

Measurements of hadronic, leptonic, and semi-leptonic B decays at Belle and Belle II

59th Rencontres de Moriond | QCD & High Energy Interactions

La Thuile, Italy

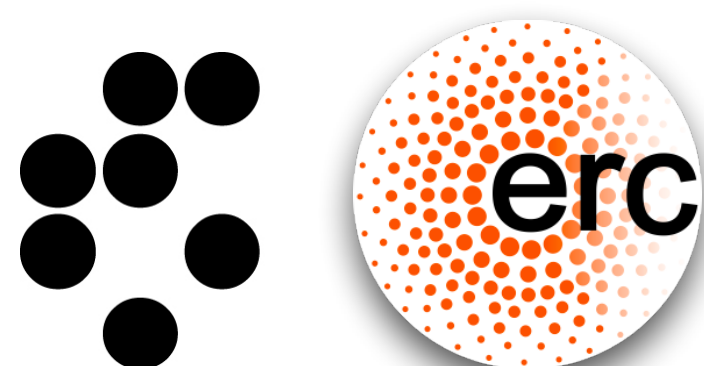
30th March - 6th April 2025

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On behalf of the Belle and Belle II collaborations



All results are new since Moriond 2024

Hadronic decays of B mesons

- $B^0 \rightarrow \rho^+ \rho^-$
- $B^0 \rightarrow \bar{\Lambda}^0 \Omega^{(*)0}$

Missing energies leptonic decays of B mesons

- $B \rightarrow \tau \nu_\tau$

Missing energies semi-leptonic decays of B mesons

- $R(D^+), R(D^{*+})$
- $|V_{cb}|$ from $B \rightarrow D \ell \nu$

Datasets for these measurements

Belle

$$\mathcal{L}_{\text{intg}}^{\Upsilon(4S)} = 711 \text{ fb}^{-1} \quad (1999 - 2010)$$

Belle II

$$\mathcal{L}_{\text{intg}}^{\Upsilon(4S)} = 365 \text{ fb}^{-1} \text{ (Run 1)} \quad (2019 - 2022)$$

All Belle (II) analyses are performed using opensourced
Belle II software framework [[Comput Softw Big Sci 3, 1 \(2019\)](#)]

Other interesting results from Belle II:

[Bianca Scavino](#), [Debjit Ghosh](#),

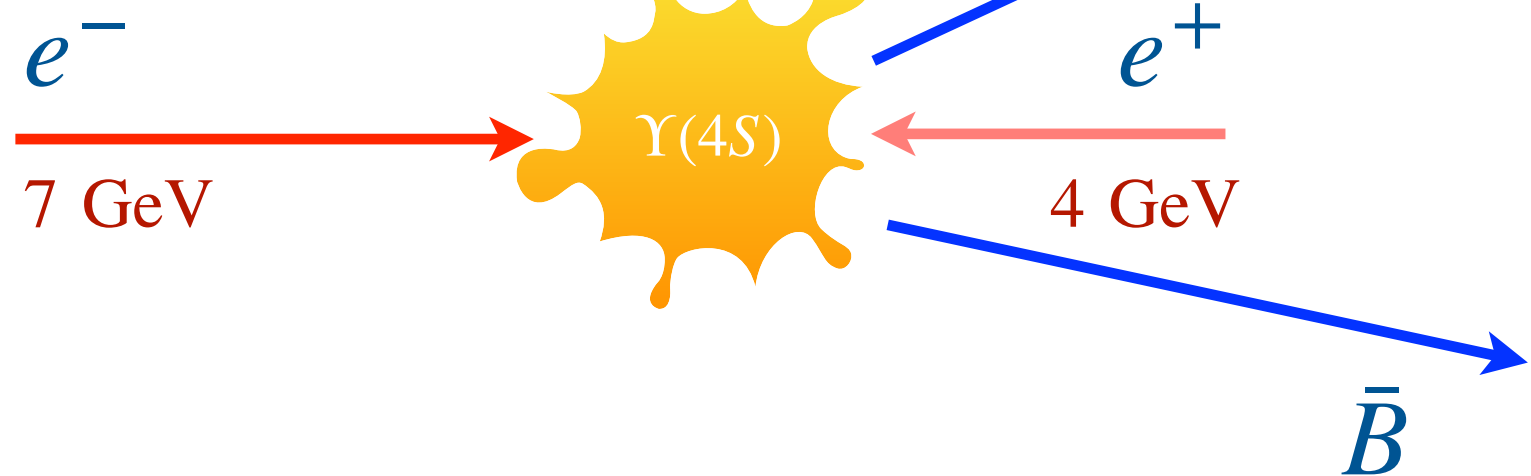
[Zuzana Gruberova](#) (tomorrow, morning session)

Kinematics at B factories

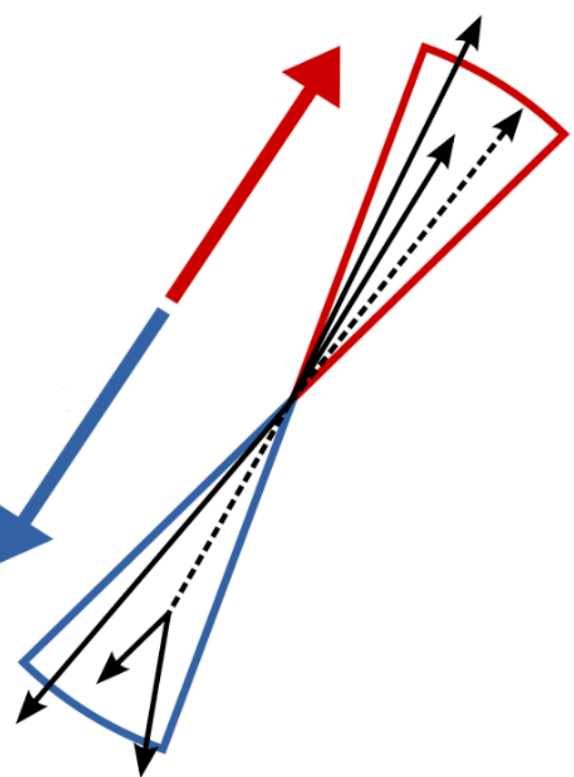


Threshold production

“clean” environment

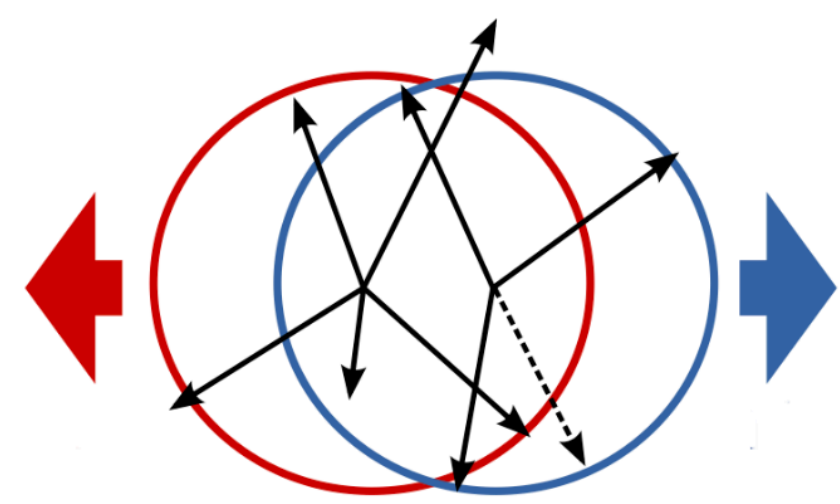


Event topology in $\Upsilon(4S)$ frame



Jet-like topology

$$e^+e^- \rightarrow q\bar{q}, q = u, d, s, c$$



Isotropic distribution of B

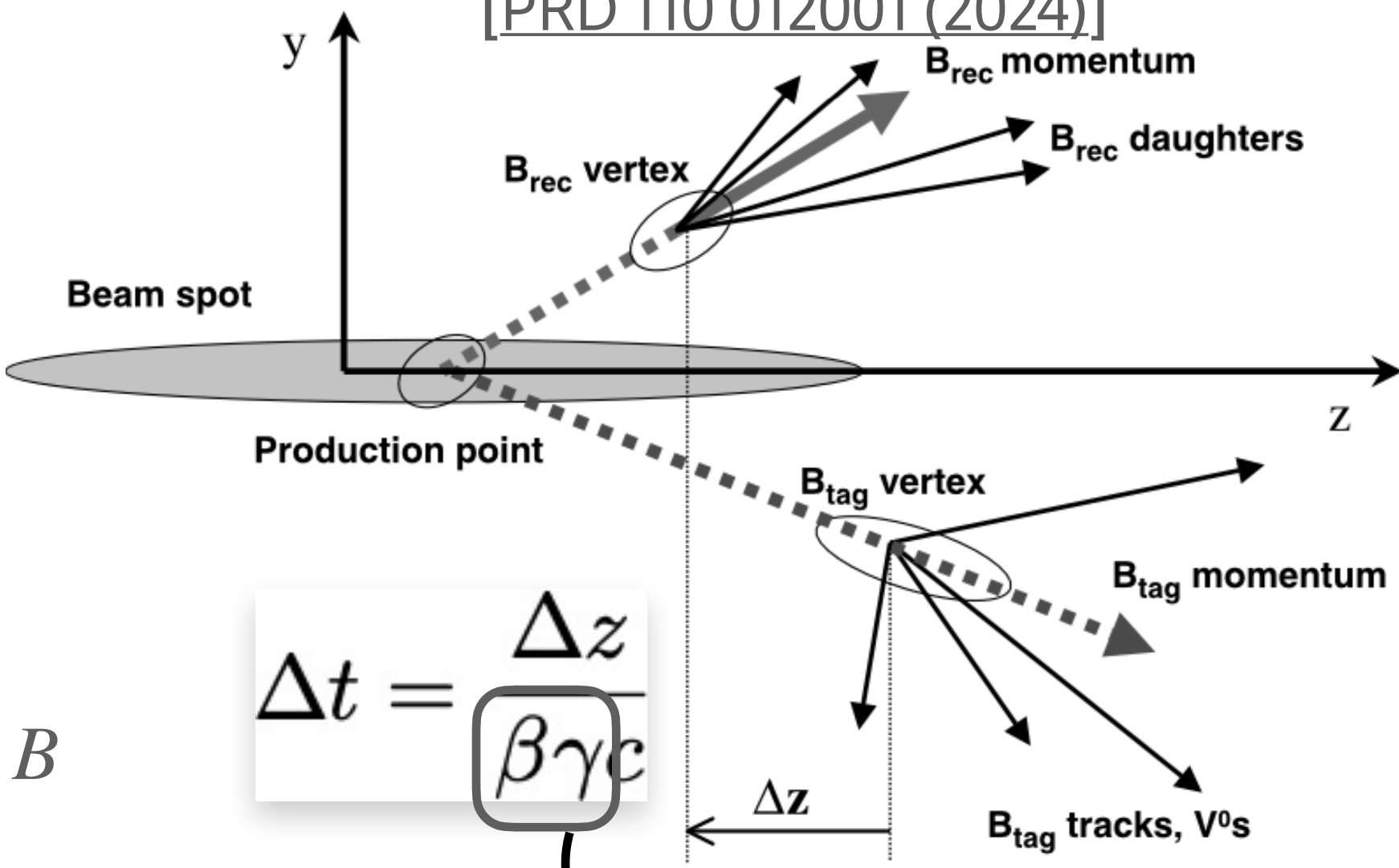
$$M_{bc} = \sqrt{E_{\text{beam}}^{*2}/c^4 - p_B^{*2}/c^2}$$

$$\Delta E = E_B^* - E_{\text{beam}}^*$$

Flavor Tagging CP analyses

[Eur. Phys. J.C 82, 283 (2022)]

[PRD 110 012001 (2024)]



$$\Delta t = \frac{\Delta z}{\beta\gamma c}$$

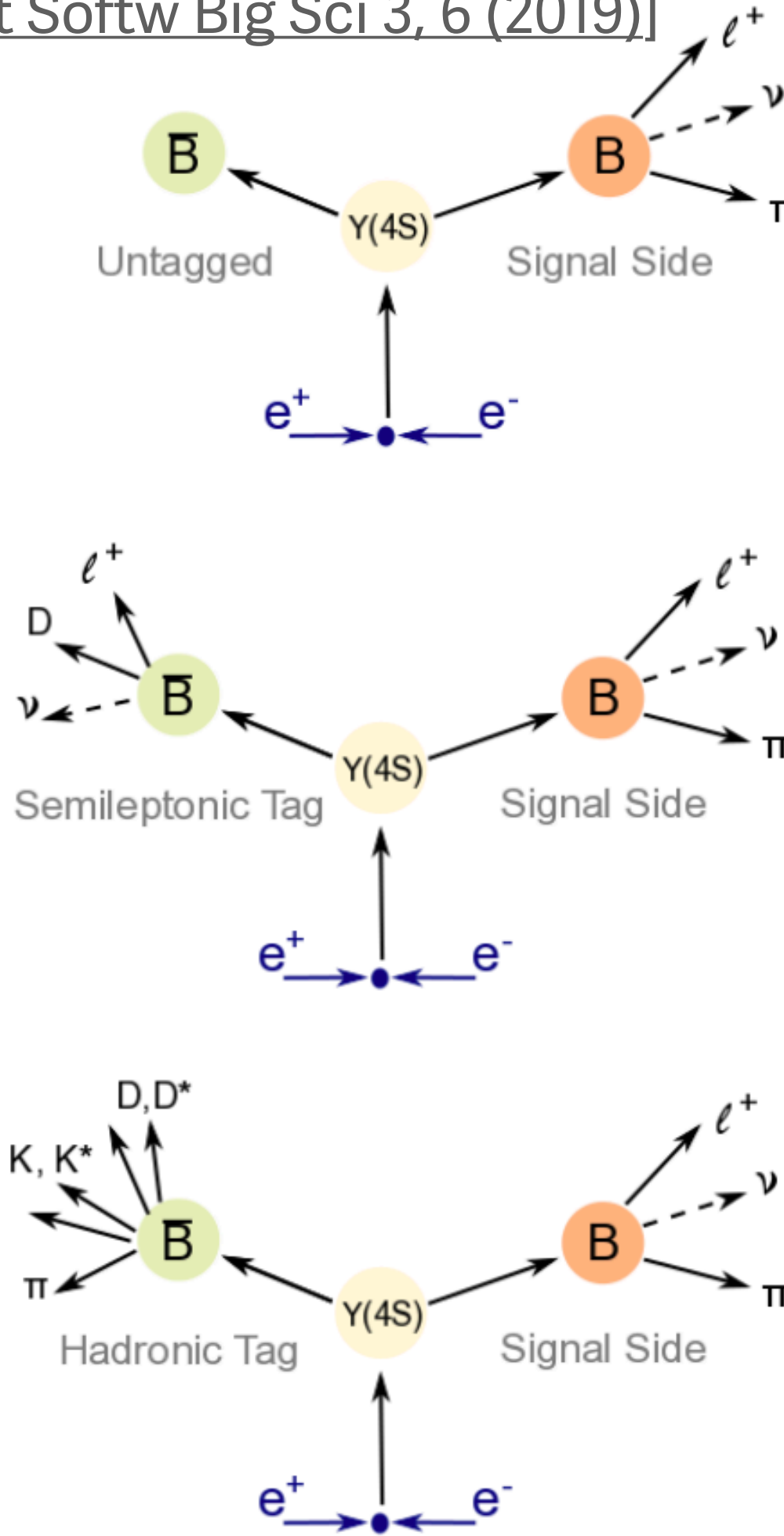
Boost = 0.29



Aid to missing energy B decays

Full Event Interpretation (FEI)

[Comput Softw Big Sci 3, 6 (2019)]



purity

efficiency

Image from L. Cao

Hadronic B decays at Belle and Belle II

$$- B^0 \rightarrow \rho^+ \rho^-$$

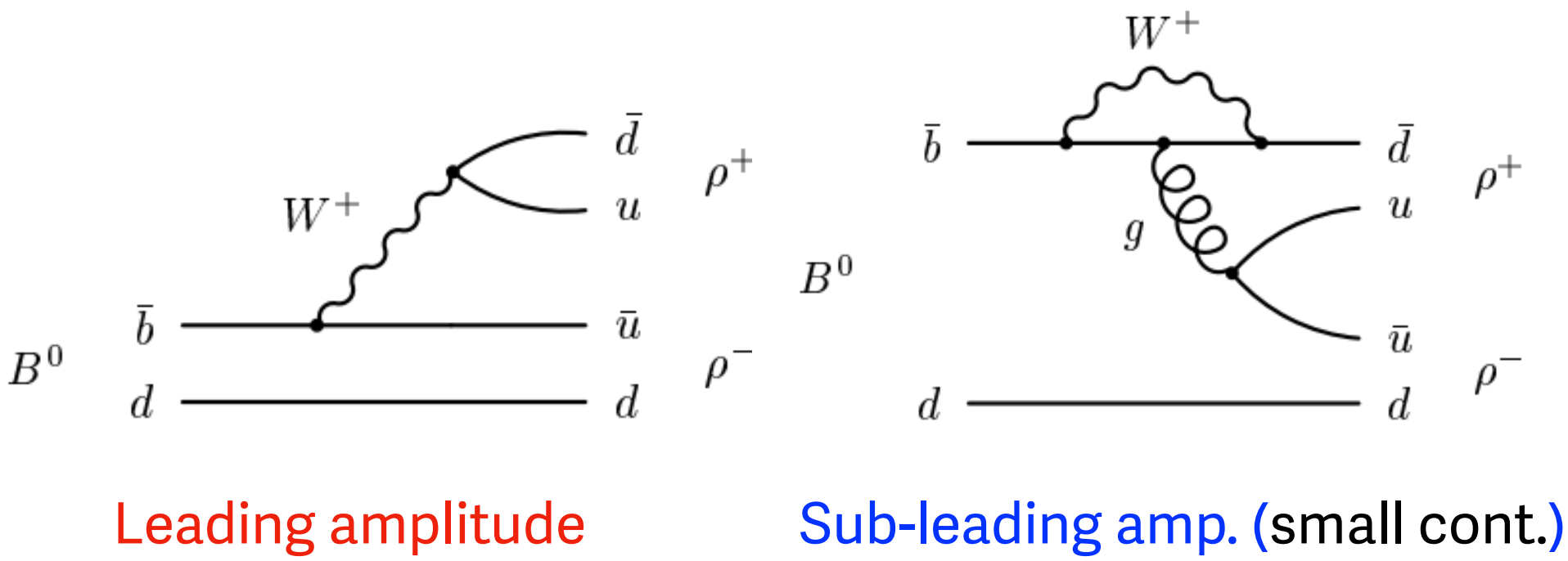
$$- B^0 \rightarrow \bar{\Lambda}^0 \Omega_c^{(*)0}$$

