	8	348	19	1.02	0.3
<b>CDC Hits min</b>	0	1	12	1	0	1	0.8	3.0	0.80	1	1	0.33	-1.6
$\mu_{ID}$ (N) cut 1	0	1	11	1	0	1	0	2.8	0.77	1	1	0.36	-1.4
$K_S^0$ exclusion	0	1	11	1	0	1	0	2.8	0.77	1	1	0.36	-1.4
$\sum E_\gamma$	0	1	11	1	0	1	0	2.8	0.77	1	1	0.36	-1.4
$E_{\pi^0}$	0	1	8	1	0	1	0	2.2	0.69	0	0	0	-3.2
$A_{sq}$	0	1	7	1	0	1	0	2.0	0.66	0	0	0	-3.0
$\mu_{ID}$ (N) cut 2	0	1	1	1	0	1	0	0.80	0.45	0	0	0	-1.8
$m_{\pi\pi}^{DV} > 0.52GeV$	0	0	0	0	0	0	0	0	0	0	0	0/0	0/0
$m_{\pi\pi}^{DV} < 0.42GeV$	0	1	1	1	0	1	0	0.80	0.45	0	0	0	-1.8
$m_+, m_-$	0	0	1	0	0	1	0	0.40	0.28	0	0	0	-1.3

TABLE VIII: Data and MC cut flows for the VRHeavySameSign and VRLightSameSign validation regions. See the Table VII caption for additional details

#### 6.4.2. VRHeavy $\pi\pi$ and VRLight $\pi\pi$

The events for this validation region are selected in the same way as SR events, except that both DV daughters are inconsistent with being leptons. This is enforced with the requirement

- both DV daughters satisfy  $MuonID < 0.01$  &&  $ElectronID < 0.01$

Containing 2 pions instead of 2 muons, these validation regions are "twice removed" from the SRs in that they contain two pions instead of two leptons, so they are less interesting than the control regions. However, they can be taken as a measure of the quality of the simulation of the  $K_S$  mass tails and of random dipion combinations.

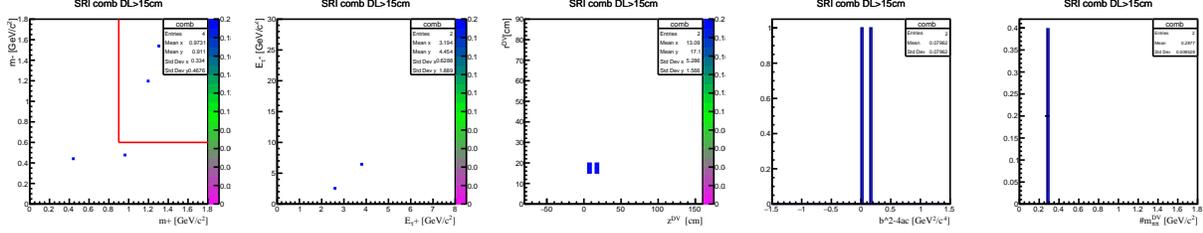


FIG. 22: Distributions of  $m_-$  vs.  $m_+$ ,  $E_{\tau^-}$  vs.  $E_{\tau^+}$ ,  $r_{DV}$  vs.  $z_{DV}$ ,  $A_{sq}$  and  $m_{\pi\pi}^{DV}$  for  $\tau^+\tau^-$  background MC events in (top plots) VRHeavySameSign and (bottom plots) VRLightSameSign.

706 The data and MC cut flows for this VR are shown in Table IX. The data/MC yield ratio  
 707 is 1.4 in VRHeavy $\pi\pi$  and 1.2 in VRLight $\pi\pi$ . The consistencies are 4.6 and 1.2, respectively.  
 708 Thus, the MC somewhat underpredicts the yields in this VR. Plots of MC and data events  
 709 in these VRs are shown in Figs. 23 and 24.

cut	$ee$	$\mu\mu$	$\tau\tau$	$4\ell$	$eeq\bar{q}$	$q\bar{q}$	$B\bar{B}$	MC	$\sigma_{MC}$	data	$\sigma_{Data}$	$\frac{data}{MC}$	cons.
textbfOnline	70969	24697	5385198	289389	53534	1522327	57573	1701280	1014	1632981	1278	0.96	-42
$N_{tracks}$	48142	13231	2615418	58702	21710	257352	3330	751236	785	774099	880	1.03	19
$DL > 15cm$	5659	3734	595090	20565	14208	89813	1148	164955	297	168794	411	1.02	8
$\chi_{prob}^2 > 10^{-5}$	3084	1638	234849	3180	1902	22457	180	62985	206	62358	250	0.99	-1.9
$LID$ prompt $\pi$	889	204	207211	2695	346	19938	110	48998	136	46712	216	0.95	-9
$\pi_{ID}$ (N) cut 1	150	20	132620	1834	104	15060	57	30434	87	30409	174	1.00	-0.13
$\sum E_\gamma$	126	10	123374	1742	96	12916	52	28055	83	27987	167	1.00	-0.36
$E_{\pi^0}$	125	10	98905	1344	95	8291	27	22148	75	22754	151	1.03	3.6
$\cos\theta_{\pi^+\pi^-}$	123	6	81426	838	34	6563	18	18176	69	18873	137	1.04	5
CDC Hits min	81	1	68673	441	5	5085	14	15104	62	15393	124	1.02	2.1
$K_S^0$ exclusion	71	1	3327	90	3	1956	2	1304	31	1613	40	1.24	6
$A_{sq}$	25	1	1393	40	1	1236	0	615	19	856	29	1.39	7
$m_{\pi\pi}^{DV} > 0.52GeV$	0	0	660	13	0	281	0	191	6	273	17	1.43	5
$m_{\pi\pi}^{DV} < 0.42GeV$	25	1	733	27	1	955	0	424	18	583	24	1.38	5
$m_+, m_-$	1	1	182	12	0	423	0	127	6	165	13	1.3	2.7

TABLE IX: Data and MC cut flows for the VRHeavy $\pi\pi$  validation regions. See the Table VII caption for additional details

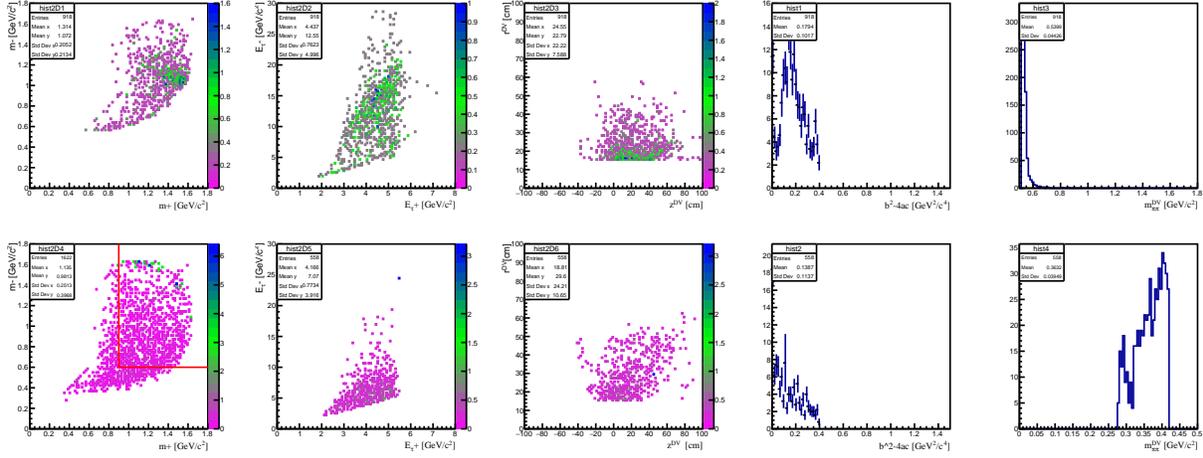


FIG. 23: Distributions of  $m_-$  vs.  $m_+$ ,  $E_{\tau^-}$  vs.  $E_{\tau^+}$ ,  $r_{DV}$  vs.  $z_{DV}$ ,  $A_{sq}$  and  $m_{\pi\pi}^{DV}$  for  $\tau^+\tau^-$  background MC events in (top plots) VRHeavy $\pi\pi$  and (bottom plots) VRLight $\pi\pi$ .

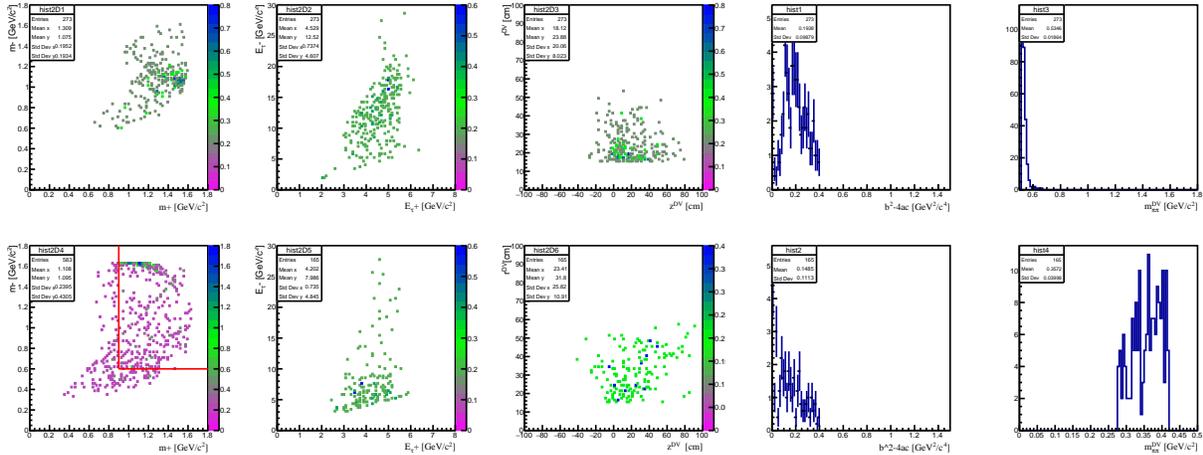


FIG. 24: Same as Fig. 23 for the background data in the pipi validation regions

### 710 6.4.3. VRK<sub>S</sub>

711 This validation region selects the decays  $\tau^- \rightarrow \pi^- K_S \nu$ ,  $K_S \rightarrow \pi^+ \pi^-$ . Candidates are  
 712 selected in the same way as the signal-region events, except that

- 713 • both DV daughters satisfy MuonID < 0.01 && ElectronID < 0.01  
 714 •  $480 < m_{\pi\pi}^{DV} < 515$  MeV.