Prologue: $B^0 \rightarrow \rho^+ \rho^-$



Theoretical motivation

- $B^0 \rightarrow \rho^+ \rho^-$ provides stringent constraint on ϕ_2 due to small contribution from the “loop” amplitude
- Angle $\phi_2/\alpha = \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$ is the least known angle of the UT



Probability distribution

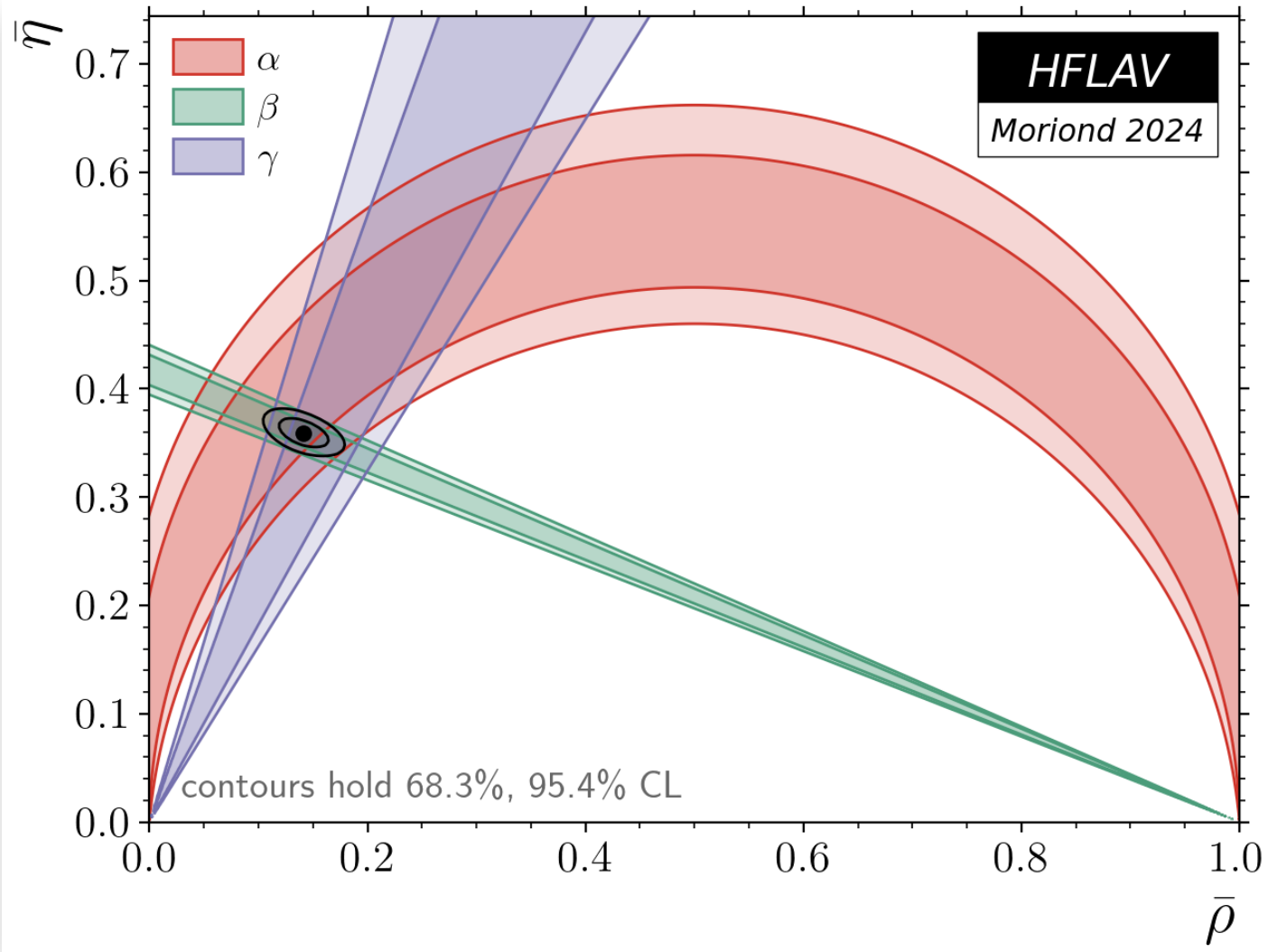
$$P(\Delta t, q) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left\{ 1 + q \left[S \sin(\Delta m_d \Delta t) - C \cos(\Delta m_d \Delta t) \right] \right\},$$

Direct CPV param.

Helicity angle distribution

$$\frac{1}{\Gamma} \frac{d^2\Gamma}{d \cos \theta_{\rho^+} d \cos \theta_{\rho^-}} = \frac{9}{4} \left[\frac{1}{4} (1 - f_L) \sin^2 \theta_{\rho^+} \sin^2 \theta_{\rho^-} + f_L \cos^2 \theta_{\rho^+} \cos^2 \theta_{\rho^-} \right],$$

Longitudinal polarisation fraction



Experiment

- Measurements dominated by BaBar and Belle (BF, CP, polarisation)
- Challenging due to presence of 4 photons in the final state; peaking bkg and combinatorics

$$\phi_1 \equiv \beta = (22.6^{+0.5}_{-0.4})^\circ \quad \phi_3 \equiv \gamma = (66.4^{+2.8}_{-3.0})^\circ \quad \phi_2 \equiv \alpha = (84.1^{+4.5}_{-3.8})^\circ$$

Decreasing order of precision

$$B^0 \rightarrow \rho^+ \rho^-$$



arXiv:2412.19624 | Submitted to Phys. Rev. D



Goal: Branching fraction (BF), polarisation, CP asymmetry, ϕ_2 measurement

STEP I: Reconstruction ($B^0 \rightarrow \rho^+[\rightarrow \pi^+\pi^0[\rightarrow \gamma\gamma]] \rho^-$)

- Boosted Decision Tree (BDT) based separation of photons from hadronic clusters

- Signals are discriminated from backgrounds via M_{bc} and ΔE

$$M_{bc} = \sqrt{E_{\text{beam}}^{*2}/c^4 - p_B^{*2}/c^2} \quad \Delta E = E_B^* - E_{\text{beam}}^*$$

- B_{tag} flavor is identified using a **GNN-based flavor tagger** [PRD 110 012001 (2024)]

- **18% improvement** over category-based Belle II FT algorithm

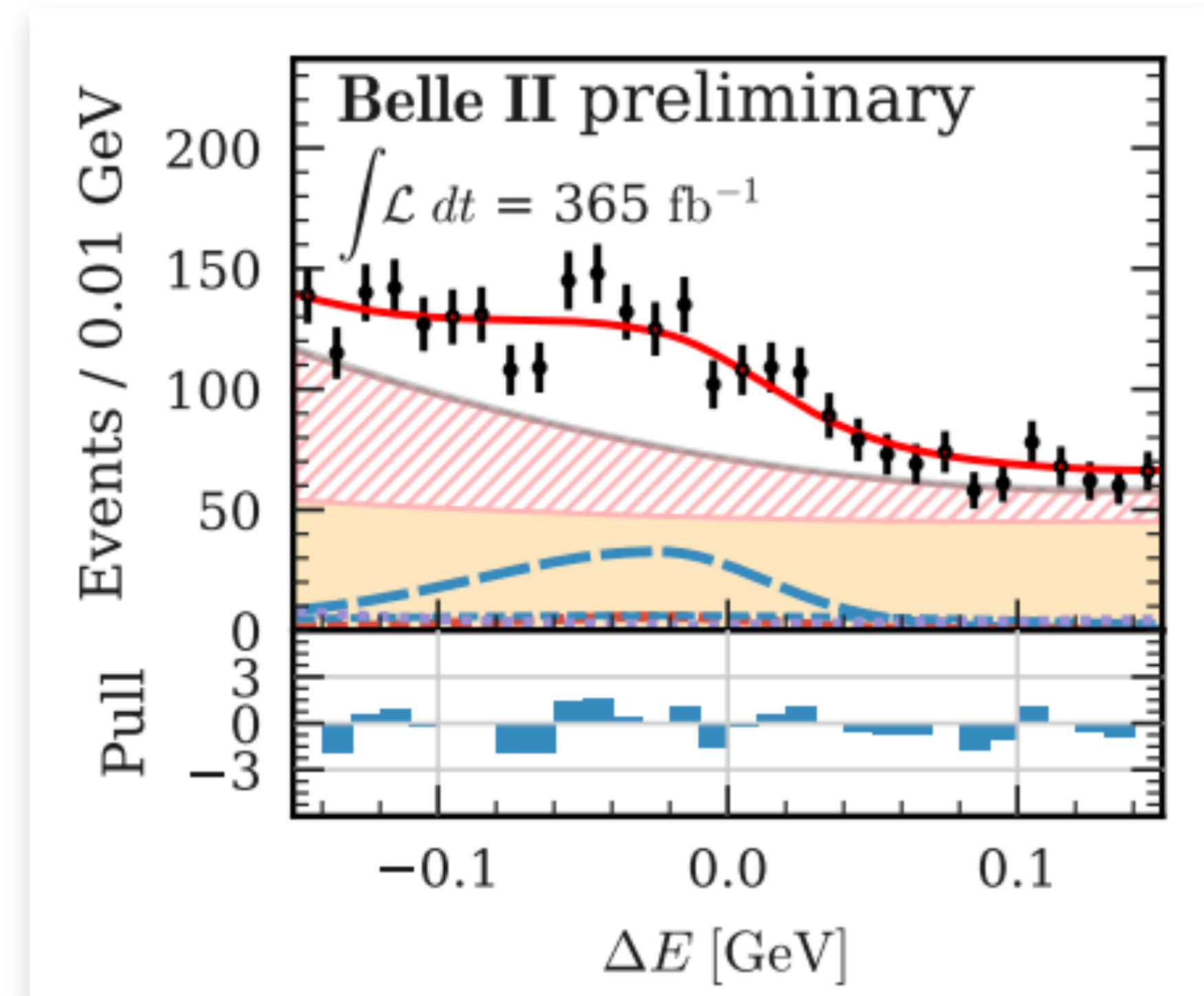
[Eur. Phys. J.C 82, 283 (2022)]

Backgrounds

- Continuum backgrounds are suppressed using a TabNet classifier
- Other irreducible backgrounds: combinatorial and peaking $B\bar{B}$, $\tau^+\tau^-$, and signal “cross-feeds” are modelled with PDFs



$$M_{bc} > 5.275 \text{ GeV}/c^2$$



$$B^0 \rightarrow \rho^+ \rho^-$$



arXiv:2412.19624 | Submitted to Phys. Rev. D



STEP II: Signal extraction fit (2 stage fit)

- Stage 1: Extended ML fit to 6 obs: $\Delta E, m_{\pi^\pm \pi^0}, T_C, \cos \theta_{\rho^\pm}$
 - Extract \mathcal{B}, f_L (longitudinal polarisation)
- Stage 2: Extended ML fit to 3 obs: $\Delta t, q, r$
 - Extract S, C

tag quality

Flavor $B(\bar{B}) \equiv 1(-1)$

Results

$$\mathcal{B}(B^0 \rightarrow \rho^+ \rho^-) = (2.88^{+0.23}_{-0.22} + {}^{+0.29}_{-0.27}) \times 10^{-5}$$

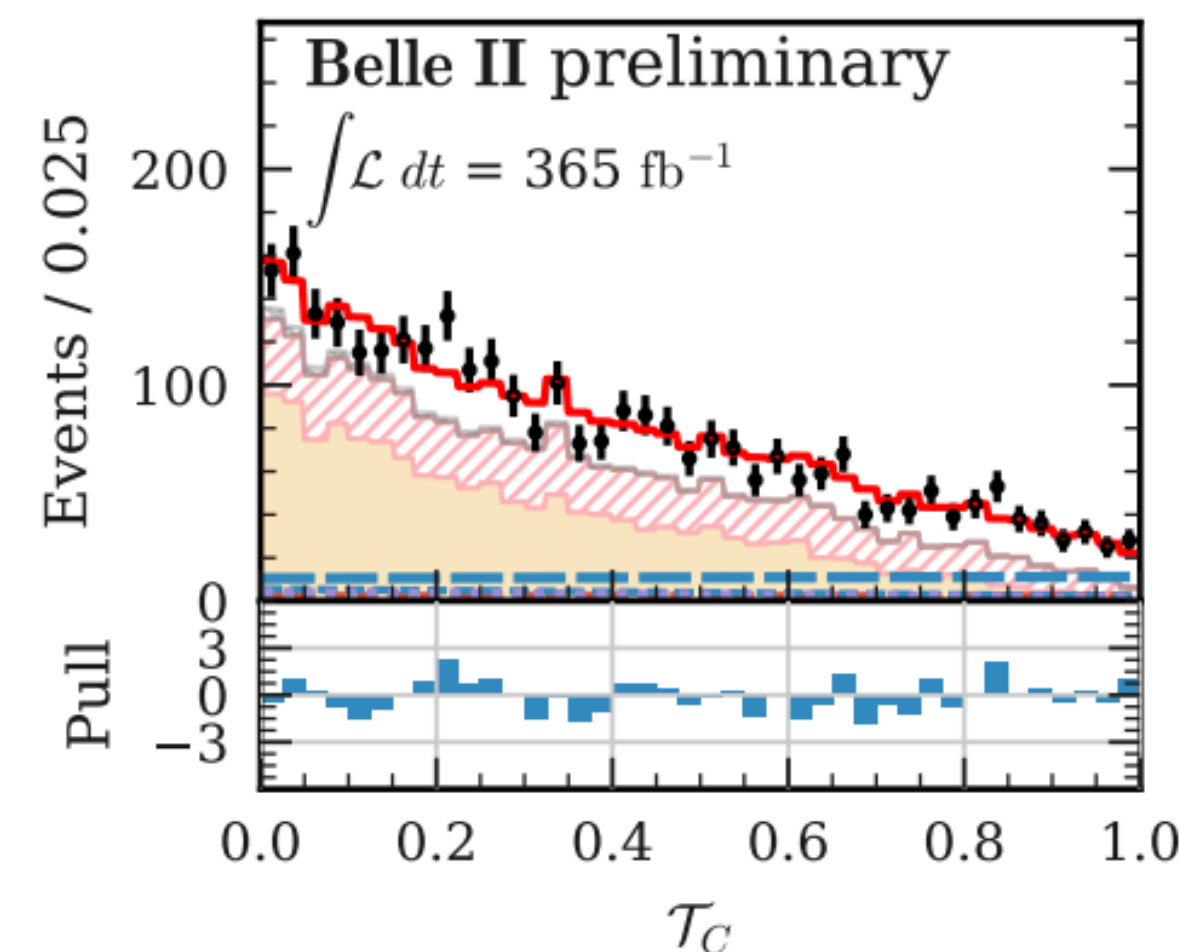
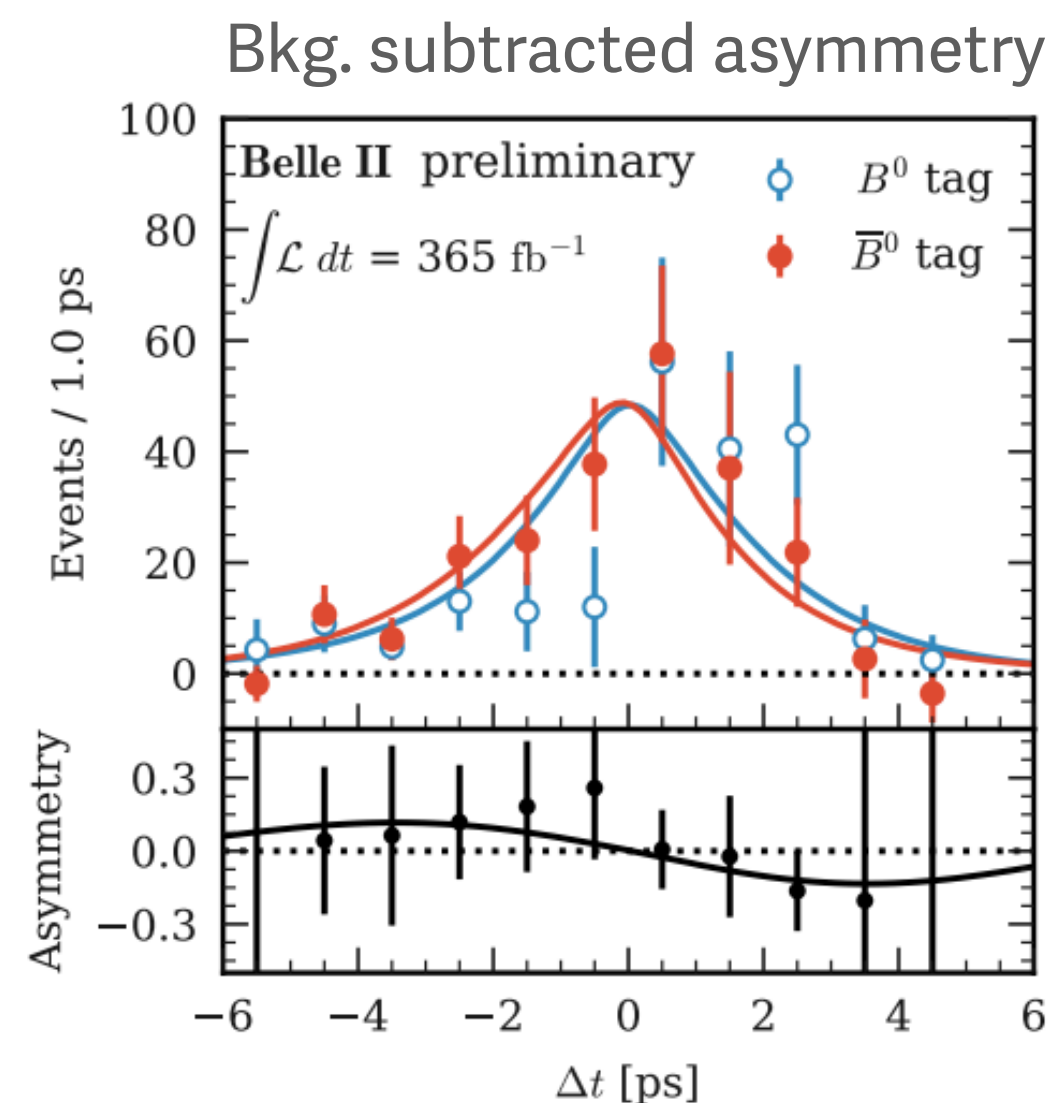
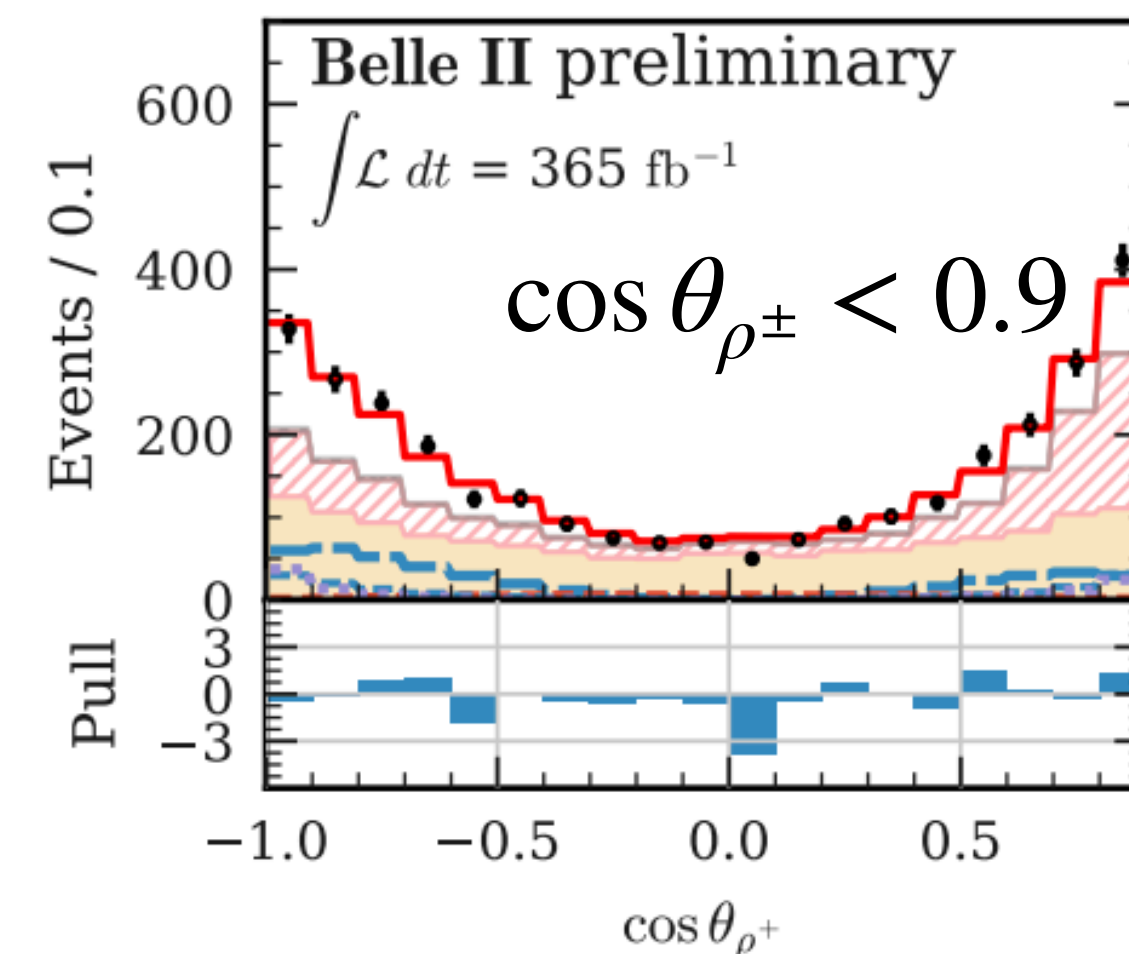
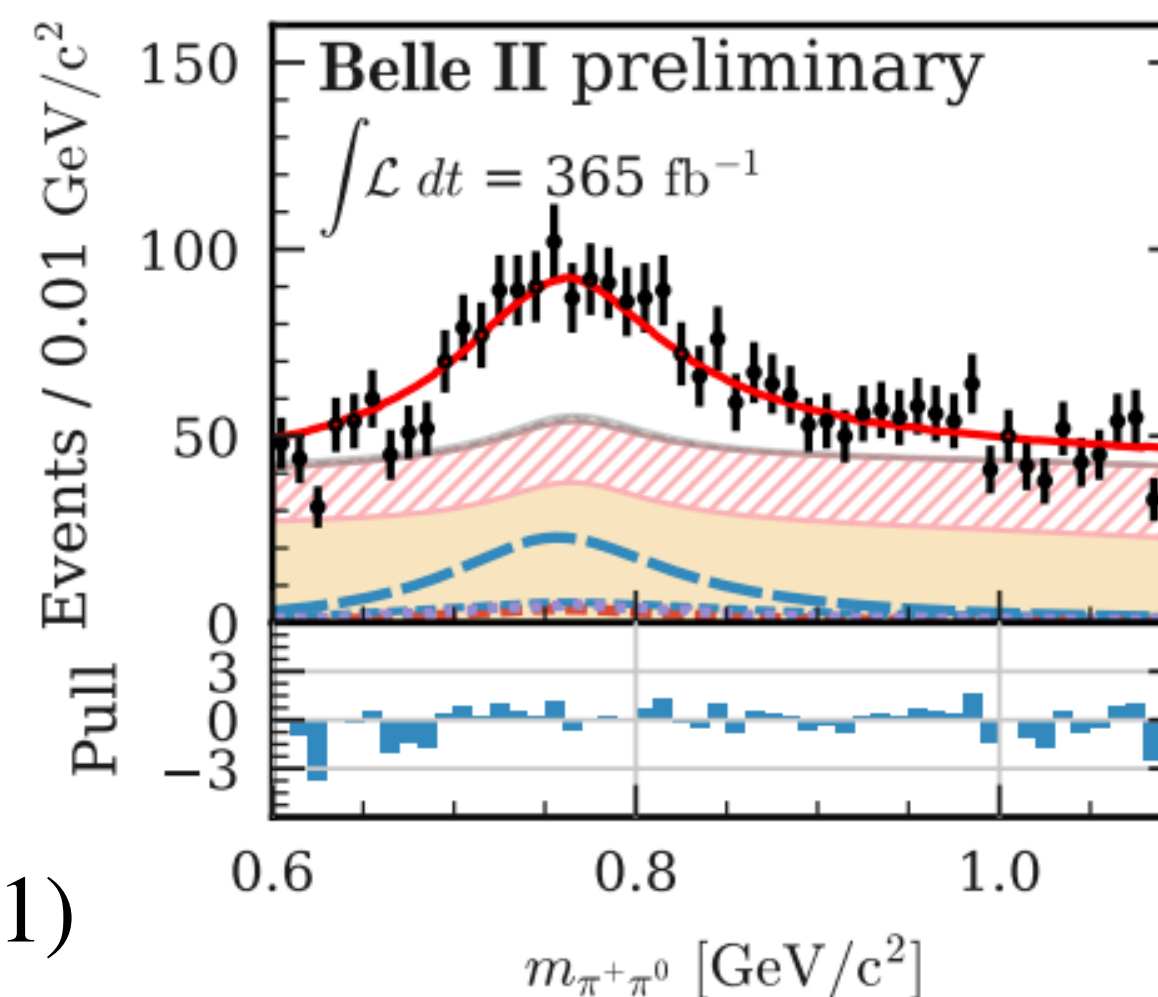
$$f_L = 0.921^{+0.024}_{-0.025} + {}^{+0.017}_{-0.015}$$

$$S = -0.26 \pm 0.19 \pm 0.08$$

$$C = -0.02 \pm 0.12 \pm 0.05$$

First uncertainty is statistical, second is systematics

(Preliminary)



Good agreement with previous BaBar (2007) and Belle (2016) expt. with equivalent BaBar and ~ 50% of Belle equivalent luminosity !

$$B^0 \rightarrow \rho^+ \rho^-$$



arXiv:2412.19624 | Submitted to Phys. Rev. D



STEP III: Constraining ϕ_2

- Perform isospin analysis based on longitudinal amplitudes, A_{ij}
- Constrain using **this** measurement + **World Averages** (BaBar, Belle, LHCb)
- Correct \mathcal{B} due to inclusion of f_{+-}/f_{00} systematics HFLAV24 (NEW)
 - Inclusion shifts the ϕ_2 value by -0.4°

$$\begin{aligned} \frac{1}{\sqrt{2}} A_{+-} + A_{00} &= A_{+0}, \\ \frac{1}{\sqrt{2}} \bar{A}_{+-} + \bar{A}_{00} &= \bar{A}_{-0}, \end{aligned}$$

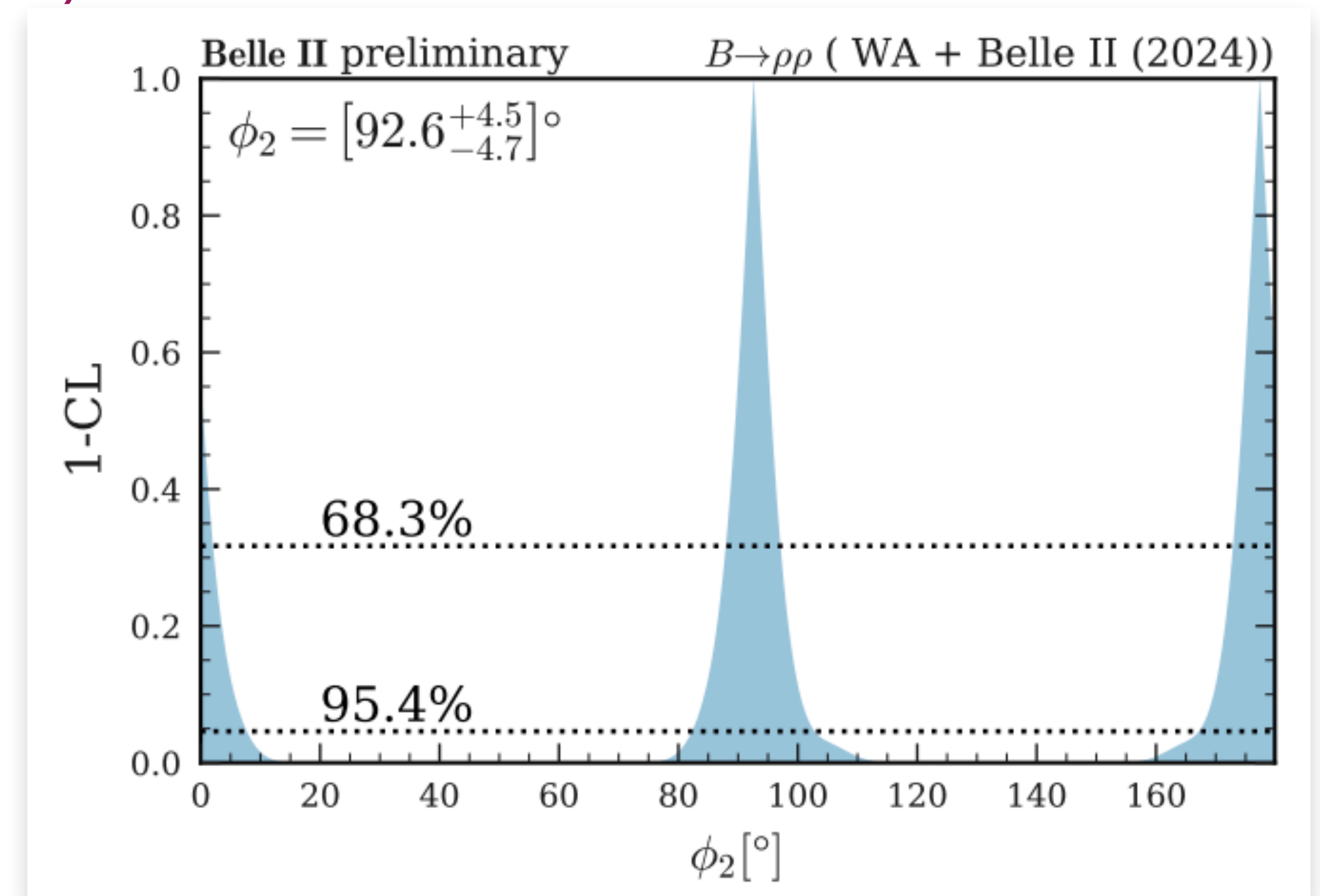
Result (from this measurement) (Preliminary)

$$\phi_2 = (91.5^{+4.8}_{-5.2})^\circ$$

$$\Delta\phi_2 = (2.4^{+4.2}_{-3.8})^\circ$$

Agrees with WA, HFLAV24

- Belle II result improves the ϕ_2 value by ~8%
- Dominant systematics (this measurement) from S [\[backup\]](#)



Second solution of ϕ_2 excluded by ϕ_1 and ϕ_3 measurements

Prologue: $B^0 \rightarrow \bar{\Lambda}^0 \Omega_c^{(*)0}$

Theoretical motivation

- Probe low-energy mechanism for baryon number violation [Phys. Rev. D 96, 075009 (2017)]

$$\bar{B}^0 \rightarrow \bar{\Lambda}^0 \Omega_c^0, \bar{B}^0 \rightarrow \bar{\Lambda}^0 \bar{\Omega}_c^0$$

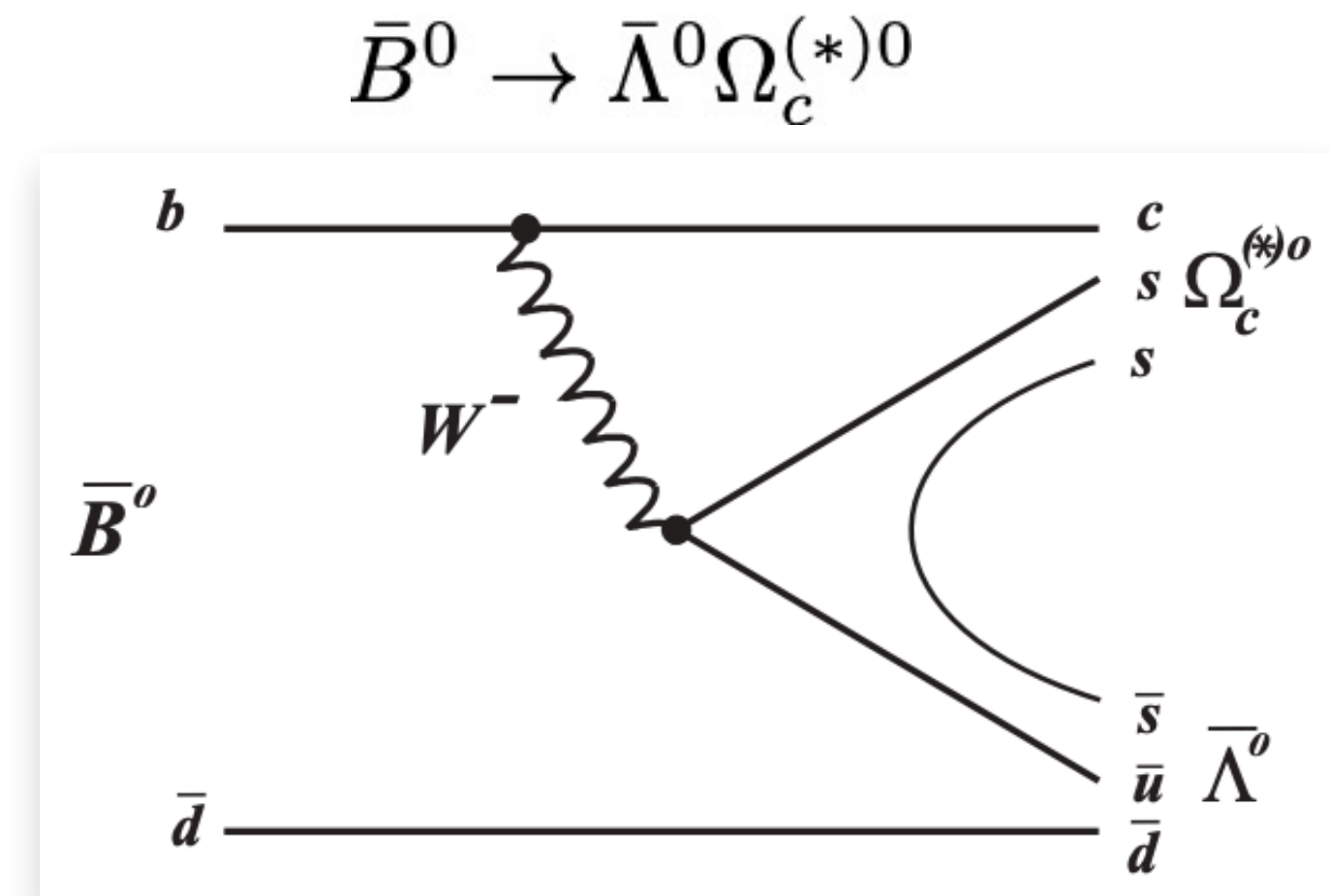
Cabibbo suppressed SM decay

Beyond SM decay

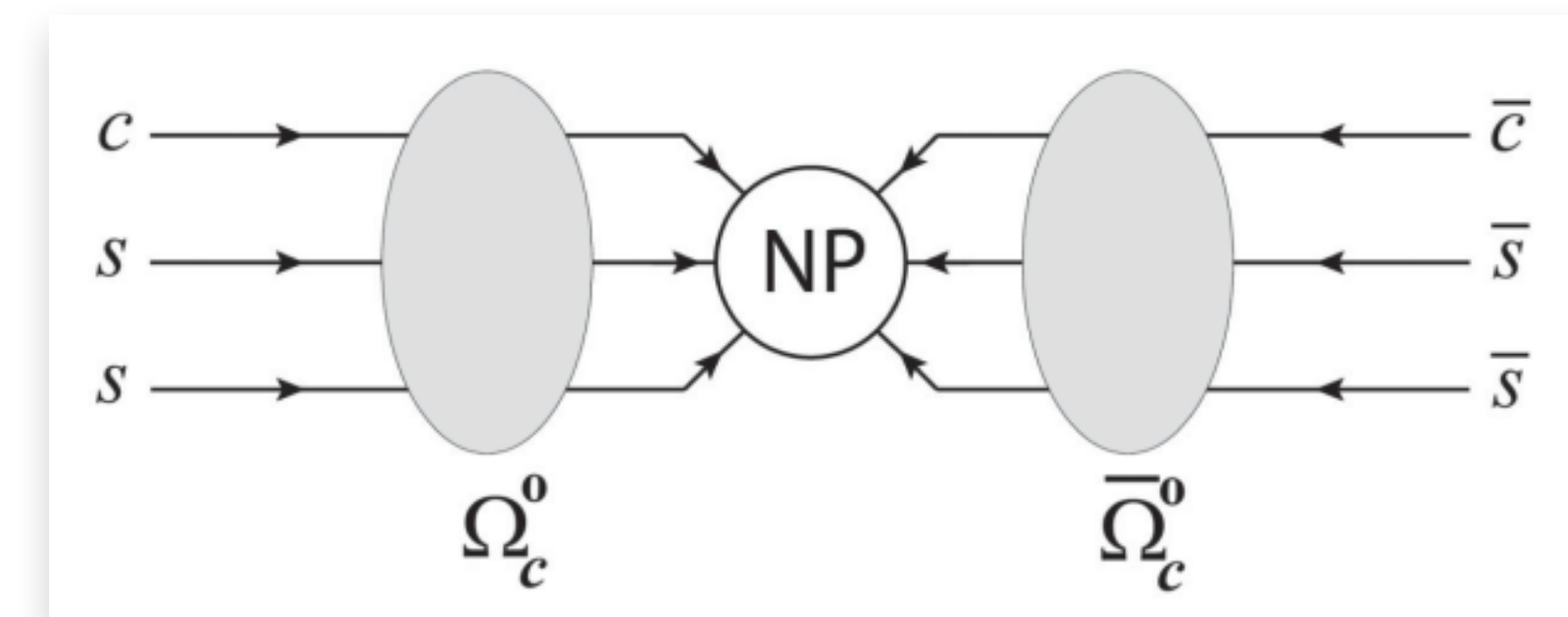
- Poorly understood due to large hadronic uncertainties