715 The data and MC cut-flow for this region are shown in Table X. The  $m_{\pi\pi}^{DV}$  distribution  
 716 for the selected events is shown in Fig. 25 for different regions of  $r_{DV}$ . We see that the  
 717  $m_{\pi\pi}^{DV}$  distribution is more shifted to the right the larger  $r_{DV}$  is. We do not understand the  
 718 source of this effect (perhaps magnetic field calibration or wrong assumption of the amount  
 719 of material traversed by highly displaced tracks), but in any case it is well simulated in the

720 MC. We note that these distributions are identical to the "untampered" mass distributions  
 721 that come out of basf2. Therefore, the shift to the right is not due to the change of track  
 722 mass hypothesis when calculating  $m_{\pi\pi}^{DV}$ . in any case, this shift does not affect our analysis.

cut	$ee$	$\mu\mu$	$\tau\tau$	$4\ell$	$ee\bar{q}q$	$q\bar{q}$	$B\bar{B}$	MC	$\sigma$ MC	data	$\sigma$ Data	$\frac{data}{MC}$	cons.
$480 < m_{\pi\pi}^{DV} < 515 GeV$	1	0	62645	289	0	2197	12	13031	51	12717	112	0.98	-2.5
$A_{sq}$	1	0	37141	79	0	1403	3	7728	39	7917	88	1.02	1.9
$m_+, m_-$	0	0	569	30	0	391	2	198	6	277	16	1.40	4.4

TABLE X: Data and MC cut flows for the  $VRK_S$  validation regions. Here we see only the last cuts of the flow. See Table IX caption for full cut flow up to these cuts, and for additional details.

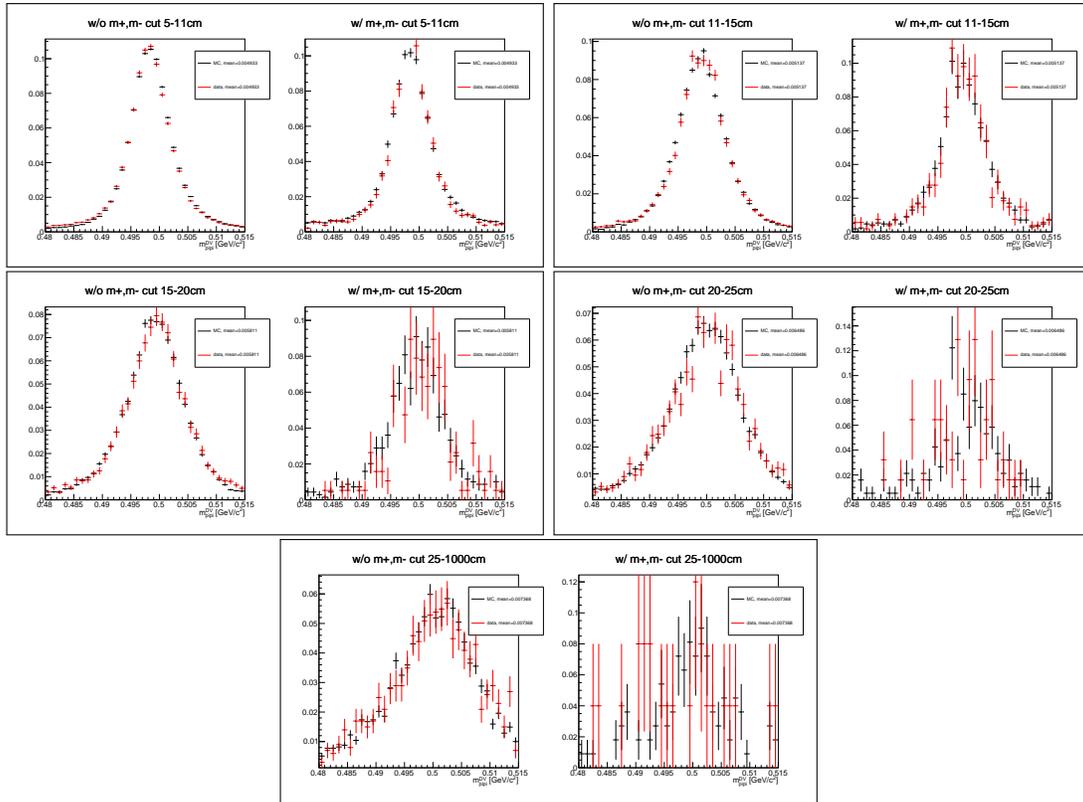


FIG. 25: Comparison of the  $m_{\pi\pi}^{DV}$  distribution of MC (black) and data (red) candidates in the  $VRK_S$  validation region, for different regions of  $r_{DV}$  as indicated on each plot. In each pair of plots, the left (right) plot is without (with) the cut on  $m_+$  and  $m_-$  used to define the SRLight and CRLight regions.

723 **7. SIGNAL EFFICIENCY AND EXPECTED NUMBER OF SIGNAL EVENTS**

724 The signal MC is generated with a particular value  $c\tau_0$  of the lifetime, so a method  
 725 for calculating the efficiency for any given lifetime  $c\tau_1$  is needed. A simple method is by  
 726 reweighting, described in Appendix B. A problem with this method is that it fails when  $\tau_1$   
 727 and  $\tau_0$  are very different and MC statistics is finite, as we show in Appendix B.

728 Therefore, we are using another approach: we use each MC sample to obtain a binned  
 729 efficiency map in terms of the DV position,

$$\epsilon_b = \frac{P_b}{G_b} \quad (22)$$

730 where  $b$  is the index of the bin centered at the radial and longitudinal DV position  $(r_{\text{DV}}, z_{\text{DV}})$ ,  
 731  $G_b$  is the number of signal events generated in this bin, and  $P_b$  is the number of these events  
 732 that passed the selection. These efficiency maps are shown in Fig. 26. Subsequently, for  
 733 each of the  $G = \sum_b G_b$  events in the signal sample we randomly draw  $R$  "toy" values of  
 734 the lifetimes  $t_r$  from an exponential distribution  $\frac{1}{\tau_1} \exp(-t_r/\tau_1)$ . For each value of  $t_r$  we  
 735 calculate a decay position

$$\begin{aligned} r_{\text{DV}}^{gr} &= \frac{p_T^g}{m_N} ct_r \\ z_{\text{DV}}^{gr} &= \frac{p_z^g}{m_N} ct_r, \end{aligned} \quad (23)$$

736 where  $p_T^g$ ,  $p_z^g$ , and  $m_N$  are the true transverse momentum, longitudinal momentum, and  
 737 invariant mass of the HNL in signal-MC event  $g$ . The total efficiency is then

$$\epsilon = \frac{1}{GR} \sum_{g=1}^G \sum_{r=1}^R \epsilon_{b(gr)}, \quad (24)$$

738 where  $b(gr)$  is the bin corresponding to position  $(r_{\text{DV}}^{gr}, z_{\text{DV}}^{gr})$  in the efficiency map. This can  
 739 be rewritten as a sum over the bins,

$$\epsilon = \frac{1}{N} \sum_b N_b \epsilon_b, \quad (25)$$

740 where  $N_b$  is the total number of toy events in bin  $b$ , and  $N = \sum_b N_b$  is the total number of  
 741 toy events. From the last expression we obtain the MC-statistical squared uncertainty on  
 742 the efficiency,

$$\sigma_\epsilon^2 = \frac{1}{N^2} \sum_b \left[ N_b^2 \frac{1}{G_b} \epsilon_b (1 - \epsilon_b) + N_b \epsilon_b^2 \right], \quad (26)$$

743 where we took the uncertainty on  $\epsilon_b$  to be binomial.

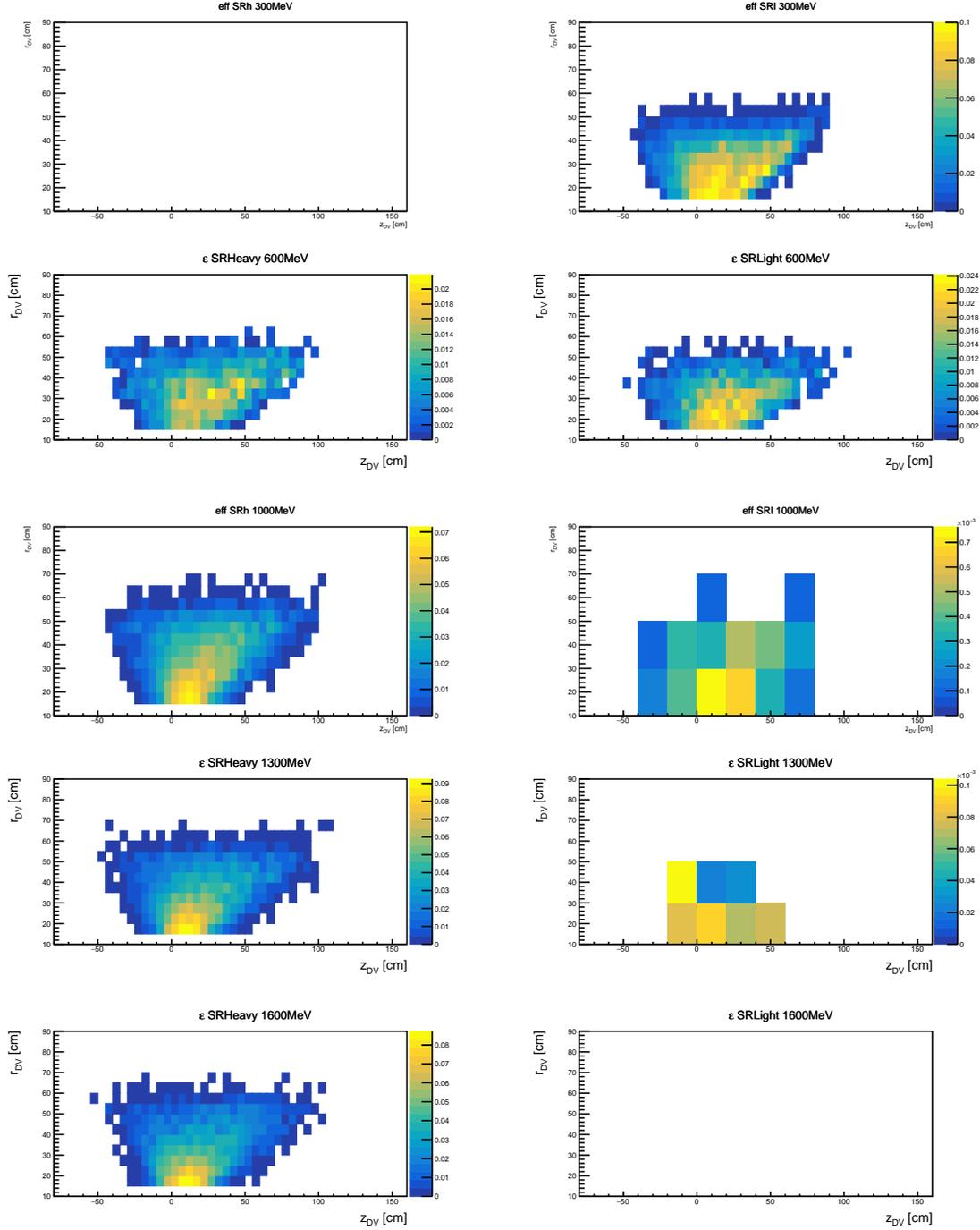


FIG. 26: Efficiency maps, calculated as described in Eq. (22), for SRHeavy (left plots) and SRLight (right plots) of signal samples of 300,600,1000,1300,1600 MeV. White bins have lower entries than the  $z$ -axis scale.

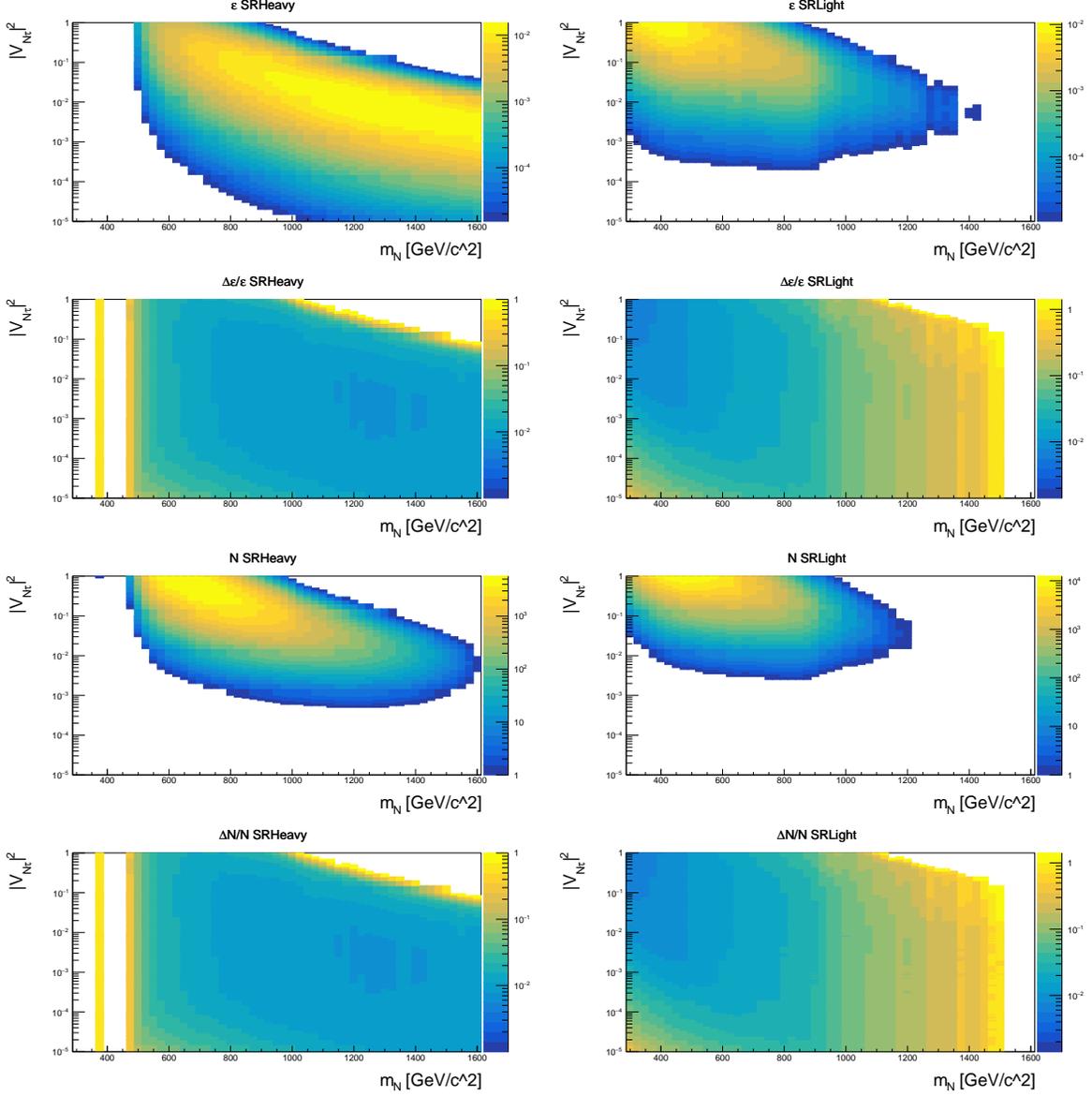


FIG. 27: The total efficiencies (first row), their relative uncertainties (second row), the expected numbers of signal events (third row), and their relative uncertainties (fourth row) as functions of  $V_{\tau N}$  and  $m_N$  for SRHeavy (left column) and SRLight (right column). White bins have lower entries than the minimal  $z$ -axis range shown.