Experiment

- No previous experimental measurements exist
- Searched for the first time using Belle dataset
- Consider two Ω_c^0 states: Ω_c^0 and $\Omega_c(2700)^0$, collectively referred to as $\Omega_c^{(*)0}$



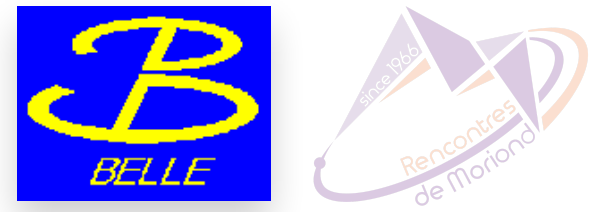
$$\Omega_c^0 \leftrightarrow \bar{\Omega}_c^0$$



$$B^0 \rightarrow \bar{\Lambda}^0 \Omega_c^{(*)0}$$



Phys. Rev. D 110, L031102 (2024)



Goal: Search for the decays $\bar{B}^0 \rightarrow \bar{\Lambda}^0 \Omega_c^0$ and $\bar{B}^0 \rightarrow \bar{\Lambda}^0 \bar{\Omega}_c^0$

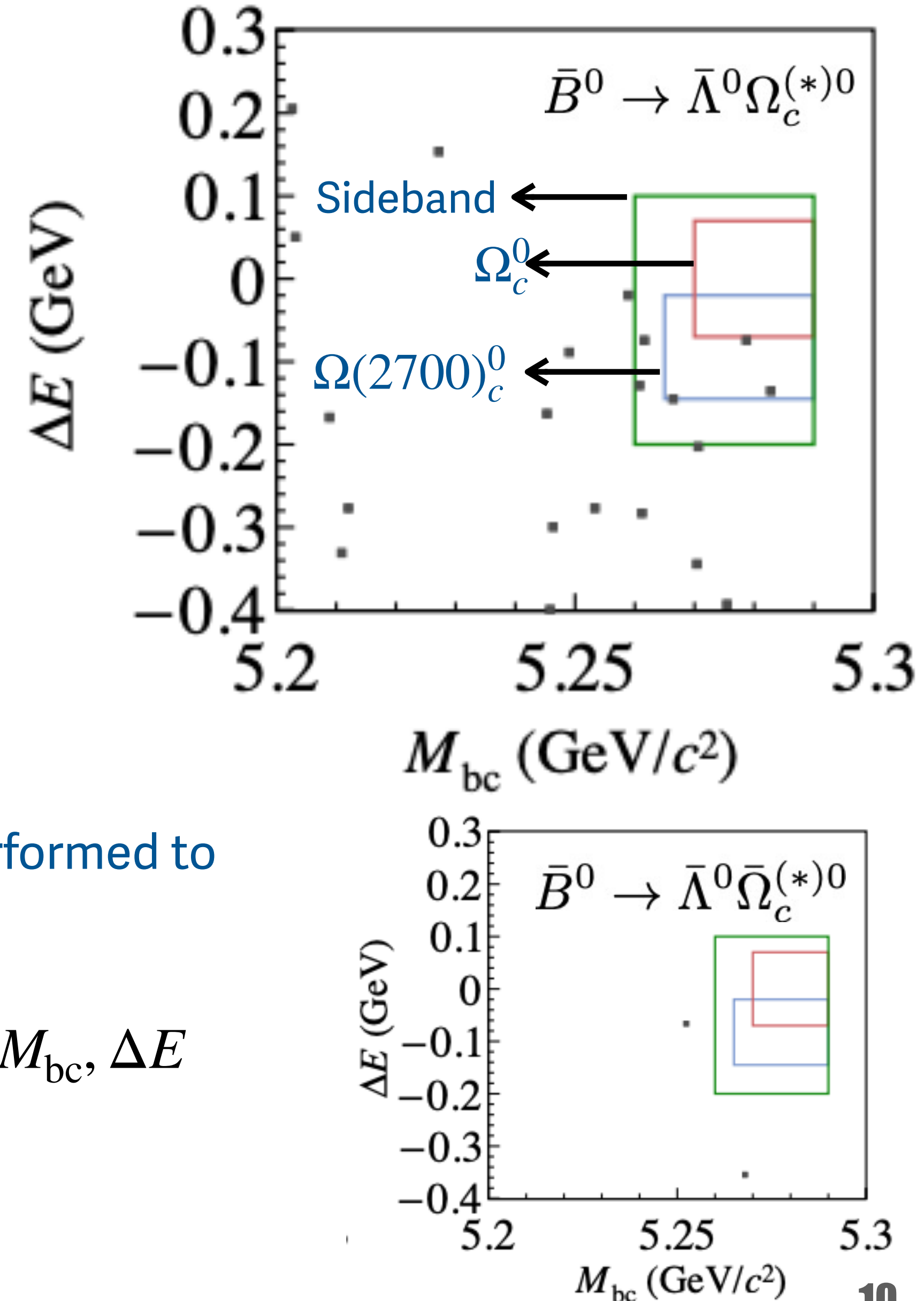
Dataset: 711 fb⁻¹ of Belle data collected at $\Upsilon(4S)$ resonance

STEP I: Reconstruction

- Signal B^0 's are reconstructed from:

$$\Lambda^0 \rightarrow p\pi^-, \Omega^- \rightarrow K^-\Lambda^0, \Omega_c^0 \rightarrow \pi^+\Omega^-, \Omega(2770)_c^0 \rightarrow \Omega_c^0\gamma \text{ (partially reco.)}$$
- Two signal categories: $\bar{\Lambda}^0 \Omega_c^{(*)0}, \bar{\Lambda}^0 \bar{\Omega}_c^{(*)0}$
- PID selections, mass / vertex-constraint fits, selection on the decay lengths performed to reduce combinatorics
- Signals are discriminated from the backgrounds using the kinematic variables: $M_{bc}, \Delta E$

$$M_{bc} = \sqrt{E_{\text{beam}}^{*2}/c^4 - p_B^{*2}/c^2} \quad \Delta E = E_B^* - E_{\text{beam}}^*$$



$$B^0 \rightarrow \bar{\Lambda}^0 \Omega_c^{(*)0}$$



STEP II: Signal extraction

- Use counting method due to low background statistics
- Simulations confer that events outside the signal (blinded) region are dominated by $q\bar{q}$ and non-signal $B\bar{B}$ events

Numbers of events	Total	Blinded region	$\bar{\Lambda}^0 \Omega_c^0$ signal region	$\bar{\Lambda}^0 \Omega_c(2770)^0$ signal region
$\bar{\Lambda}^0 \Omega_c^0$ data	21	5	0	3
Background	N/A	1.6 ± 0.7	0.44 ± 0.45	0.44 ± 0.45
$\bar{\Lambda}^0 \bar{\Omega}_c^0$ data	2	0	0	0
Background	N/A	0.18 ± 0.17	0.00 ± 0.12	0.12 ± 0.15

Results

First upper limits on 2-body BNV decays

Quantity ($\times \mathcal{B}(\Omega_c^0 \rightarrow \Omega^- \pi^+)$)	Upper limit (at 95% CL)
$\mathcal{B}(B \rightarrow \bar{\Lambda}^0 \Omega_c^0)$	9.7×10^{-8}
$\mathcal{B}(B \rightarrow \bar{\Lambda}^0 \Omega_c(2770)^0)$	31.2×10^{-8}
$\mathcal{B}(B \rightarrow \bar{\Lambda}^0 \bar{\Omega}_c^0)$	9.5×10^{-8}
$\mathcal{B}(B \rightarrow \bar{\Lambda}^0 \bar{\Omega}_c(2770)^0)$	10.0×10^{-8}

B flavor blind measurements

Source	Uncertainty (%)
Track reconstruction (overall)	2.9
π^+ PID (for $\Omega_c^0 \rightarrow \pi^+ \Omega^-$)	0.8
K^- PID (for $\Omega^- \rightarrow K^- \Lambda^0$)	1.4
p PID (for Λ^0 decays)	2×1.0
Decay length (Ω^-)	2.0
Reconstructed masses	4×0.5
Vertex fits (χ^2)	1.5
M_{bc} and ΔE	0.5
$\mathcal{B}(\Omega^- \rightarrow \Lambda^0 K^-)$	1.0
$\mathcal{B}(\Lambda^0 \rightarrow p \pi^-)$	2×0.7
$N_{B^0 \bar{B}^0}$	2.9
Detector charge asymmetry	0.8
Polarization of baryons	0.5
MC statistics	0.7
Overall (σ_r)	6.2

Missing energies leptonic B decays at Belle II

$$- B^+ \rightarrow \tau^+ \nu_\tau$$

Prologue: $B^+ \rightarrow \tau^+ \nu$



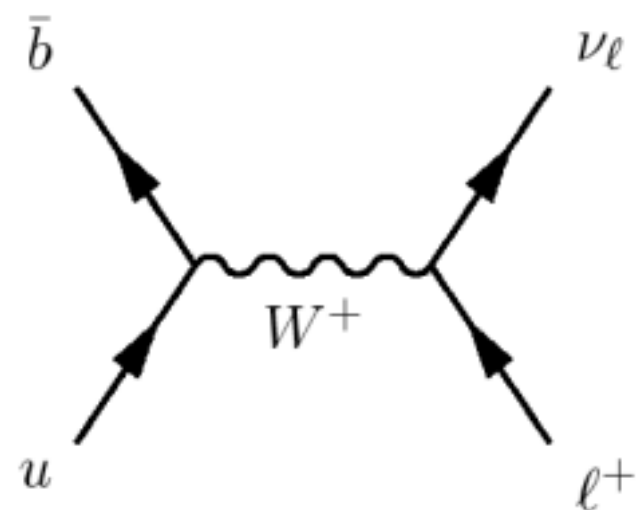
Theory

- Precise BF value is important to check consistency with SM predictions / constrain new physics

$$\mathcal{B}(B \rightarrow \tau \nu) > \mathcal{B}(B \rightarrow \mu \nu) > \mathcal{B}(B \rightarrow e \nu)$$

\longrightarrow

BF decreases with decrease in m_ℓ and increase in helicity suppression



SM Feynman diagram

- Potential modes to precisely measure $|V_{ub}|$

$$\mathcal{B}(B^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2 m_B m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

encodes $b \rightarrow u$ annihilation info. (theory input)

Experiment

- Challenging (particularly, τ mode) due to undetected neutrinos in the final state
- At present, the measurements are statistically limited

Experiment	Tag	$\mathcal{B}(10^{-4})$
Belle	Hadronic	$0.72^{+0.27}_{-0.25} \pm 0.11$
BABAR	Hadronic	$1.83^{+0.53}_{-0.49} \pm 0.24$
Belle	Semileptonic	$1.25 \pm 0.28 \pm 0.27$
BABAR	Semileptonic	$1.8 \pm 0.8 \pm 0.2$
PDG		1.09 ± 0.24

$$B^+ \rightarrow \tau^+ \nu$$



arXiv: 2502.04885 | Submitted to Phys. Rev. D



Goal: BF measurement using hadronic tagging (*First from Belle II*)

STEP I: Reconstruction

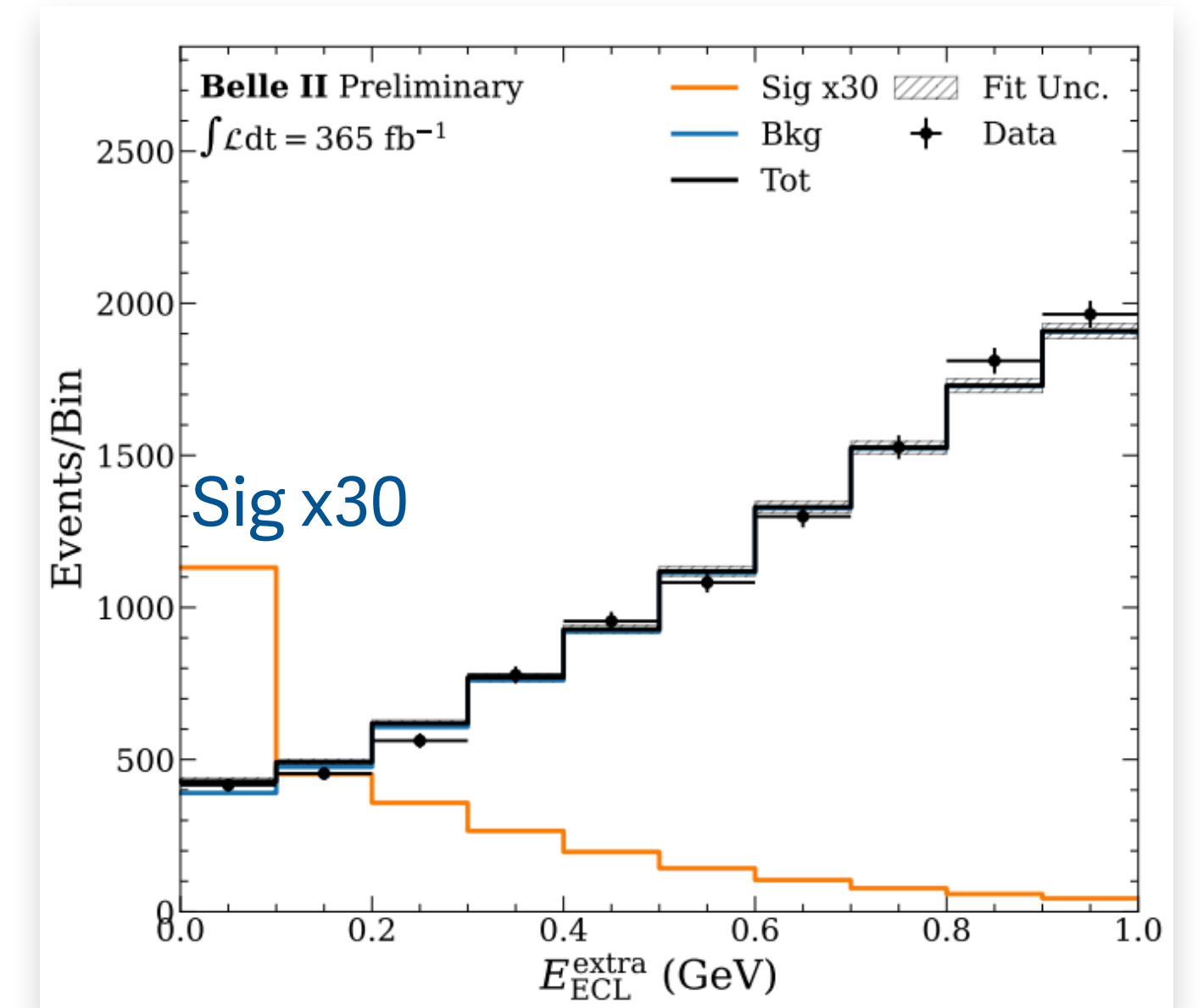
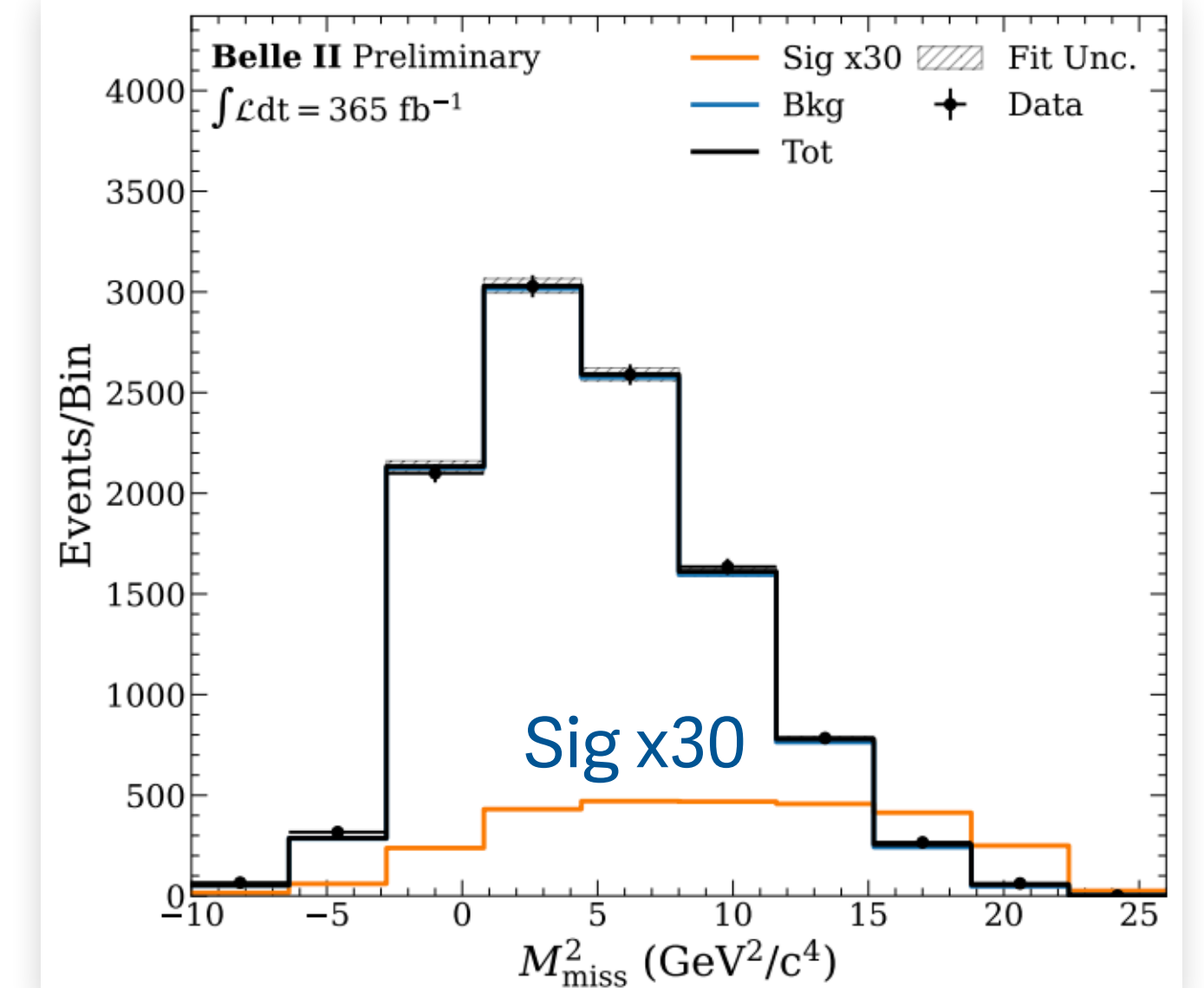
- Use hadronic FEI to reconstruct the companion B , the B_{tag}
- Reconstruct B_{sig} from the remaining tracks and clusters
- Use both leptonic (e, μ) and **hadronic** channels (π, ρ) of τ 's ($\sim 70\%$ BF coverage)
- Use two most discriminating variables, $M_{\text{miss}}^2, E_{\text{ECL}}^{\text{extra}}$:

$$M_{\text{miss}}^2 = (p_{\text{beam}}^* - p_{\text{tag}}^* - p_{\text{sig}}^* - p_{\text{ROE}}^*)^2$$

$E_{\text{ECL}}^{\text{extra}} \equiv$ Total residual energy from neutral clusters †

† use of BDTs to clean the neutral clusters $\notin B_{\text{tag}}, B_{\text{sig}}$

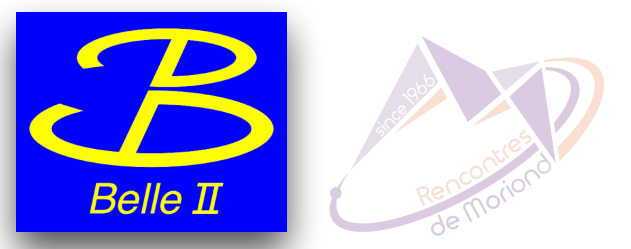
Signal signature: high M_{miss}^2 and low $E_{\text{ECL}}^{\text{extra}}$



$$B^+ \rightarrow \tau^+ \nu$$



arXiv: 2502.04885 | Submitted to Phys. Rev. D



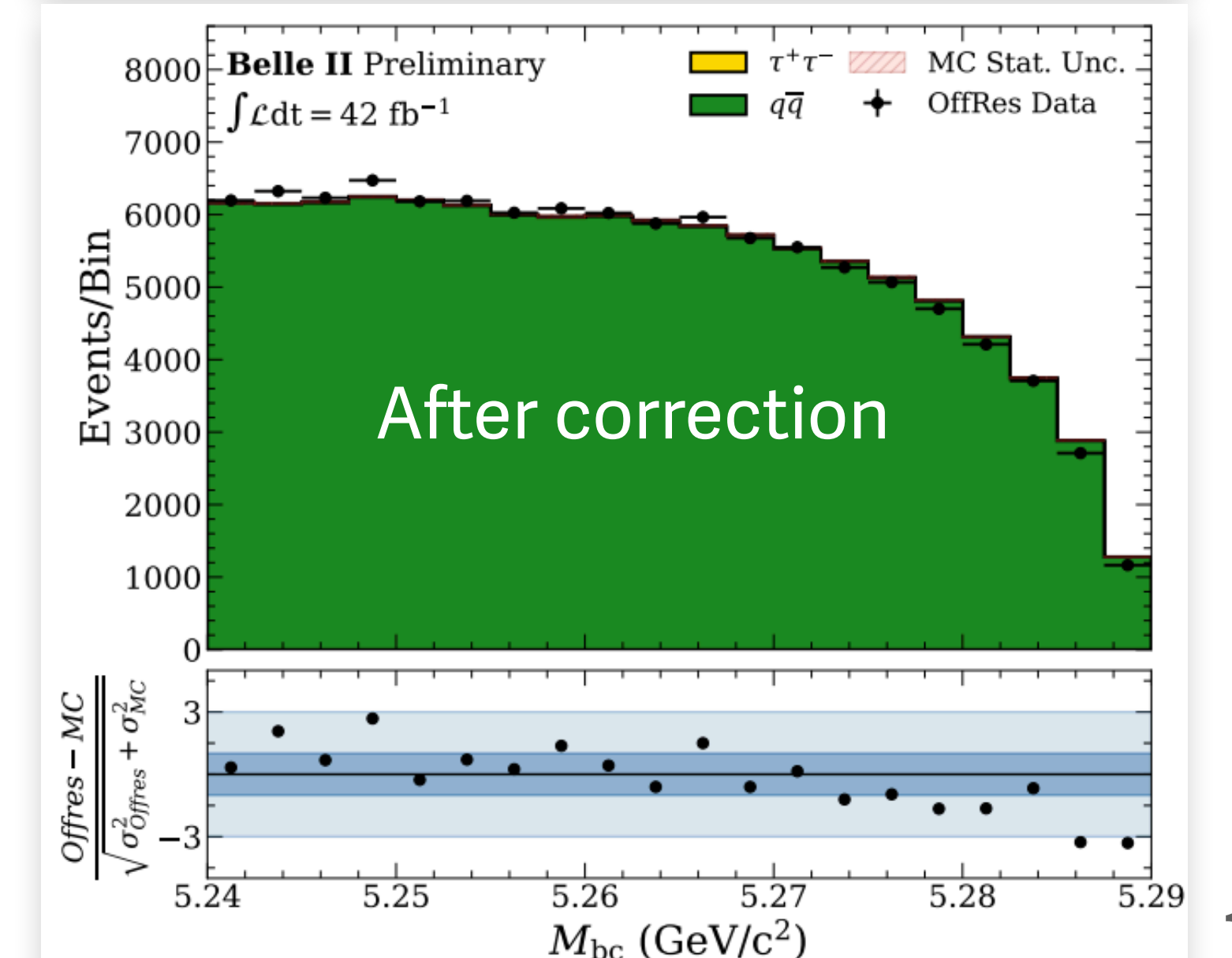
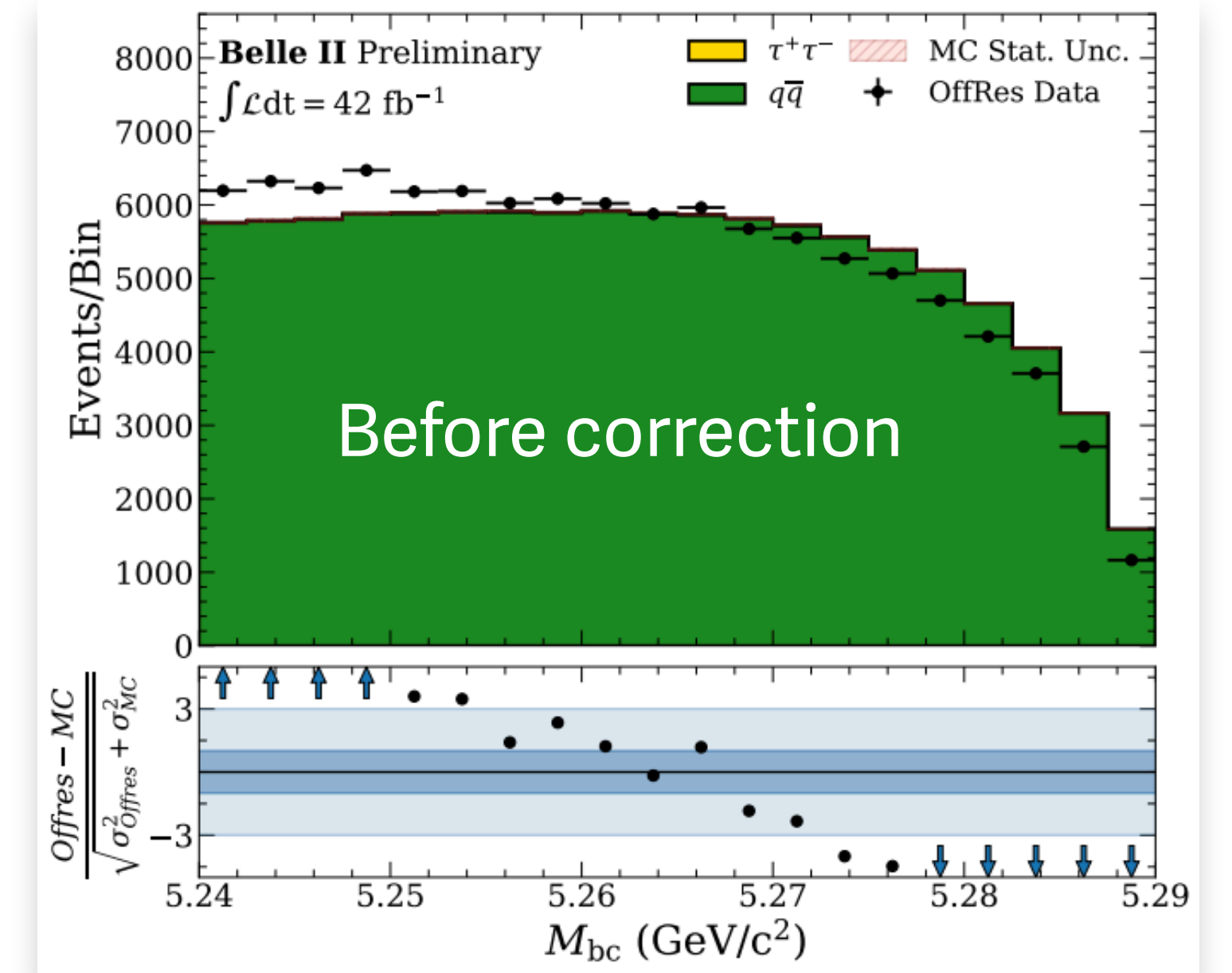
STEP II: Background suppression

- 2-stage BDTs
 - Continuum backgrounds ($e^+e^- \rightarrow q\bar{q}/\tau\bar{\tau}$) (dominant)
 - Non-signal $B\bar{B}$ backgrounds

$$\mathcal{L}_{\text{intg}}^{\text{OffRes}} = 42 \text{ fb}^{-1}$$

STEP III: Calibration and validation

- Calibration:
 - FEI efficiency correction using data-driven methods (off-resonance data)
 - 2 control channels: $B \rightarrow X\ell\nu, B \rightarrow D^{(*)}\pi^+$
 - Cluster multiplicity corrections between data - simulation
 - Additional corrections include: misID, photon eff., continuum re-weighting
- Validation:
 - Validated using $B^+ \rightarrow D^{*0}\ell^+\nu_\ell$ control mode



$$B^+ \rightarrow \tau^+ \nu$$



arXiv: 2502.04885 | Submitted to Phys. Rev. D



STEP IV: Signal extraction

- Simultaneous **binned** ML 2D fit to $M_{\text{miss}}^2, E_{\text{ECL}}^{\text{extra}}$

Results (Preliminary)

3σ significance

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu) = (1.24 \pm 0.41 \pm 0.19) \times 10^{-4}$$

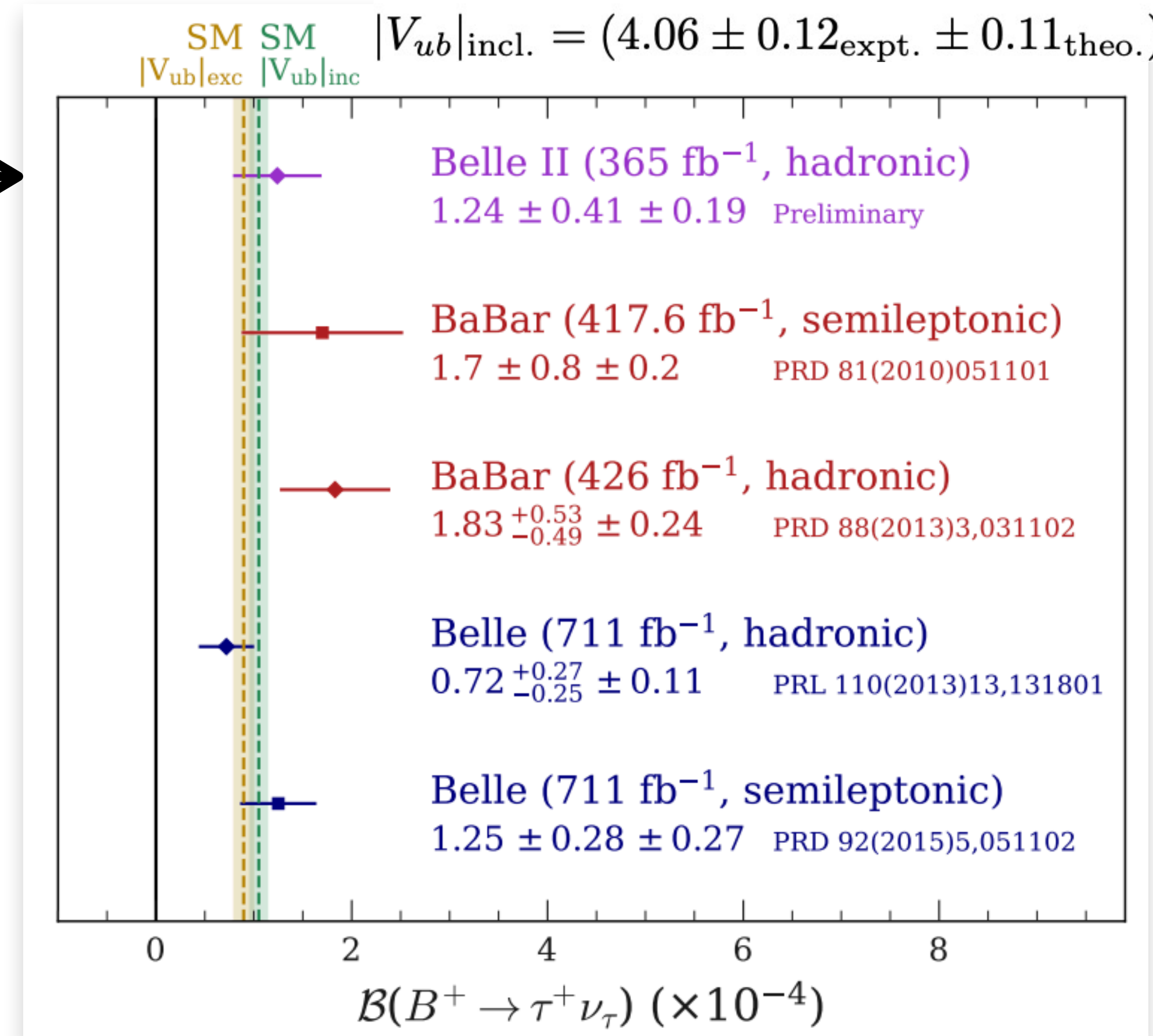
First uncertainty is statistical, second is systematics

Assuming SM and $f_B = (190.0 \pm 1.3)$ MeV from FLAG24

$$|V_{ub}|_{B^+ \rightarrow \tau^+ \nu} = (4.41_{-0.89}^{+0.74}) \times 10^{-3}$$

$$|V_{ub}|_{\text{excl.}} = (3.75 \pm 0.6_{\text{expt.}} \pm 0.19_{\text{theo.}}) \times 10^{-3}$$

$$|V_{ub}|_{\text{incl.}} = (4.06 \pm 0.12_{\text{expt.}} \pm 0.11_{\text{theo.}}) \times 10^{-3}$$



More precise than BaBar (had. tag), with equivalent dataset

First had-tagged results from Belle II

- Consistent with world average and SM predictions
- Dominant systematics from limited statistics of simulations [\[backup\]](#)

Missing energies semi-leptonic decays at Belle II

- $R(D^{(*)})$ semi-leptonic tag
- Untagged $B \rightarrow D\ell\nu$ and $|V_{cb}|$