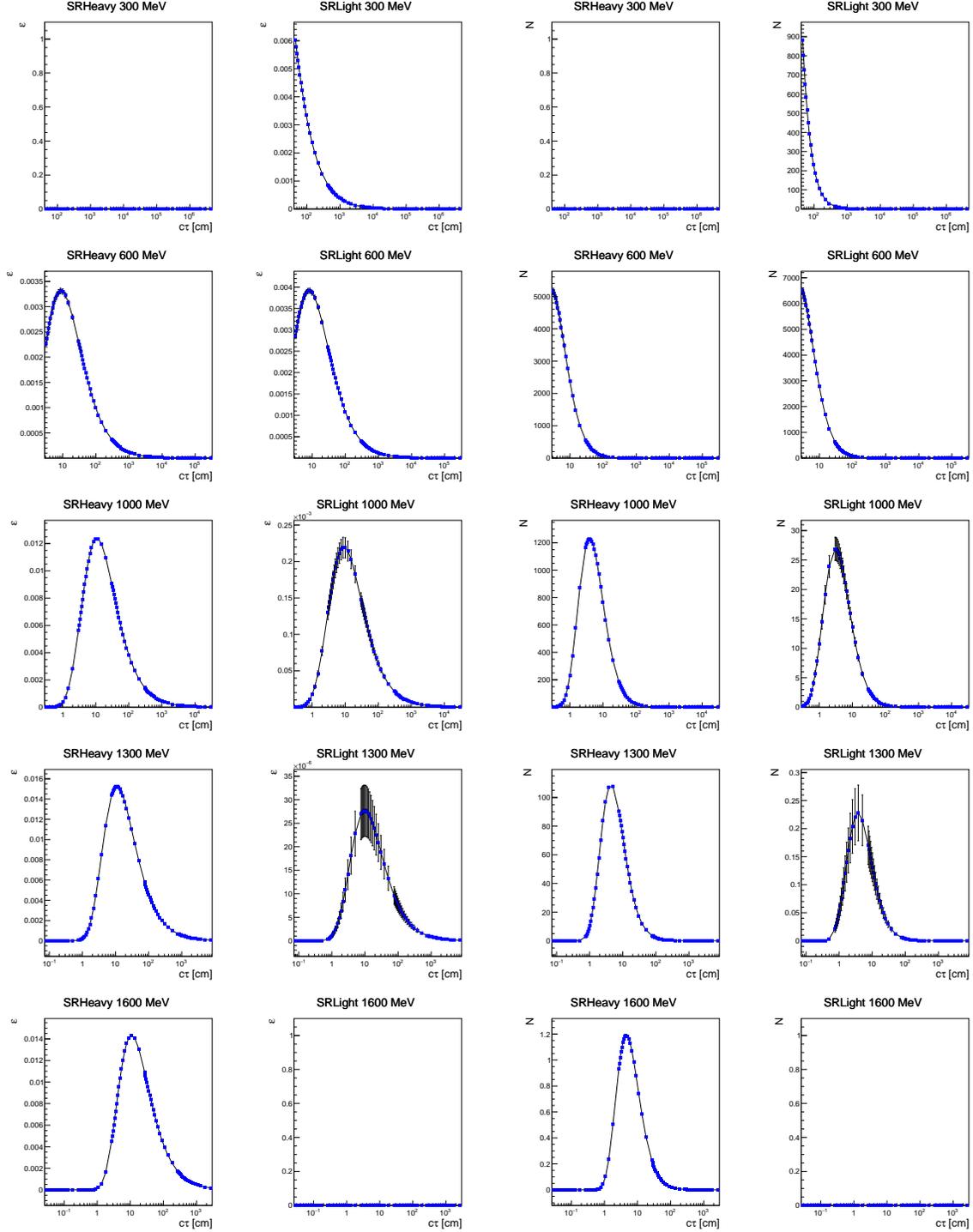


FIG. 28: Efficiency  $\varepsilon$  and expected number of signal events  $N$  as functions of  $c\tau_1$  in the SRHeavy and SRLight for the signal samples of  $m_N = 300, 600, 1000, 1300, 1600$  MeV

744 **8. SYSTEMATIC UNCERTAINTIES**

745 All systematic uncertainties are treated with Gaussian nuisance parameters applied to  
 746 the signal or background model in the statistical analysis described in Sec. [10](#). These un-  
 747 certainties are as follows:

- 748 • Background prediction: we take the relative systematic uncertainty to be the percent-  
 749 age change in the background model needed to bring the data and MC to  $1\sigma$  agreement  
 750 in the CRs and the  $\pi\pi$  VRs). Specifically,

$$\sigma(N_{\text{bgd}}) = \left( \frac{N_{\text{data}} - N_{\text{MC}}}{\sigma(N_{\text{data}} - N_{\text{MC}})} - 1 \right) \frac{\sigma(N_{\text{data}} - N_{\text{MC}})}{N_{\text{MC}}}, \quad (27)$$

751 where

$$\sigma(N_{\text{data}} - N_{\text{MC}}) = \sqrt{\sigma_{\text{MC}}^2 + N_{\text{data}}} \quad (28)$$

752 is the statistical uncertainty on  $N_{\text{data}} - N_{\text{MC}}$ . If Eq. [\(27\)](#) yields a negative value,  
 753 the data and MC are consistent to within the available statistics, and we take the  
 754 uncertainty to be 0. The values of  $\sigma(N_{\text{bgd}})$  extracted from the CRs and VRs are  
 755 shown in Table [X1](#). For the final systematic we use the most conservative value of  
 756 34%, taken from VRHeavy $\pi\pi$ . This uncertainty is taken to be uncorrelated among  
 757 the bins (see Sec. [10](#)).

- 758 • MC statistical errors, as described in Eq. [\(26\)](#). Uncorrelated among the bins.
- 759 • Signal model: our signal MC is generated with a phase-space distribution. To deter-  
 760 mine the impact of this on the efficiency, we use events with MadGraph. MadGraph  
 761 is another generator, which consider that proper matrix element for the decay, so the  
 762 distributions might not be flat [\[55\]](#). The MadGraph samples generated for us by Nico-  
 763 las Neil, a co-author of Ref. [\[33\]](#). The model used in MadGraph is the SM + Majorana  
 764 neutrinos [\[56\]](#) modified with an effective vertex of the form  $\partial_\mu \pi \bar{\tau} \gamma^\mu (1 - \gamma_5) N + h.c$  to  
 765 generate the  $\tau^- \rightarrow \pi^- N$  decay.

766 Distributions of the MadGraph-generated events are shown in Figs. [29](#) and [30](#). In  
 767 Fig. [29](#) we see that the distribution for  $\cos \theta_d^\tau$  of the decay angle of the  $\tau$ , defined as  
 768 the angle between the CM frame and the HNL in the  $\tau$  rest frame, is flat, as it is in  
 769 the phase-space model used in our KKMC+PYTHIA events. This is also the case for  
 770 the distribution for  $\cos \theta_d^N$  of the decay angle of the HNL, defined as the angle between

Sample	Uncertainty
CRHeavy	0.15
CRLight	0
VRHeavy $\pi\pi$	0.34
VRLight $\pi\pi$	0.19

TABLE XI: Options for the relative systematic uncertainty on the background yield extracted from different samples according to Eq. [\(27\)](#). The final systematic used is the most conservative value, from VRHeavy $\pi\pi$

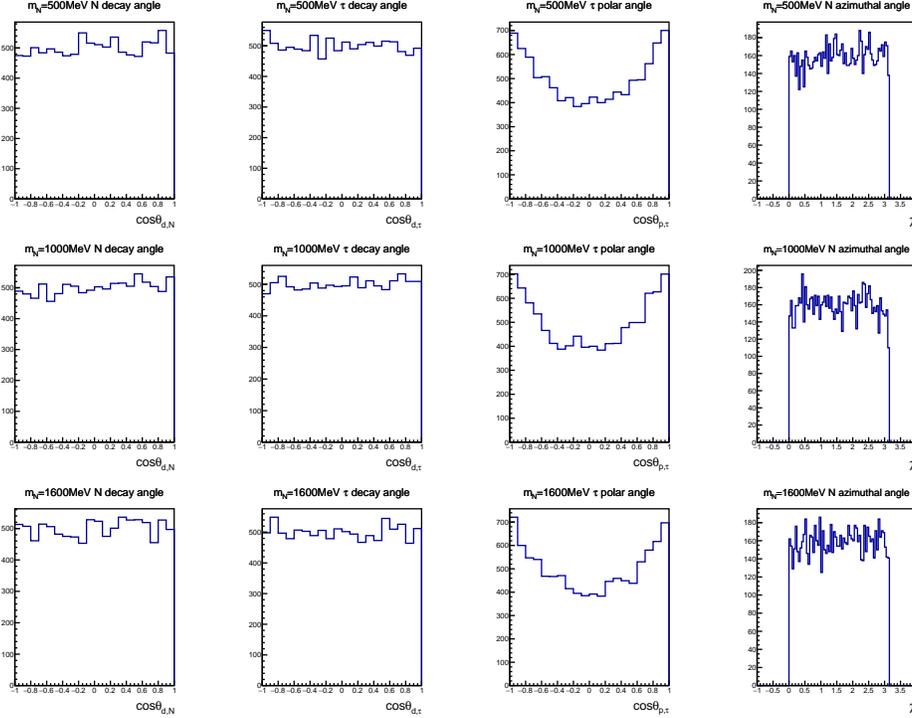


FIG. 29: Distributions of the cosines of the decay angle of the HNL (left column), the decay angle of the  $\tau$  (2nd column), the polar angle of the  $\tau$  (3rd column), and the angle between the decay planes (right column) for HNL masses of 500 (top row), 1000 (middle row), and 1600 MeV (bottom row) in events generated with MadGraph.

771 the  $\tau$  and the plane of the 3-body HNL decay products in the HNL rest frame. We  
 772 also show in Fig. 29 the flatly (i.e., phase-space) distributed angle  $\chi$  between the plain  
 773  $\vec{p}_\tau \times \vec{p}_\pi$  defined by the  $\tau$  and pion momenta and the plain  $\vec{p}_\tau \times \vec{p}_\nu$  defined by the  $\tau$  and  
 774 neutrino momenta, all in the HNL rest frame. The events are flat in  $\chi$ , as Fig. 29 also  
 775 shows the distribution of the  $\tau$  polar angle in the CM frame, demonstrating that it is  
 776 generated correctly according to a  $1 + \cos^2 \theta$  distribution.

777 In contrast to these angles, Fig. 30 shows that while the Dalitz plots of the KKMC  
 778 events are flat, as expected, this is not the case for the MadGraph events. Specifically,  
 779 MadGraph generates more low- $m_{\mu^+\mu^-}$  events. As a result, the MadGraph events tend  
 780 to also have lower values of  $m_{\pi\pi}^{DV}$ , causing a shift of events from SRHeavy to SRLight.

781 Therefore, to calculate the systematic uncertainty, we weight our signal events by the  
 782 ratio between the MadGraph and KKMC Dalitz-plot histograms and recalculate the  
 783 change in efficiency.

784 This uncertainty is taken to be uncorrelated among the bins.

785 It should be mentioned that not all MadGraph files have been generated yet, so at this  
 786 moment we cannot apply this uncertainty. Instead, a conservative constant uncertainty  
 787 of 20% was used in the final fit, but it will be changed in the near future.