Prologue: $R(D^{(*)})$

Theory

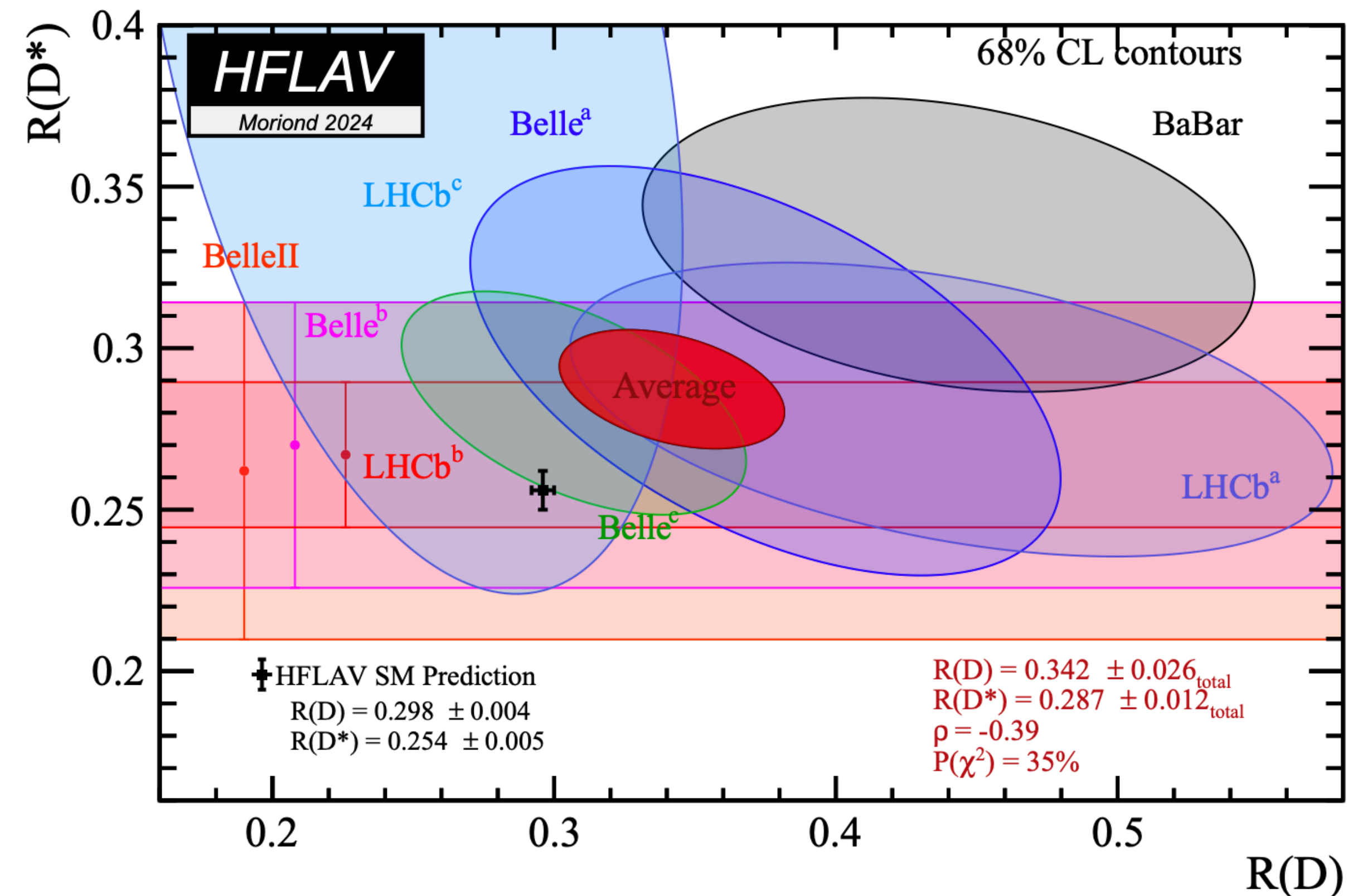
- Lepton Flavor Universality (LFU) is an “accidental symmetry” within the SM broken only by charged lepton masses
- Highly sensitive to non-SM physics and can be probed by precise LFU ratios

$$\mathcal{R}(D^{(*)+}) = \frac{\mathcal{B}(\bar{B}^0 \rightarrow D^{(*)+} \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B}^0 \rightarrow D^{(*)+} \ell^- \bar{\nu}_\ell)}$$

Experiment

- Observe $\sim 3\sigma$ excess by BaBar, Belle (II), and LHCb experiments
- [HFLAV24](#) average hints at potential new physics

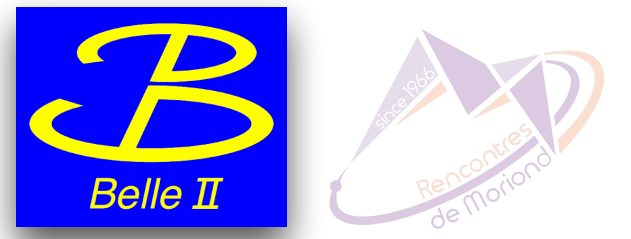
Combined deviation from SM stands at 3.3σ



$R(D^+)$ and $R(D^{*+})$



To be submitted to Phys. Rev. D



Goal: $R(D^+)$ and $R(D^{*+})$ measurement using semi-leptonic tagged approach (*First results*)

STEP I: Reconstruction

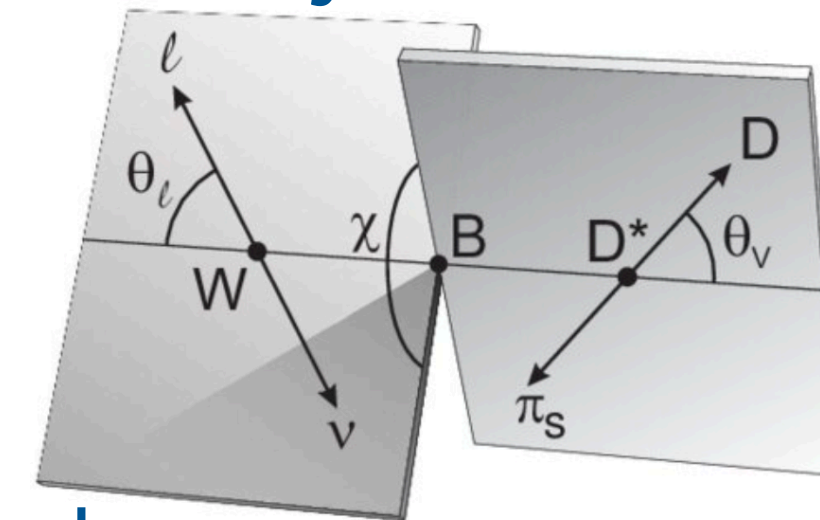
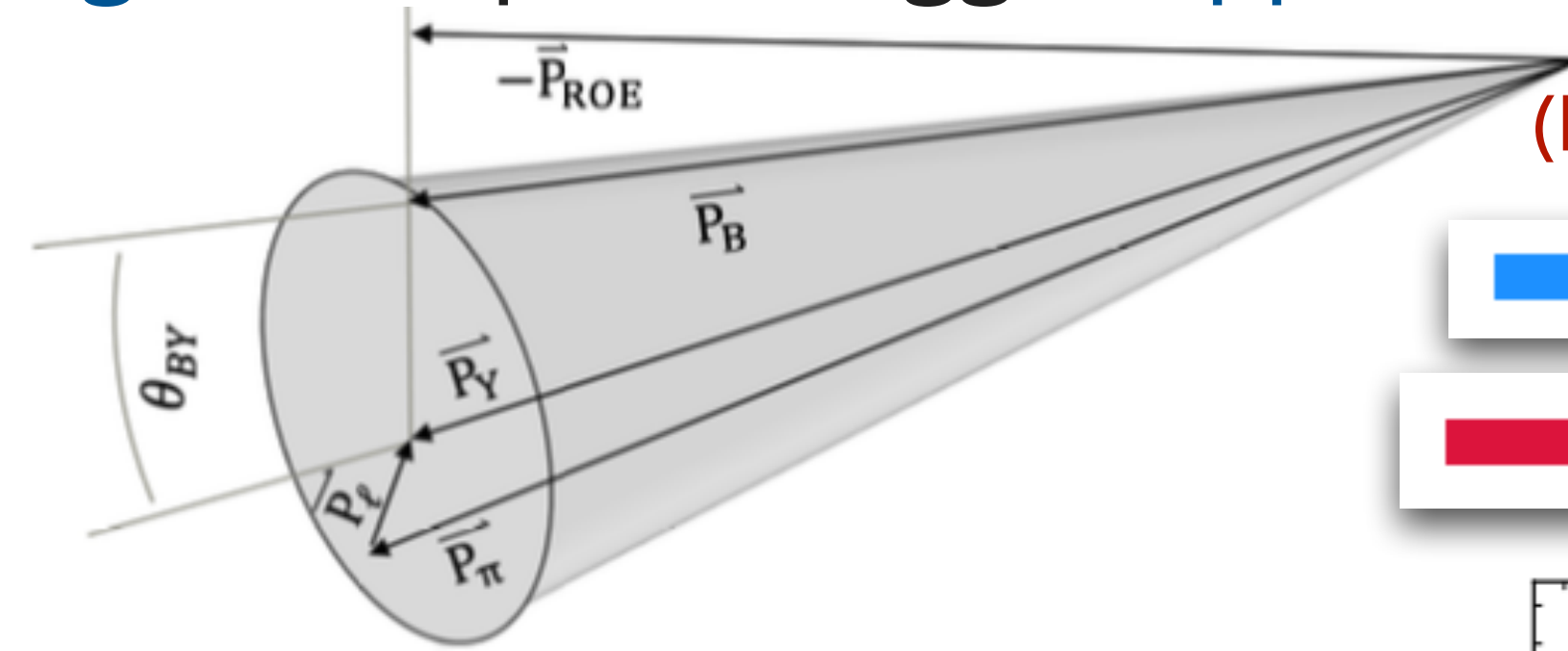
- Use semi-leptonic FEI to reconstruct the B_{tag}
- B_{sig} (B^0) is reconstructed from $D^{(*)}$, light leptons, and leptonic τ decays

STEP II: MVA-based event classification

- BDT trained to classify semi-leptonic, semi-tauonic, background
- Input BDT variables: angular (2), momenta of ℓ , D (2), and $E_{\text{ECL}}^{\text{extra}}$ (backup)

$$\cos \theta_{BY} = \frac{2E_{\text{beam}}E_Y - m_B^2 - m_Y^2}{2|\vec{p}_B||\vec{p}_Y|} \quad (\text{Most discriminating})$$

- Output scores: z_ℓ , z_τ , and z_{bkg}

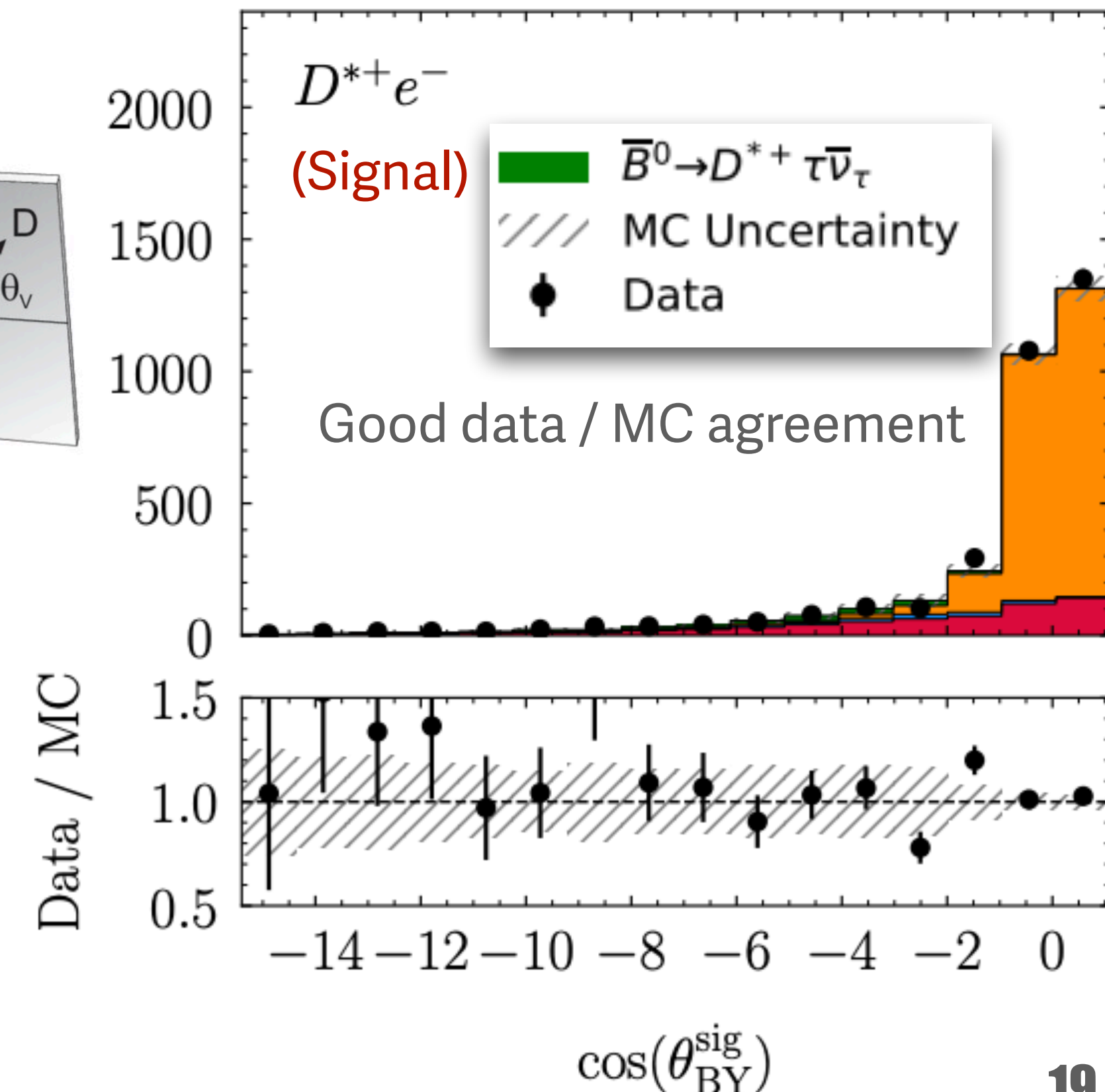


(Normalisation)

$\bar{B}^0 \rightarrow D^{*+} \ell \bar{\nu}_\ell$

$B\bar{B}$ and Continuum Bkg. in $D^{*+} \ell$

$\bar{B}^0 \rightarrow D^{**+} \ell \bar{\nu}_\ell + \bar{B}^0 \rightarrow D_{\text{gap}}^{**+} \ell \bar{\nu}_\ell$ in $D^{*+} \ell$



STEP III: Signal extraction

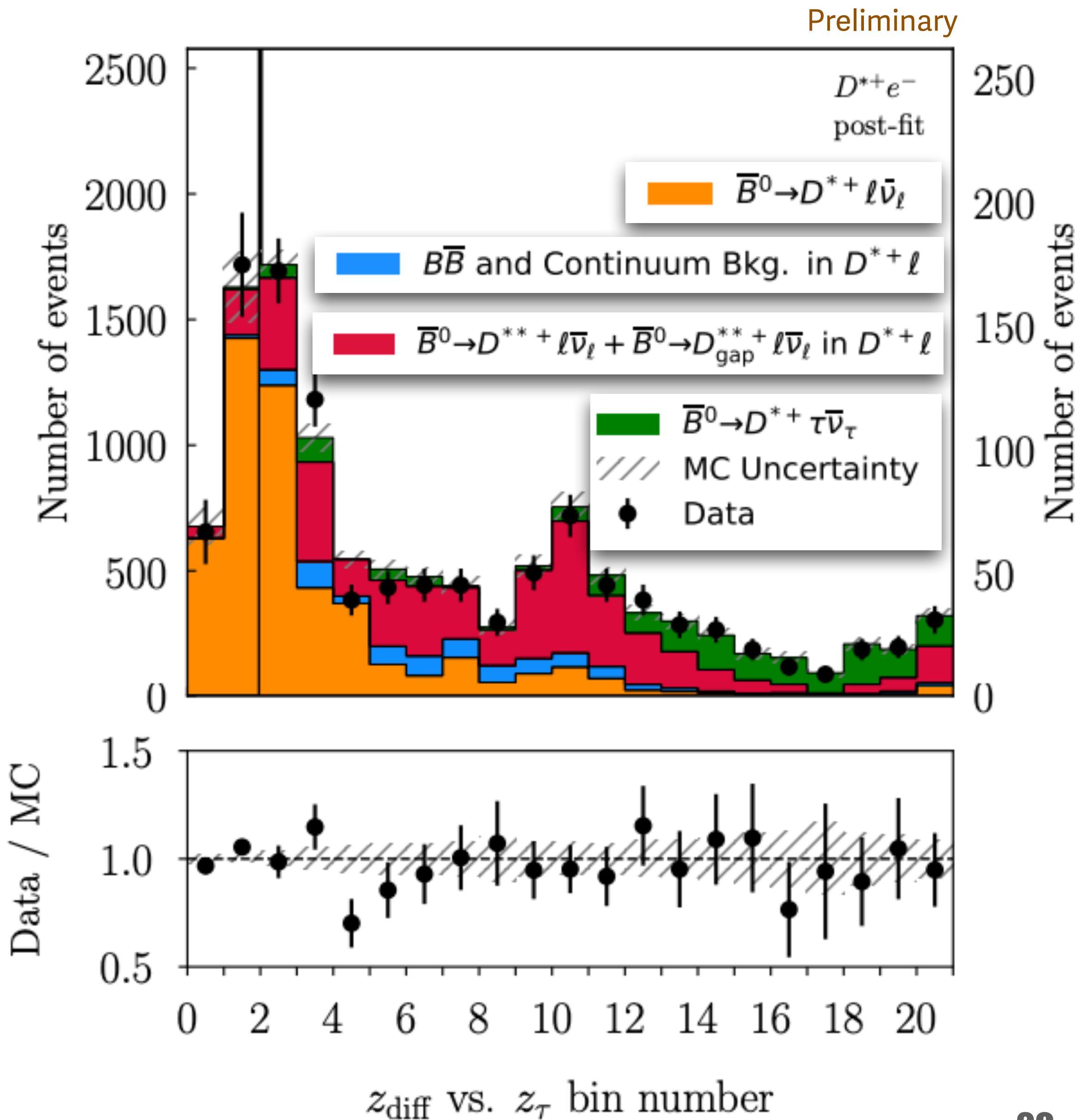
- 2D binned log-likelihood fit to z_τ and $z_{\text{diff}} = z_\ell - z_{\text{bkg}}$

Signal yields across different fit categories

Sample	D^+e	$D^+\mu$	$D^{*+}e$	$D^{*+}\mu$
$\bar{B}^0 \rightarrow D^+ \ell \bar{\nu}_\ell$	2519 ± 68	2233 ± 61		
$\bar{B}^0 \rightarrow D^{*+} \ell \bar{\nu}_\ell$	2486 ± 63	2323 ± 58	2344 ± 51	1961 ± 44
$\bar{B}^0 \rightarrow D^+ \tau \bar{\nu}_\tau$	191 ± 41	155 ± 65		
$\bar{B}^0 \rightarrow D^{*+} \tau \nu$	106 ± 14	84 ± 11	155 ± 19	111 ± 14
$\bar{B} \rightarrow D^{**} \ell \bar{\nu}_\ell / \bar{B} \rightarrow D_{\text{gap}}^{**} \ell \bar{\nu}_\ell$	653 ± 112	586 ± 102	87 ± 55	75 ± 46
$B\bar{B}$ and Continuum Bkg.	2177 ± 145	1582 ± 149	611 ± 95	497 ± 83
Data	8219	6854	3241	2621

Stability checks agree with the nominal values [\[backup\]](#)

- Redetermine $R(D^{(*+)})$ using different sample splits: lepton flavor, charge, lepton polar angle, # tracks, $D^{(*+)}$ channels
- Simultaneous fit to account for correlations in common syst.