- 788 • Luminosity: 1.4% [57]. Correlated for signal and background and among all regions.
- 789 •  $\sigma(e^+e^- \rightarrow \tau^+\tau^-)$ : 0.3% [57]. Correlated for signal and background and among all

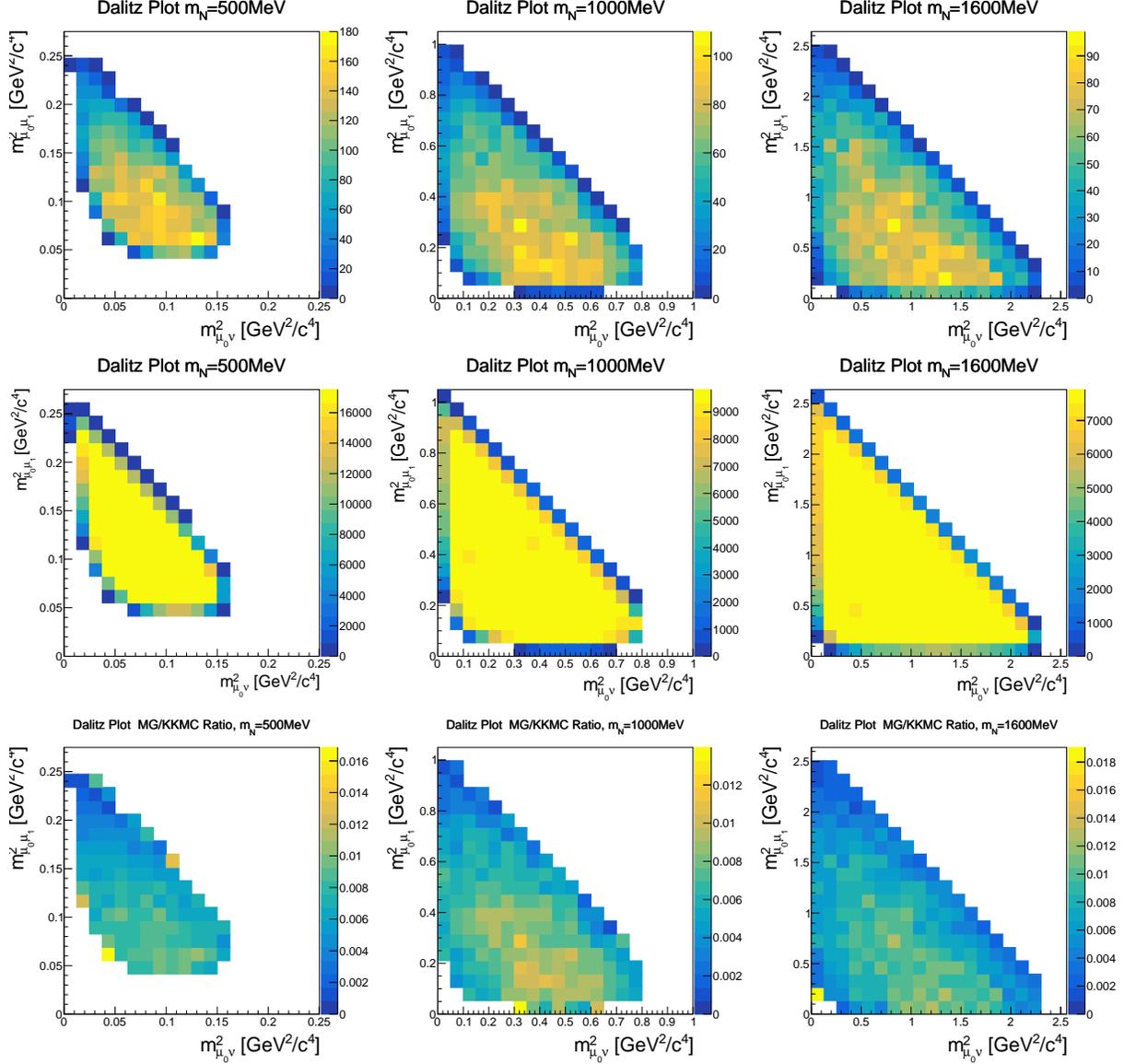


FIG. 30: Dalitz plots for the HNL decay for samples generated with MadGraph (top row) and KKMC (middle row) with  $m_N = 500, 1000, 1600$  MeV, and the ratio between the histograms (bottom row), which is used to weight the KKMC-generated events to obtain the signal-model systematic uncertainty.

790 regions. For the background, it is scaled by the  $\tau\tau$  contribution as predicted by the  
 791 MC.

792 •  $\mathcal{B}(\tau^+ \rightarrow 1 - \text{prong})$ :  $(85.24 \pm 0.06)\%$  (see Table [I](#))

793 •  $\sigma(e^+e^- \rightarrow q\bar{q})$ : it is anticipated that the continuum MC may not be accurate in  
 794 the very restricted phase space of our selection, but it is not clear what the relevant  
 795 systematic uncertainty should be. Therefore, we do not associate a specific value with  
 796 this uncertainty. Rather, its effect is covered by the uncertainty extracted from the  
 797 data-MC agreement in the VRs (above).

- 798 • Tracking: 0.35% per prompt track [57], totalling 0.7% for the two tracks. Correlated  
799 for signal and background and among all regions.
- 800 • displaced-track tracking and vertexing: we apply the method of Eq. (27) to the  $VRK_S$ ,  
801 which contains a high-purity sample of  $K_S$  decays. The resulting uncertainty is 1.2%.
- 802 • Muon identification: 2% [57] for the SRs, 1% for the CRs. Correlated for signal and  
803 background and among all regions.
- 804 • Trigger: 1.2% [58]. Correlated for signal and background and among all regions.

## 805 9. UNBLINDING PLAN

806 We propose to first unblind the data up to cut #10, where we require that there is no  $\pi^0$   
807 on the signal side. From the MC (Table VI, the expected number of background events at  
808 that point is  $19.8 \pm 2.0$ . Therefore, observation of, e.g., 30 events would be consistent with  
809 this expectation at the  $2.1\sigma$  level. Next, we apply cut #11,  $A_{sq} < 0.4$  while also requiring  
810 that events fail cut #12 (which is that both muon candidates have muon ID  $> 0.9$ ). Table VI  
811 predicts  $3 \pm 0.8$  background events. This muon ID veto suppresses the signal efficiency is  
812 suppressed down to 12.1%, 12.2%, and 10.2% for the 300, 1000, and 1600 MeV samples.  
813 While this is evaluated at the generated lifetime, this suppression is not expected to change  
814 significantly for other lifetimes, since muID is dominated by the KLM.

Sample	$ee$	$\mu\mu$	$\tau\tau$	$4\ell$	$eeq\bar{q}$	$q\bar{q}$	$B\bar{B}$	MC	$\sigma$ MC	Sig300	Sig1000	Sig1600
<b>NOT <math>\mu_{ID}</math> (N) cut 2</b>	0	0	57	0	0	5	0	12.4	1.57	2433	2403	1656
$A_{sq}$	0	0	20	0	0	2	0	4.4	0.94	2433	2391	1622
$m_{\pi\pi}^{DV} > 0.52\text{GeV}$	0	0	5	0	0	0	0	1	0.45	0	2102	1567
$m_{\pi\pi}^{DV} < 0.42\text{GeV}$	0	0	15	0	0	2	0	3.4	0.82	2433	289	55
$m_+, m_-$	0	0	0	0	0	1	0	0.2	0.2	2431	33	0

TABLE XII: Changes in the SR and Table VI (starting from  $E_{\pi^0}$  cut line) for the unblinding plan.

815 Next, we will unblind the SR but without the muon ID cut on the second muons. This  
816 will allow detailed comparison between the data and generic MC before final unblinding. If  
817 some inconsistency is seen, we will consider whether any additional validation studies are  
818 needed before unblinding the SR.

## 819 10. STATISTICAL ANALYSIS AND RESULTS

820 The expected number of signal events is calculated from the product of the luminosity,  
821 cross section, tag-side branching fraction, signal branching fractions, and efficiency:

$$N_{\text{sig}} = 2 \mathcal{L} \sigma(e^+e^- \rightarrow \tau^+\tau^-) \mathcal{B}(\tau \rightarrow 1 - \text{prong}) \mathcal{B}(\tau \rightarrow \pi N) \mathcal{B}(N \rightarrow \mu^+\mu^-\nu) \epsilon. \quad (29)$$

822 We use pyhf (see Section 33.3 for more details about this software) to calculate exclusion  
 823 limits based on this expectation and on the observed numbers of events in the two SRs and  
 824 two CRs. The signal and background models for the SRs and CRs are obtained from the  
 825 yields in MC. The observed yields are taken from the data (before unblinding, we take the  
 826 observed data yield to be 0 in the two SRs).

827 We define a grid in the  $m_N$  vs.  $|V_{N\tau}|^2$  parameter space. In  $m_N$ , grid points are separated  
 828 by 25 MeV. In  $|V_{N\tau}|^2$ , we use 20 points per decade, separated equidistantly in log scale. For  
 829 each point in this grid we determine the expected number of signal events in SRHeavy and  
 830 SRLight using the external inputs given in Appendix C and the signal efficiencies in Fig. 27.  
 831 The expected signal yields in the two signal regions are used as the signal model for pyhf.  
 832 The background model is taken from Eq. (17). In addition, pyhf uses the observed yields  
 833 in CRHeavy and CRLight, with a model obtained from the expected MC yields given in  
 834 Eq. (21). No scaling of the MC is applied by hand. We use nuisance parameters to include  
 835 all systematic uncertainties in the fit. We use the general HistFactory template to estimate  
 836 the probability distribution function.

$$P = \underbrace{\prod_{\text{bin } R} \text{Pois}(n_R | \lambda_R(\eta, \chi))}_{\text{main}} \underbrace{\prod_{\text{constraint } \chi} c_\chi(a_\chi | \chi)}_{\text{auxiliary}} \quad (30)$$

837 where:

- 838 •  $R$  is the index for the four regions:
  - 839 – SRHeavy
  - 840 – SRLight
  - 841 – CRHeavy
  - 842 – CRLight
- 843 • Pois is the Poisson function.
- 844 •  $n_R$  is the number of events observed in region  $R$ .
- 845 •  $\lambda_R(\eta, \chi)$  is the expected number of events in region  $R$ , obtained from a sum over signal  
 846 and background MC.
- 847 •  $\eta$  denotes a scaling factor used internally by pyhf.
- 848 •  $\chi$  denotes all the systematics-related parameters (Sec. 8) and the background expect-  
 849 tations.
- 850 •  $c_\chi(a_\chi | \chi)$  is a constraint term (Gaussian or Poisson) that constrains  $\chi$  to a known value  
 851  $a_\chi$  to within a given uncertainty.