$R(D^+)$ and $R(D^{*+})$



To be submitted to Phys. Rev. D



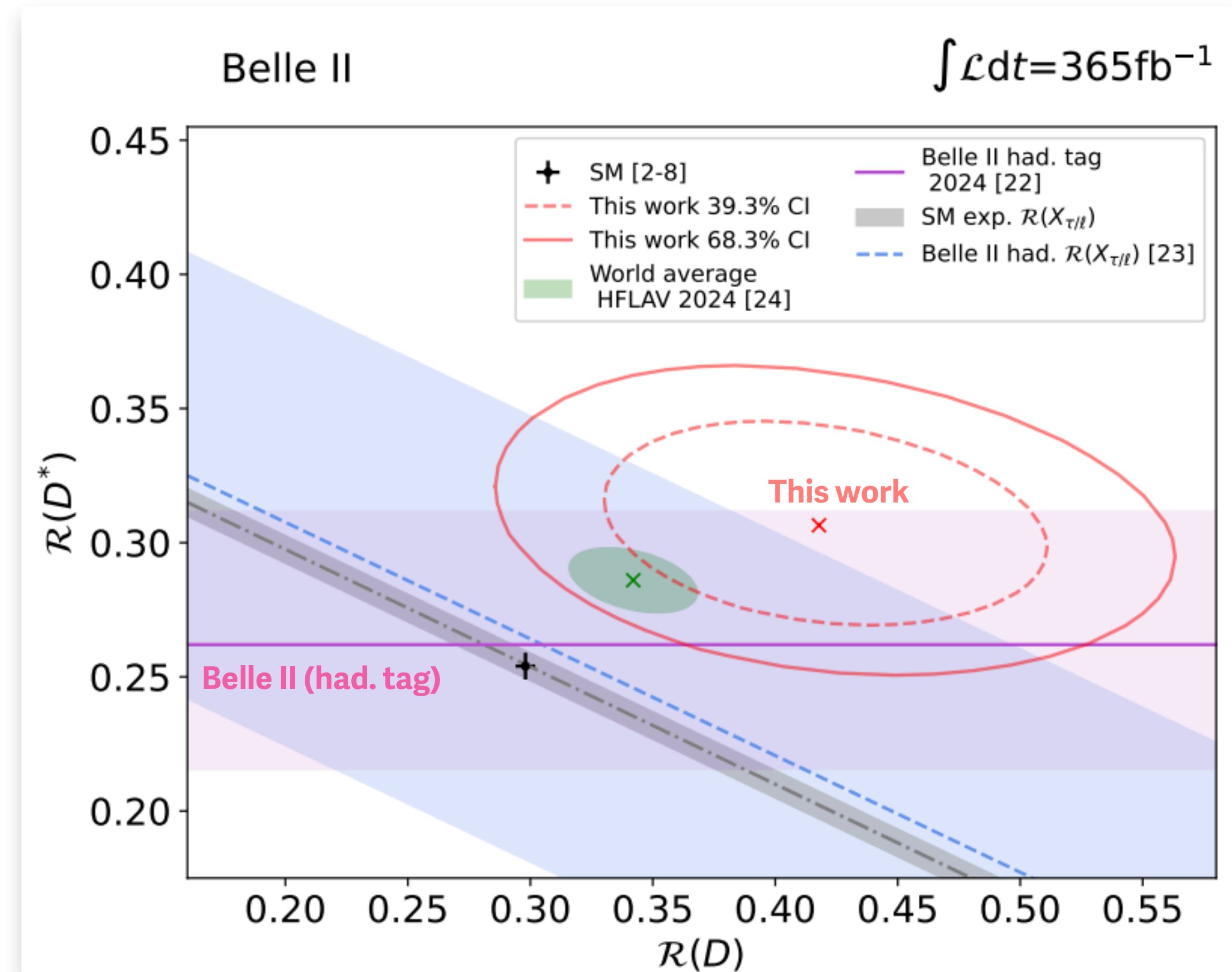
Results (Preliminary)

$$\mathcal{R}(D^+) = 0.418 \pm 0.074 \text{ (stat)} \pm 0.051 \text{ (syst)}$$
$$\mathcal{R}(D^{*+}) = 0.306 \pm 0.034 \text{ (stat)} \pm 0.018 \text{ (syst)}$$

with a correlation of $\rho = -0.24$

- Results are compatible with SM within 1.7σ
- Agrees with HFLAV24 average *
- Measurements dominated by statistical uncertainty
- Systematics dominated by limited statistics of simulations, MVA training, and template shapes [\[backup\]](#)

First SL-tagged results on $R(D^{(*+)})$ from Belle II



HFLAV24

$$\star \quad \mathcal{R}(D) = 0.342 \pm 0.026$$
$$\mathcal{R}(D^*) = 0.287 \pm 0.012$$

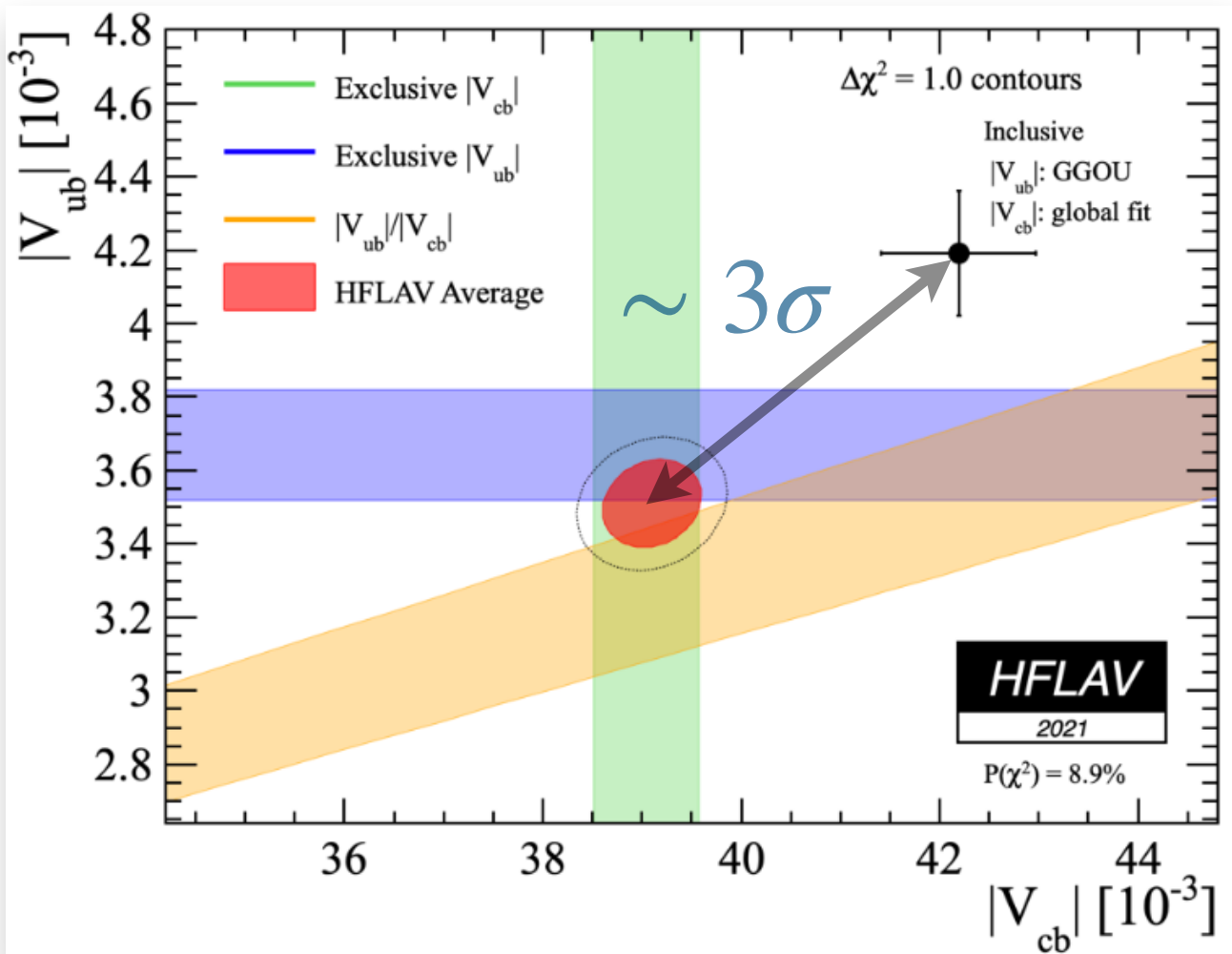
Prologue: $|V_{cb}|$ measurement



Theory

- Method: Exclusive ($B \rightarrow D^{(*)}\ell\nu$) and Inclusive ($B \rightarrow X_c\ell\nu$)
- Limitations:
 - Exclusive: Knowledge of the FFs (CLN, BCL) | Systematics dominate (expt.)
 - Inclusive: Higher order terms in HQE | Theoretical uncertainties dominate

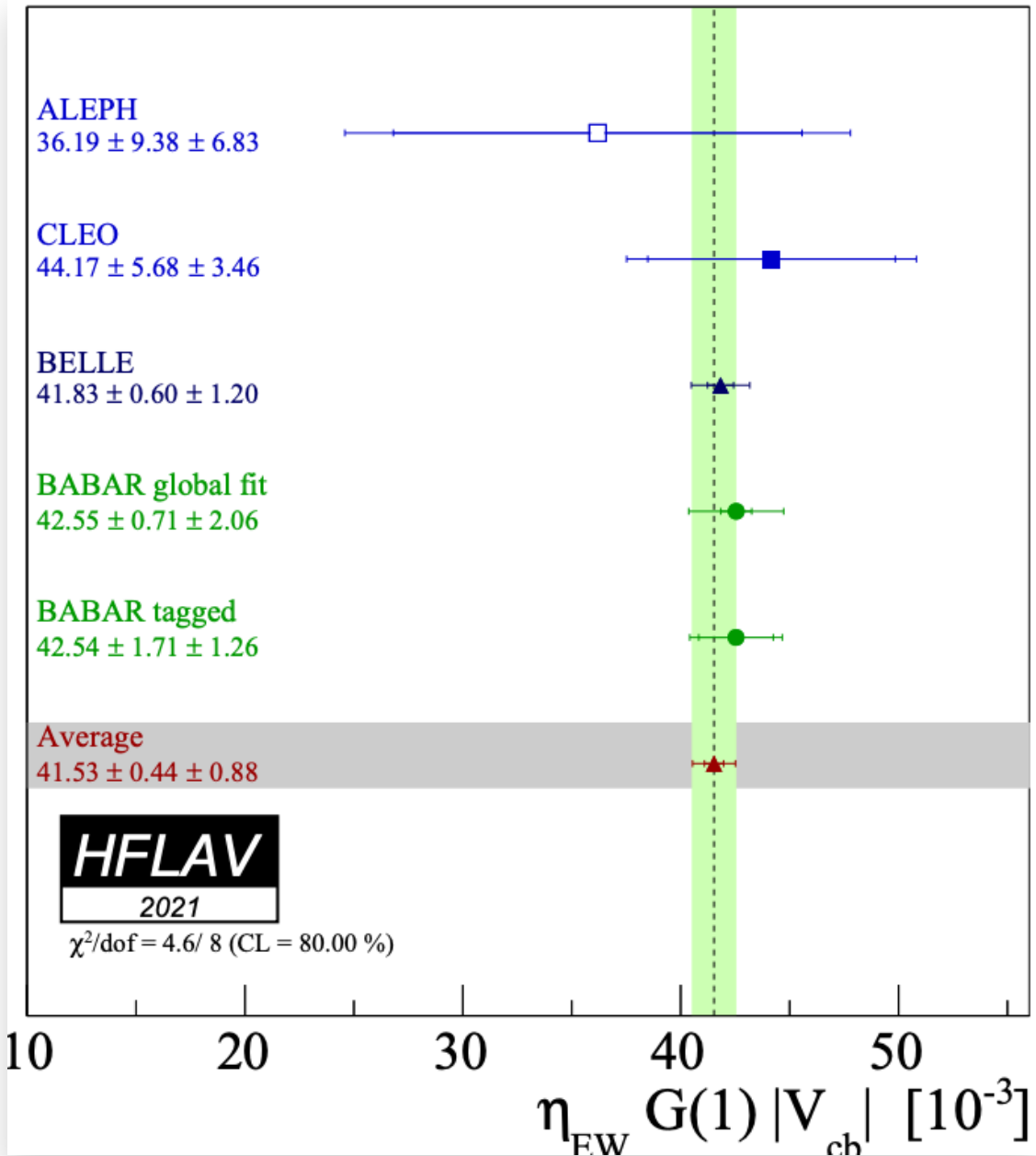
$|V_{cb}| = (42.2 \pm 0.5) \times 10^{-3} \quad (\text{inclusive})$ $|V_{cb}| = (39.8 \pm 0.6) \times 10^{-3} \quad (\text{exclusive})$



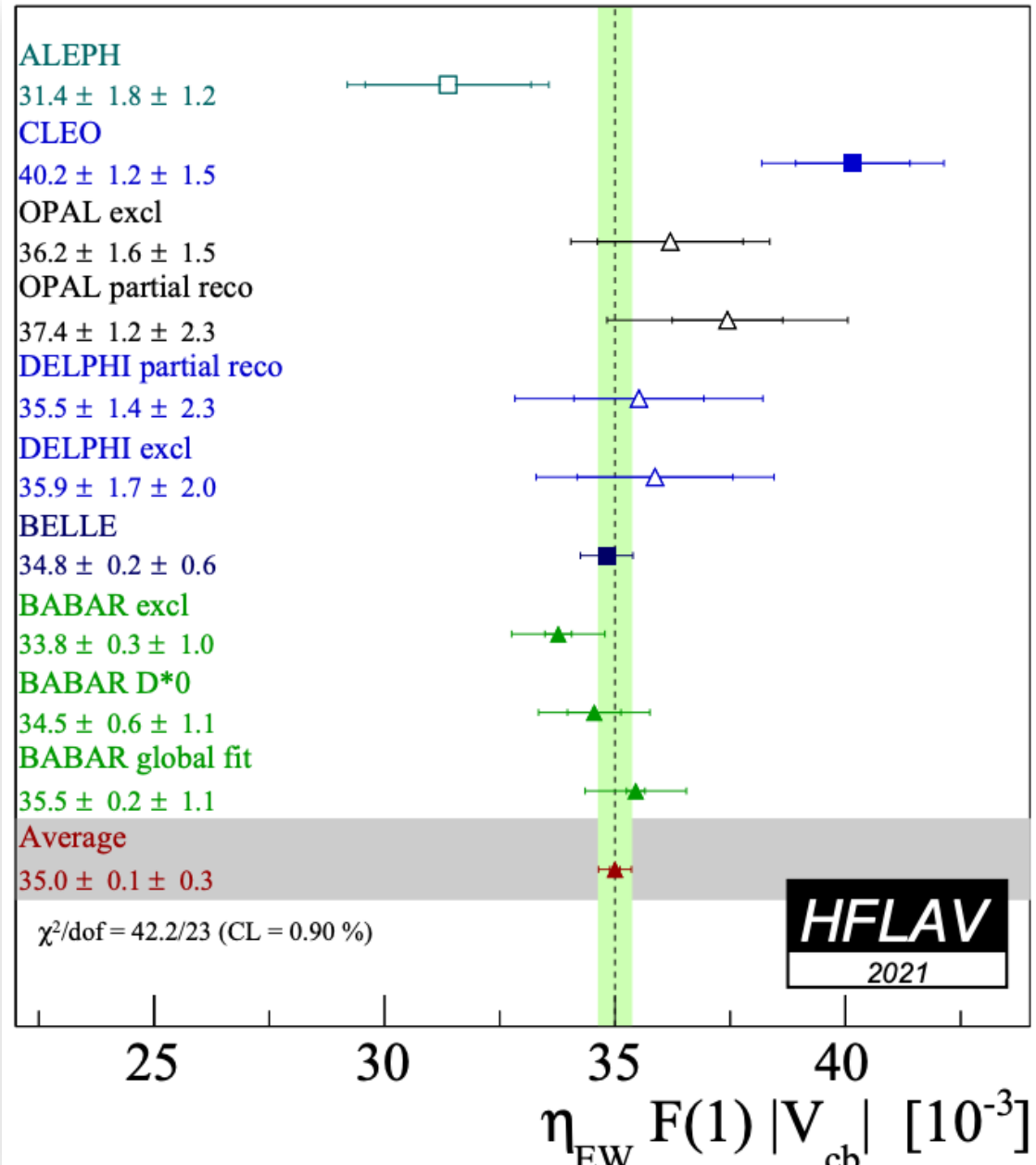
Experiment

- Consistently observe $\sim 3\sigma$ difference between exclusive and inclusive $|V_{cb}|, |V_{ub}|$ measurements
- Exclusive $|V_{cb}|$ from $B \rightarrow D\ell\nu$ can be advantageous: less theory unc. then $B \rightarrow D^*$ and do not suffer from slow-pion systematics

$|V_{cb}|$ from $B \rightarrow D\ell\nu$



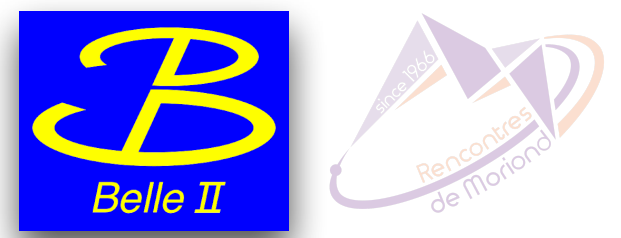
$|V_{cb}|$ from $B \rightarrow D^*\ell\nu$