852 The main and auxiliary components are labelled in the equation.

853 We perform hypothesis test with 10000 toy experiments for each of these points in the  
 854 mesh. We use  $\tilde{q}$  test statistics. For each point, we calculate, with the help of pyhf, observed  
 855  $CL_s$  values, expected  $CL_s$  values, and expected  $CL_s$  values at  $\pm 1\sigma$  and  $\pm 2\sigma$ . We then draw

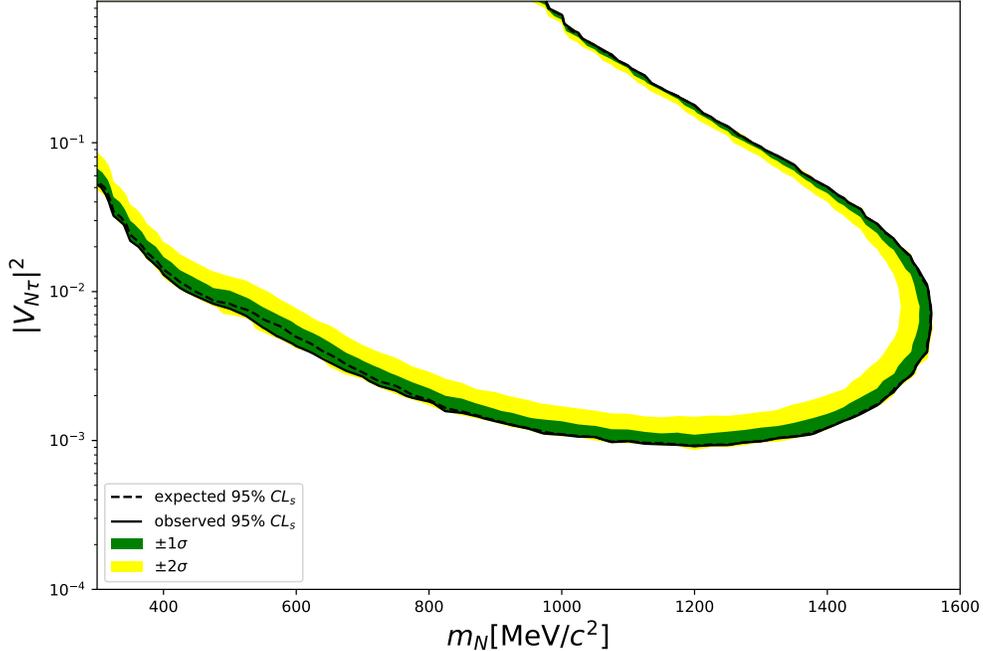


FIG. 31: The expected (dashed) and observed (solid) 95% CL limits on  $|V_{N\tau}|^2$  vs.  $m_N$ . The green and yellow bands show the  $1\sigma$  and  $2\sigma$  bands for the expected limits.

856 six separate contours which visually represent the 95%  $CL_s$  limits. Then we fill between  
 857  $\pm 2\sigma$  contours with yellow and between  $\pm 1\sigma$  contours with green as shown in Fig. 31. It  
 858 also shows the expected and observed (**before unblinding**, the observed curve assumes 0  
 859 events observed in the signal regions) 95% CL limits in the plane of  $|V_{N\tau}|^2$  vs.  $m_N$ .

860 This is the main result of the analysis.

## 861 11. CONCLUSION

862 This research sought to discover the HNL, if it exists, with the parameters for which  
 863 the research is sensitive, and otherwise, to exclude this parameter space. Fig. 31 presents  
 864 such limits (in  $|V_{\tau N}|$  vs.  $m_N$  parameter space). Although this thesis does not include the  
 865 signal data analyses (but only the MC), we can already bring up some points to discuss.  
 866 By comparing our result to the expectation of Ref. 33 (see Fig. 11), we can notice that  
 867 the DELPHI result was not significantly improved upon, and for certain, not in the order  
 868 of magnitude predicted. This is due to the low than expected efficiency we received for  
 869 our MC samples: The pheno paper's estimates were validated against Belle II MC, but the  
 870 efficiency at Belle turns out to be significantly smaller. This is particularly the case for muon  
 871 identification and tracking efficiency of highly displaced tracks. In addition to the efficiency  
 872 issue, the pheno paper assumed that  $\tau \rightarrow \pi\pi^0 N$  would be used and also  $N \rightarrow e^+e^-\nu_\tau$ . Since  
 873 we haven't yet included these, the total sensitivity is lower than expected. Nonetheless, this

874 is the first application of this new method, and with the additional channels and use of Belle  
875 II data, the sensitivity is expected to increase to the level predicted in the pheno paper. Of  
876 course, the signal model uncertainty that is described in Sec. 8 is still need to be applied  
877 and will affect uncertainties bandwidth.

878 In order to understand this issue better, we work these days on expanding our research,  
879 and we additionally analyze the signal side decay chain, where  $N \rightarrow e^+e^-$ . It means that  
880 the mass range of interest can go below 300 MeV. Dealing with the  $e^+e^-$  channel requires  
881 different managing of the selection criteria, which are now studied for the upcoming final  
882 decision about them. Hence, results for the  $e^+e^-$  channel has yet to be achieved.

883 As for the unblinding the data of the  $\mu^+\mu^-$  channel, we are waiting for the final approval  
884 from the recently formed review committee, so we can continue with the plan as described  
885 in Sec. 9. If the data and MC are consistent in this region, we will proceed to unblind the  
886 SR. If some inconsistency is seen, we will consider whether any additional validation studies  
887 are needed before unblinding the SR.

888 **Appendix A: HNL mass calculation**

889 We start with 4-momentum conservation in the  $\tau$  decay:

$$p_\tau = p_N + p_x . \quad (\text{A1})$$

890 Solving the above equation gives

$$m_\tau^2 = m_N^2 + m_x^2 + 2E_N E_x - 2|\vec{p}_N||\vec{q}_x| , \quad (\text{A2})$$

891 where

$$|\vec{q}_x| \equiv |\vec{p}_x| \cos \theta_{Nx} \quad (\text{A3})$$

892 Similarly, 4-momentum conservation in the HNL ( $N$ ) decay:

$$p_N = p_y + p_{\nu_\tau} . \quad (\text{A4})$$

893 Solving the above equation gives

$$0 = m_N^2 + m_y^2 - 2E_N E_y + 2|\vec{p}_N||\vec{q}_y| , \quad (\text{A5})$$

894 where

$$|\vec{q}_y| \equiv |\vec{p}_y| \cos \theta_{Ny} \quad (\text{A6})$$

895 Comparing Eqs. (A2) and (A5) gives the solution of HNL energy ( $E_N$ ) in terms of the  
896 magnitude of HNL three momentum ( $|\vec{p}_N|$ )

$$E_N = \frac{m_\tau^2 + m_y^2 - m_x^2}{2(E_x + E_y)} + \frac{(|\vec{q}_y| + |\vec{q}_x|)|\vec{p}_N|}{(E_x + E_y)} \quad (\text{A7})$$

897

$$\Rightarrow E_N = A + B|\vec{p}_N| \quad (\text{A8})$$

898 where

$$A = \frac{m_\tau^2 + m_y^2 - m_x^2}{2(E_x + E_y)}, \quad B = \frac{(|\vec{q}_y| + |\vec{q}_x|)}{(E_x + E_y)} \quad (\text{A9})$$

899 are the two known quantities. 4-momentum relation of HNL is

$$m_N^2 = E_N^2 - |\vec{p}_N|^2 \quad (\text{A10})$$

900 Expressing the mass of HNL ( $m_N$ ) in terms of A, B and  $|\vec{p}_N|$

$$\Rightarrow m_N^2 = (A + B|\vec{p}_N|)^2 - |\vec{p}_N|^2 \quad (\text{A11})$$

901 Using Eq. (A8) in (A2) gives

$$\Rightarrow m_N^2 = \frac{(\frac{E_y}{E_x})(m_\tau^2 - m_x^2) - m_y^2}{(1 + \frac{E_y}{E_x})} + \frac{2(\frac{E_y}{E_x}|\vec{q}_x| - |\vec{q}_y|)}{(1 + \frac{E_y}{E_x})}|\vec{p}_N| \quad (\text{A12})$$

$$\Rightarrow m_N^2 = C + D|\vec{p}_N| \quad (\text{A13})$$

903 where

$$C = \frac{(\frac{E_y}{E_x})(m_\tau^2 - m_x^2) - m_y^2}{(1 + \frac{E_y}{E_x})}, \quad D = \frac{2(\frac{E_y}{E_x}|\vec{q}_x| - |\vec{q}_y|)}{(1 + \frac{E_y}{E_x})} \quad (\text{A14})$$

904 are the two known quantities.

905 Comparing Eq. (A11) and Eq. (A13) gives a quadratic equation of the form

$$(B^2 - 1)|\vec{p}_N|^2 + (2AB - D)|\vec{p}_N| + (A^2 - C) = 0 \quad (\text{A15})$$

906 This gives solution of  $|\vec{p}_N|$  as

$$|\vec{p}_N| = \frac{-(2AB - D) \pm \sqrt{(2AB - D)^2 - 4(B^2 - 1)(A^2 - C)}}{2(B^2 - 1)} \quad (\text{A16})$$

907 Using Eq. (A16) in Eq. (A13) gives the solution of  $m_N$  with 2-fold ambiguity.

## 908 Appendix B: The lifetime-reweighting efficiency-calculation method

909 To calculate the efficiency for a lifetime  $\tau_1$  using an MC sample generated with lifetime  
910  $\tau_0$ , one gives each event a weight

$$w_i = \frac{\tau_0 \exp(-t/\tau_1)}{\tau_1 \exp(-t/\tau_0)}, \quad (\text{B1})$$

911 where  $t$  is the true decay time of the HNL in event  $i$ . One then obtains the efficiency from  
912 the sum of weights:

$$\epsilon = \frac{\sum_p w_p}{\sum_p w_p + \sum_f w_f}, \quad (\text{B2})$$

913 where the index  $p$  runs over the events that passed the cuts, and  $f$  runs over all the events  
914 that failed the cuts. Writing this as  $\epsilon = P/(P + F)$ , one can obtain the MC-statistical  
915 uncertainty on the efficiency,

$$\sigma_\epsilon^2 = \frac{1}{(P + F)^4} \left[ F^2 \sum_p w_p^2 + P^2 \sum_f w_f^2 \right]. \quad (\text{B3})$$

916 The problem with this method is that it is biased for large when one has finite MC statistics  
917 and  $\tau_1$  is very different from  $\tau_0$ . To see this, we write Eq. (B2) explicitly:

$$\epsilon = \frac{\sum_p \exp(-t_p \alpha)}{\sum_p \exp(-t_p \alpha) + \sum_f \exp(-t_f \alpha)}, \quad (\text{B4})$$

918 where we define

$$\alpha = \frac{1}{\tau_1} - \frac{1}{\tau_0}. \quad (\text{B5})$$

919 When statistics is infinite, the sums become integrals, and there is no problem. But for  
 920 finite samples, we do not have enough statistical precision to differentiate between  $\alpha$  and  
 921 the larger of  $1/\tau_0$  or  $1/\tau_1$ . If we have  $\tau_1 \gg \tau_0$ , then

$$\alpha \approx -\frac{1}{\tau_0}, \quad (\text{B6})$$

922 and then

$$\epsilon \approx \frac{\sum_p \exp(t_p/\tau_0)}{\sum_p \exp(t_p/\tau_0) + \sum_f \exp(t_f/\tau_0)}, \quad (\text{B7})$$

923 which is independent of  $\tau_1$ . Therefore, at high  $\tau_1$ , the efficiency becomes  $\tau_1$  independent,  
 924 which is clearly wrong (more events decay outside the detector as  $\tau_1$  grows, so the efficiency  
 925 should drop). In the opposite case,  $\tau_1 \ll \tau_0$ , we have

$$\epsilon \approx \frac{\sum_p \exp(-t_p/\tau_1)}{\sum_p \exp(-t_p/\tau_1) + \sum_f \exp(-t_f/\tau_1)}, \quad (\text{B8})$$

926 so the efficiency is independent of how we generated the events ( $\tau_0$ ), which is again obviously  
 927 wrong.

## 928 **Appendix C: Cross section, branching fractions, and lifetimes**

929 This appendix shows the model parameters used to calculate the number of signal events  
 930 produced as a function of  $M_N$  and  $|V_{\tau N}|^2$ .

931 We use the cross section [38]

$$\sigma(e^+e^- \rightarrow \tau^+\tau^-) = 0.919 \pm 0.003 \text{ nb}. \quad (\text{C1})$$

932 All other parameters are from Ref. [46]. Tables XIII, XIV, and XV show values of  
 933  $Br(\tau^- \rightarrow N\pi^-)$ ,  $Br(N \rightarrow \mu^+\mu^-\nu_\tau)$ , and the HNL lifetime  $c\tau$ , respectively. The number of  
 934 digits is far below the actual uncertainties.

## 935 **Appendix D: Comparison of efficiency between basf2 vs b2bii for 1 GeV HNL of $c\tau$** 936 **= 30 cm**

937 This appendix shows a comparison of efficiency using events generated via KKMC fol-  
 938 lowed by Belle vs Belle II detector simulation

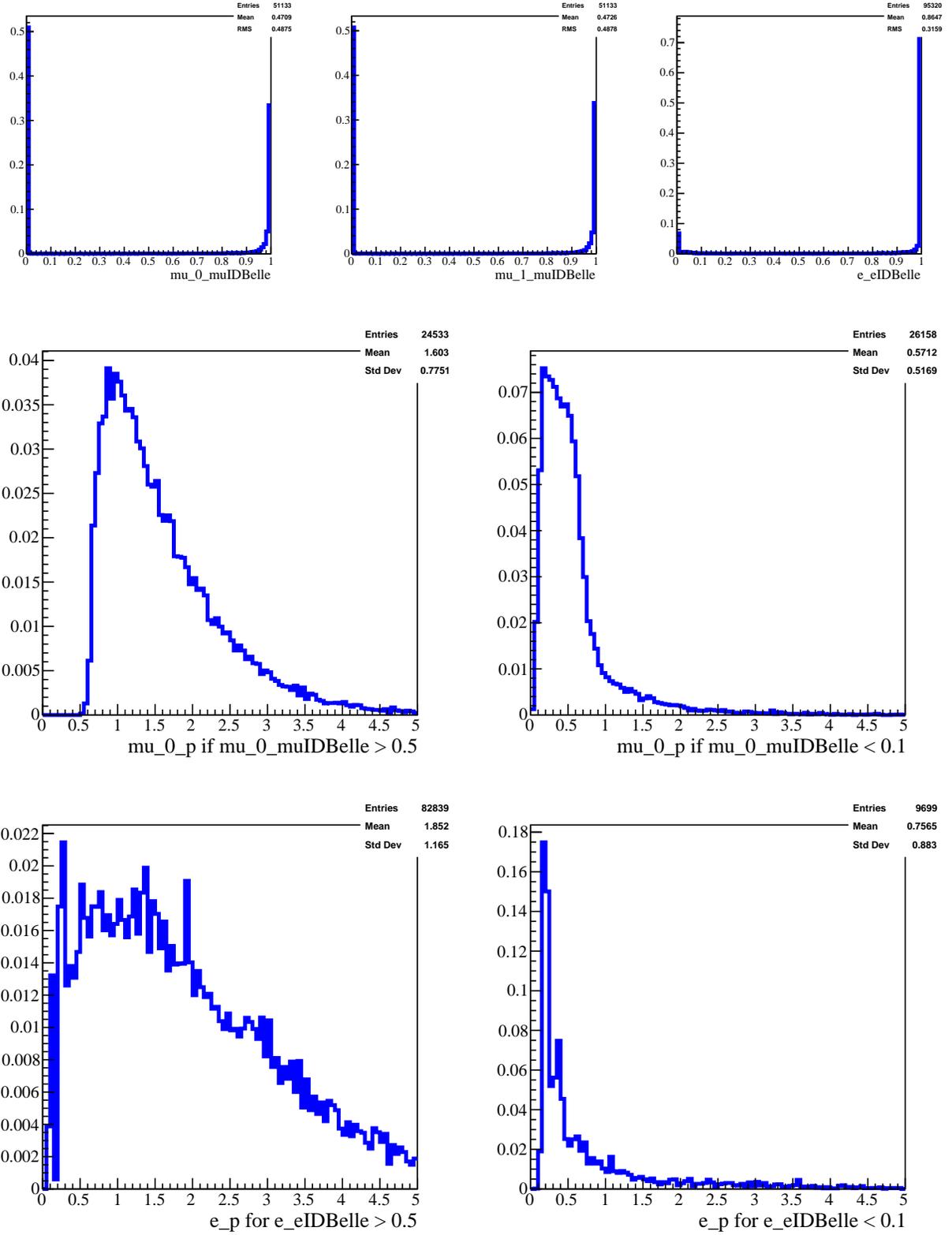


FIG. 32: B2BII lepton ID performances for the signal samples of  $m_N = 1000$  MeV

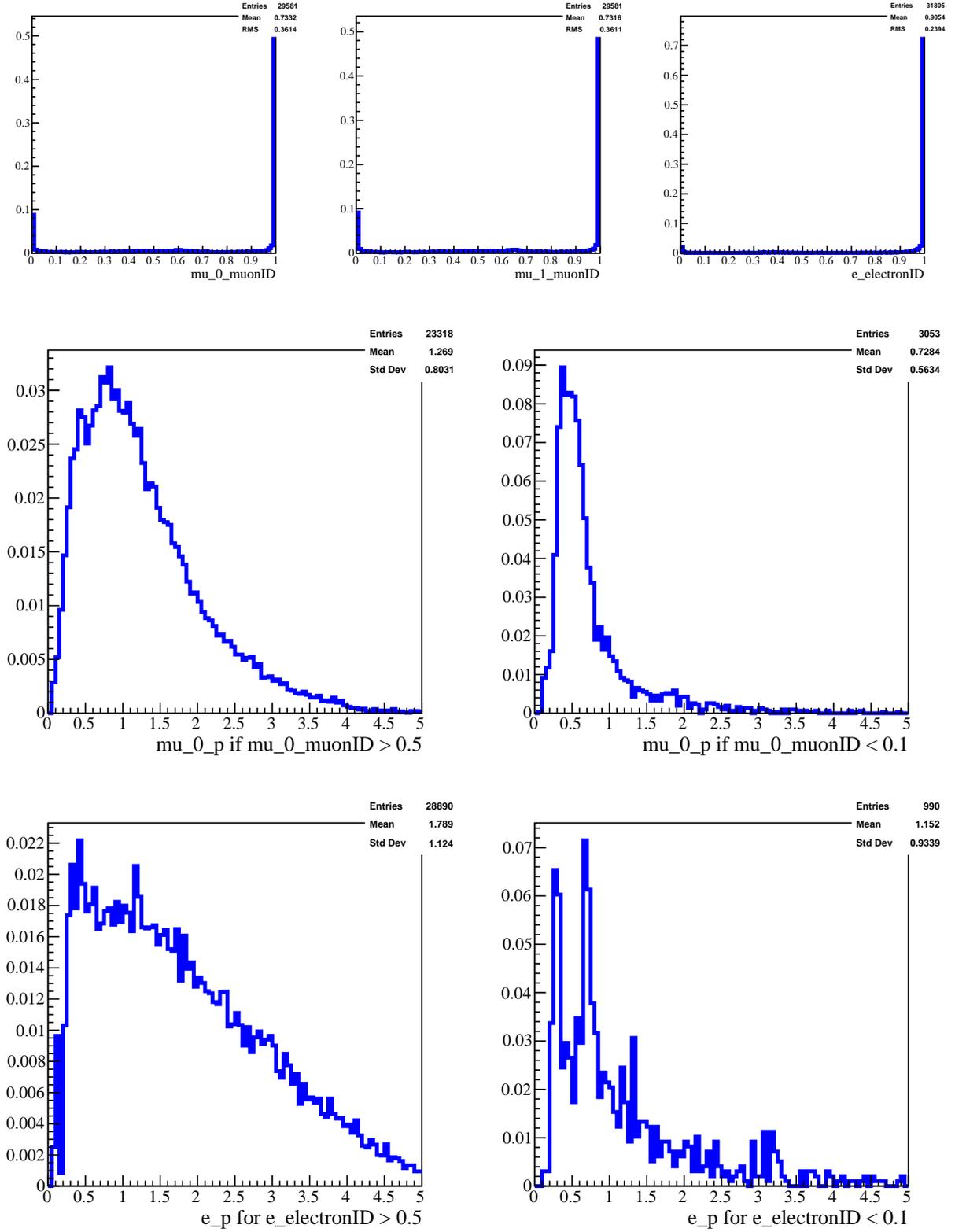


FIG. 33: Belle II lepton ID performances for the signal samples of  $m_N = 1000$  MeV

TABLE XIII: The branching fraction  $Br(\tau^- \rightarrow N\pi^-)$  as a function of the HNL mass for  $|V_{\tau N}|^2 = 1$

HNL mass (GeV)	$br(\tau^- \rightarrow N\pi^-)$
0.2	250.10716611
0.25	0.10592627
0.275	0.104567
0.3	0.10309155
0.325	0.10150349
0.35	0.099806623
0.375	0.098005062
0.4	0.096103157
0.425	0.094105516
0.45	0.092016986
0.475	0.089842646
0.5	0.087587795
0.525	0.085257939
0.55	0.08285878
0.575	0.080396201
0.6	0.077876256
0.625	0.075305152
0.65	0.072689236
0.675	0.070034978
0.7	0.067348957
0.725	0.064637841
0.75	0.061908372
0.775	0.05916735
0.8	0.056421606
0.825	0.053677993
0.85	0.050943358
0.875	0.048224525
0.9	0.045528274
0.925	0.042861318
0.95	0.040230279
0.975	0.037641667
1.	0.035101856
1.025	0.03261706
1.05	0.030193305
1.075	0.027836405
1.1	0.025551938
1.125	0.023345215
1.15	0.021221255
1.175	0.019184758
1.2	0.01724007
1.225	0.015391161
1.25	0.013641592
1.275	0.011994482
1.3	0.010452482
1.325	0.0090177381
1.35	0.0076918632
1.375	0.0064759024
1.4	0.0053703008
1.425	0.0043748704
1.45	0.0034887568
1.475	0.0027104063
1.5	0.0020375327
1.525	0.0014670828
1.55	0.0009519505
1.575	0.00061711744
1.6	0.00032688486

TABLE XIV: The branching fraction  $Br(N \rightarrow \mu^+ \mu^- \nu_\tau)$  as a function of the HNL mass for  $|V_{\tau N}|^2 = 1$

HNL mass (GeV)	$Br(\tau^- \rightarrow \mu^+ \mu^- \nu_\tau)$
0.225	0.0000037764628
0.25	0.000094673938
0.275	0.00038099178
0.3	0.00089464024
0.325	0.001638705
0.35	0.002587494
0.375	0.003713401
0.4	0.0049975021
0.425	0.0064250842
0.45	0.0079446983
0.475	0.009559162
0.5	0.011272705
0.525	0.012991204
0.55	0.014794949
0.575	0.016582111
0.6	0.01837722
0.625	0.020055846
0.65	0.021790102
0.675	0.023393541
0.7	0.025041682
0.725	0.026631279
0.75	0.028147725
0.775	0.029697193
0.8	0.030922257
0.825	0.031866706
0.85	0.032785284
0.875	0.033681067
0.9	0.03445172
0.925	0.035115506
0.95	0.03575332
0.975	0.036367441
1.	0.03525189
1.025	0.03291937
1.05	0.033283246
1.075	0.033628892
1.1	0.033957696
1.125	0.034270923
1.15	0.034564315
1.175	0.034835458
1.2	0.035093944
1.225	0.035340668
1.25	0.03557645
1.275	0.035802041
1.3	0.036014904
1.325	0.036211503
1.35	0.036399689
1.375	0.036580009
1.4	0.036752964
1.425	0.036919017
1.45	0.037078599
1.475	0.037235705
1.5	0.037396646
1.525	0.037552345
1.55	0.037703114
1.575	0.037849242
1.6	0.037990999

TABLE XV: The product  $c\tau$  of the speed of light and the HNL lifetime as a function of the HNL mass for  $|V_{\tau N}|^2 = 1$

HNL mass (GeV)	$c\tau$
0.2	3168.9416
0.225	1635.1219
0.25	968.10727
0.275627.83044	0.3 432.12832
0.325	312.45594
0.35	232.93065
0.375	177.89561
0.4	138.81736
0.425	110.38016
0.45	88.771277
0.475	72.333378
0.5	59.712532
0.525	49.526423
0.55	41.55885
0.575	35.021382
0.6	29.699644
0.625	25.187928
0.65	21.558764
0.675	18.457172
0.7	15.928967
0.725	13.793066
0.75	11.977082
0.775	10.466808
0.8	9.0952638
0.825	7.87625
0.85	6.852706
0.875	5.9886415
0.9	5.2394944
0.925	4.5912124
0.95	4.0379663
0.975	3.5637968
1.	3.009908
1.025	2.4586761
1.05	2.1825495
1.075	1.9429384
1.1	1.734321
1.125	1.5521106
1.15	1.392263
1.175	1.2515194
1.2	1.1275544
1.225	1.0180768
1.25	0.92114613
1.275	0.83511389
1.3	0.75850663
1.325	0.69006244
1.35	0.62891329
1.375	0.5741669
1.4	0.52505366
1.425	0.48090796
1.45	0.44115266
1.475	0.40532523
1.5	0.37303789
1.525	0.3438001
1.55	0.31728033
1.575	0.29318769
1.6	0.27126637

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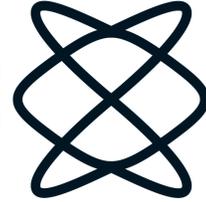
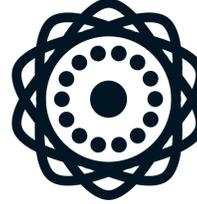
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## תקציר

קיימת מוטיבציה גבוהה למציאת לפטונים ניטרלים כבדים (Heavy Neutral Leptons - HNLs), שיכולים להיות הפתרון לבעיות המסה של הניטרונים, החומר האפל והאסימטריה הבריונית ביקום. ניסויים קודמים הצליחו לקבוע גבולות צפופים על ערבוב (mixing) של HNL עם חלקיקי ניטרינו אלקטרוני ומואוני. היות שכך, אנו מחפשים את HNL המתערבב בעיקר עם חלקיקי ניטרינו טאואוני. HNL שכזה יכול להיות מיוצר בדעיכות של חלקיקי טאו, והמקום הטוב ביותר לחקור אותם הוא במפעלי B (B-factories). החיפוש שלנו מתבצע בדגימות של  $10^8$  אירועים של  $e^+e^- \rightarrow \tau^+\tau^-$  עם אנרגיית מרכז מסה של 10.58 GeV, אשר נאספו ע"י בל (Belle). החיפוש מתמקד בחלקיקי HNL בעלי זמן חיים ארוך, ובטווח מסה של 0.3-1.6 GeV, כך שאנו מנצלים חתימה של קודקוד מרוחק (Displaced Vertex) כדי לצמצם את הרקע. אנו מציבים צפי לגבול חדש על הערבוב בין HNL לניטרינו טאואוני.

הפקולטה למדעים  
מדויקים ע"ש ריימונד  
ובברלי סאקלר  
אוניברסיטת תל אביב



בית הספר לפיזיקה ואסטרונומיה

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חיבור זה הוגש כחלק מהדרישות לקבלת התואר  
M.Sc. – "מוסמך אוניברסיטה"

על ידי

**אורי ישראל פוגל**

העבודה נכתבה בהדרכתו של

**פרופ' אבנר סופר**