$|V_{cb}|$ from $B \rightarrow D\ell\nu$



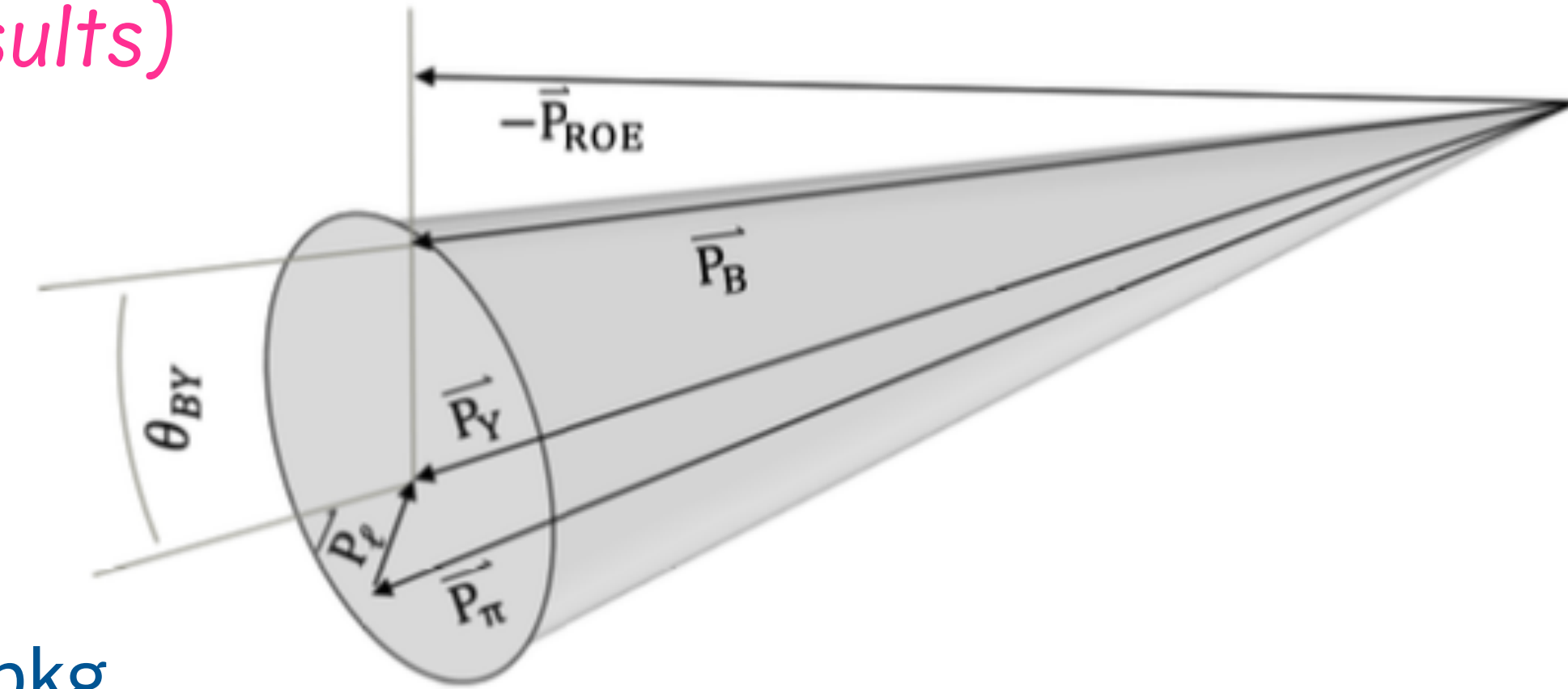
To be submitted to Phys. Rev. D



Goal: $|V_{cb}|$ from $B \rightarrow D\ell\nu$ using **untagged** approach (*First results*)

STEP I: Reconstruction

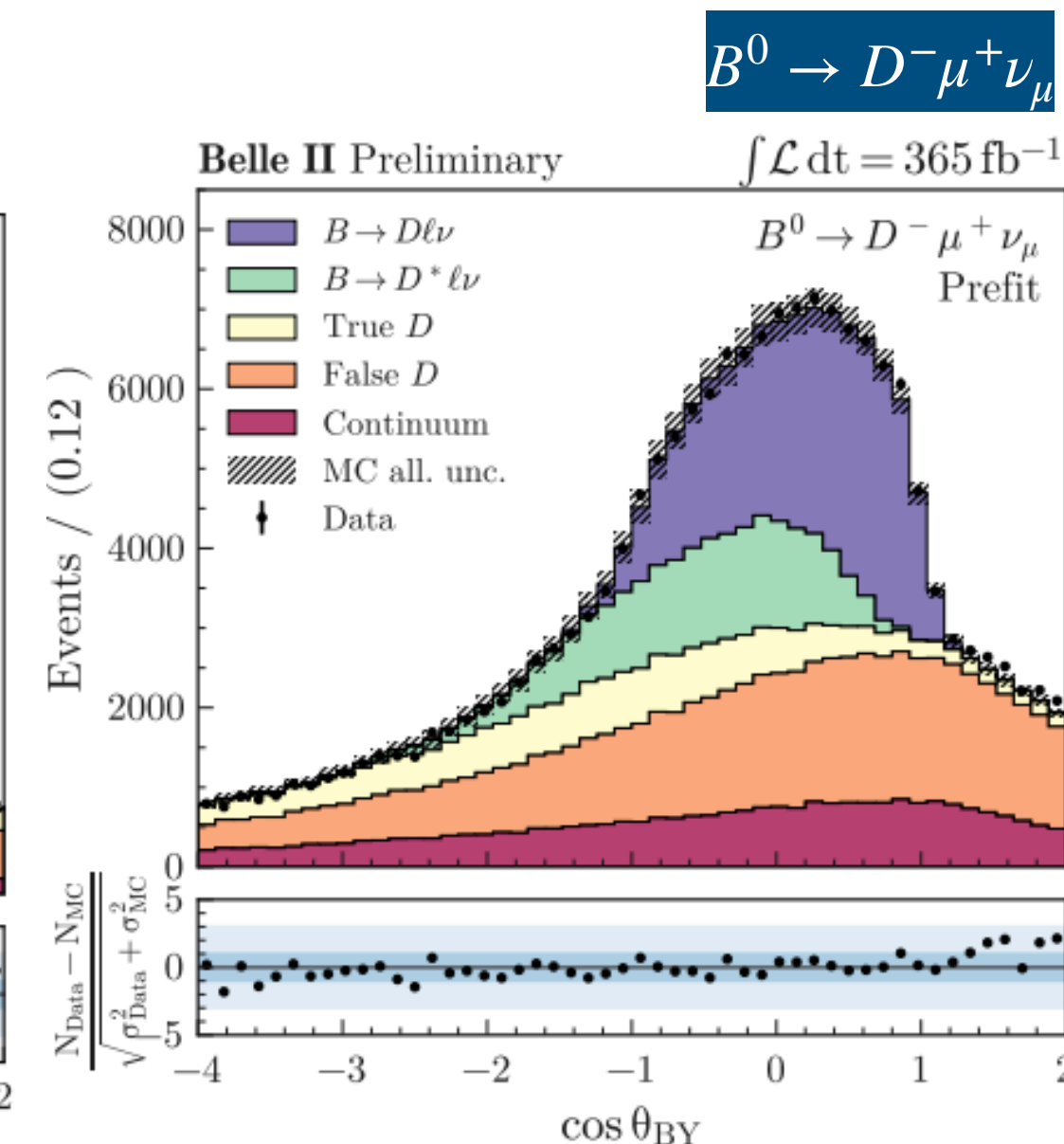
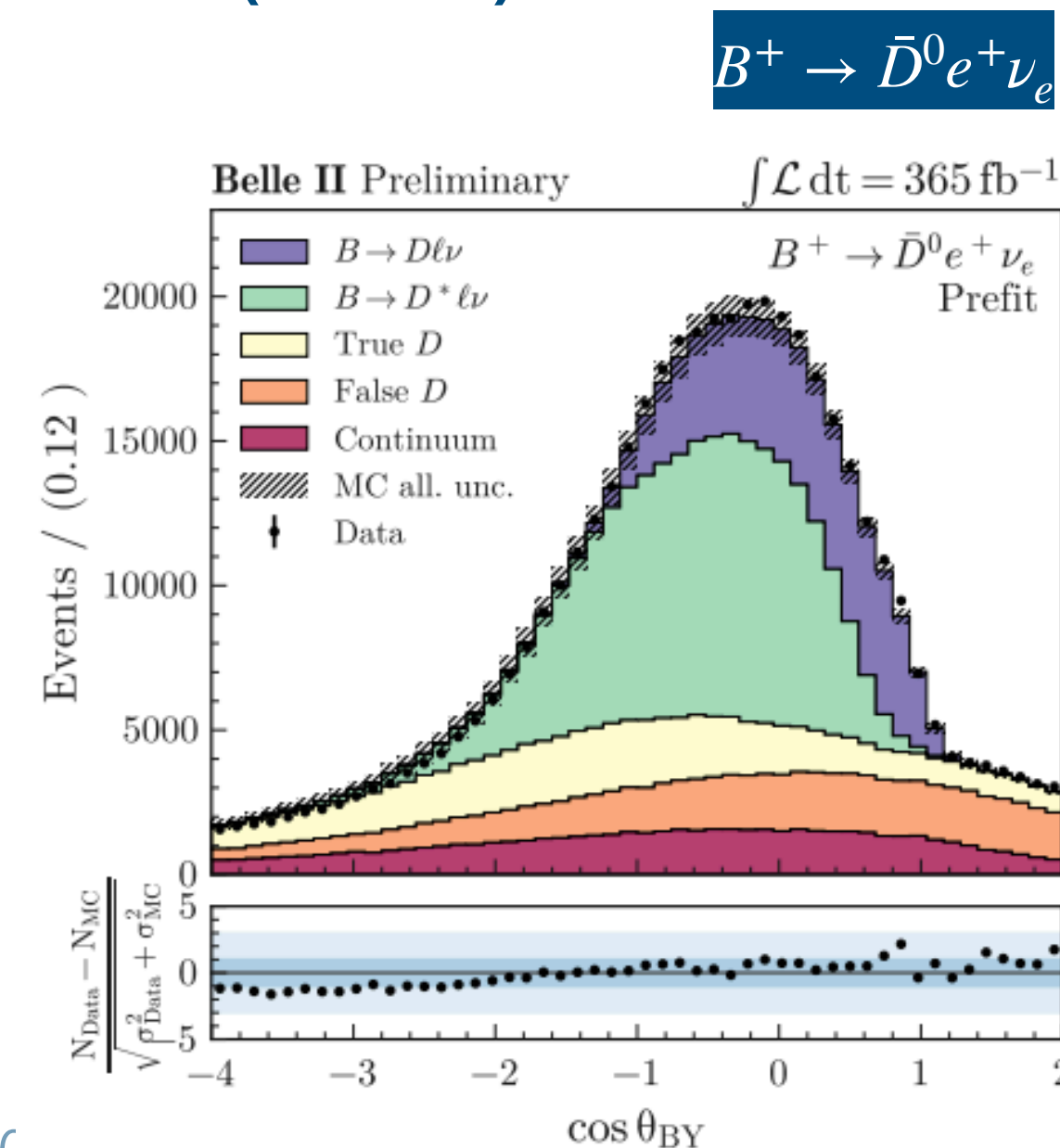
- Candidate $B \rightarrow D\ell\nu$'s are formed from ℓ (e, μ) and a D ($D \rightarrow K\pi, K\pi\pi$)
- $p_{\ell,D}^*$ selections are applied to select primary leptons and reject hadronic bkg.
- p_B^{miss} is estimated based on Diamond Frame (BaBar's) and ROE method (Belle's)



Good data / MC agreement

$$\cos \theta_{BY} = \frac{2E_B^* E_Y^* - M_B^2 - M_Y^2}{2|p_B^*||p_Y^*|}$$

- Finally, w ($\equiv v_B \cdot v_D$, 4-vel.) is accessed from p_B^{miss}
- "Feed-downs" from $B \rightarrow D^*\ell\nu$ are vetoed while continuum bkg. are suppressed using several kinematic selections



$|V_{cb}|$ from $B \rightarrow D\ell\nu$



To be submitted to Phys. Rev. D

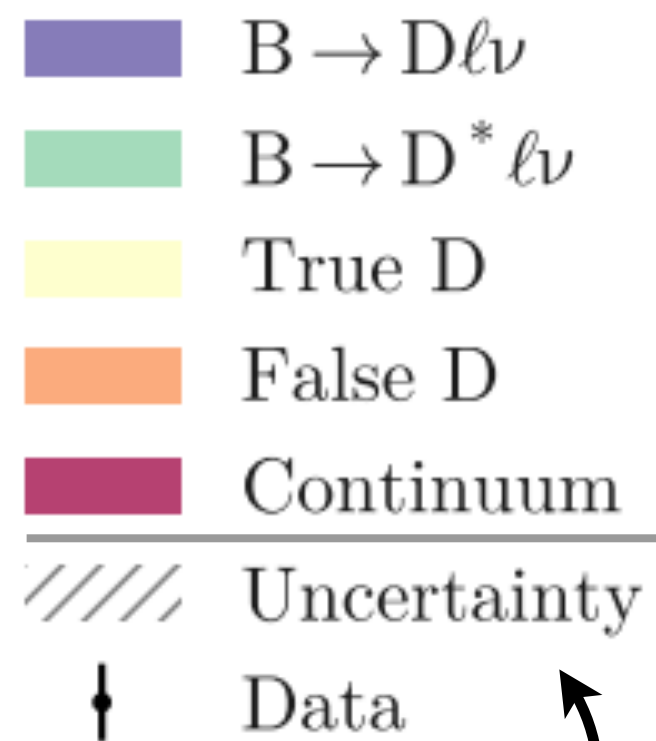


STEP II: Signal extraction

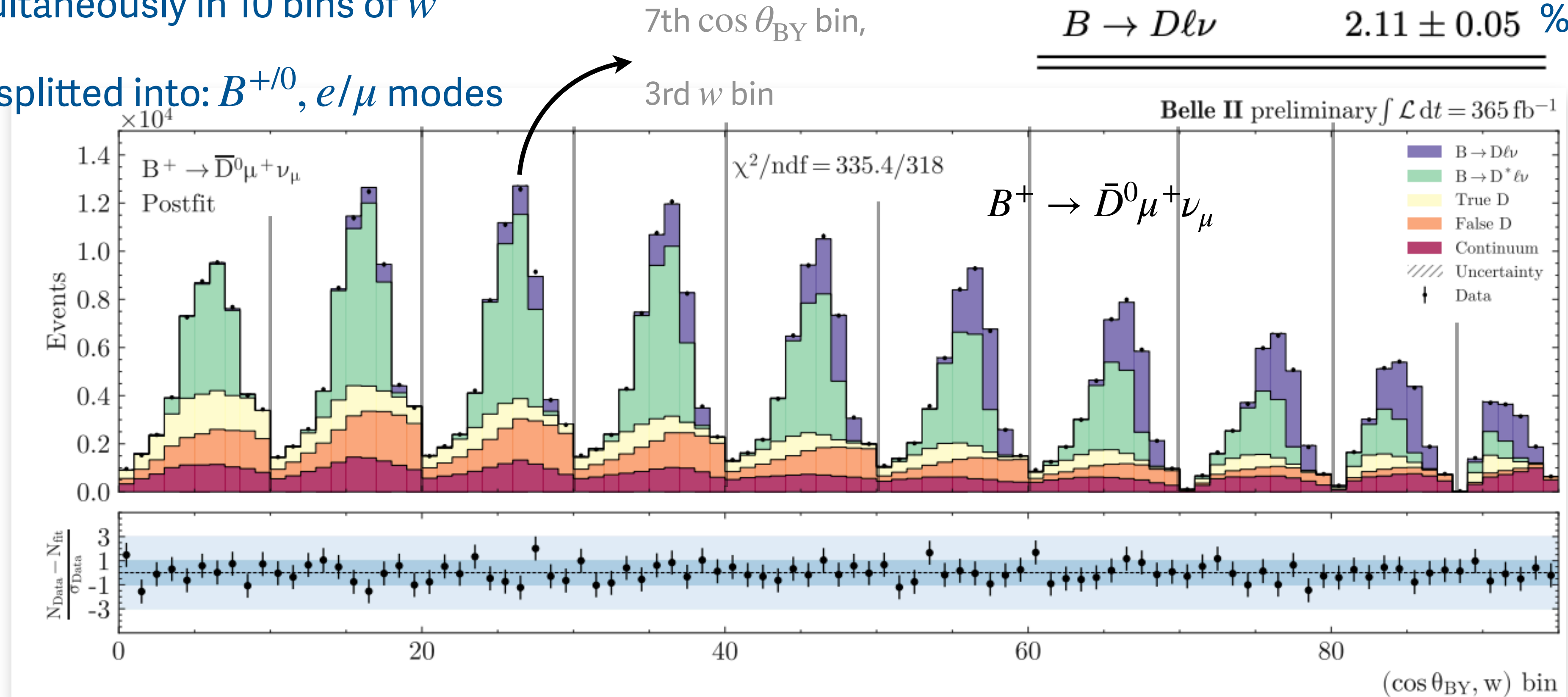
- Signal yield is extracted from a ML fit to a 10 **bin** dist. of $\cos \theta_{BY}$ and is performed simultaneously in 10 bins of w
- Results of BF are splitted into: $B^{+/0}, e/\mu$ modes

	HFLAV
$B^+ \rightarrow \bar{D}^0 \ell^+ \nu_\ell$	2.21 ± 0.06 %
$B^0 \rightarrow D^- \ell^+ \nu_\ell$	2.12 ± 0.06 %
$B \rightarrow D\ell\nu$	2.11 ± 0.05 %

Fit templates



Total



[backup]

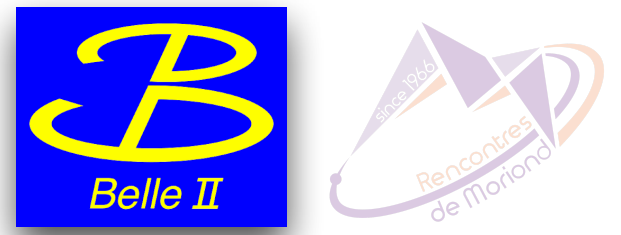
$$\mathcal{B}(B^+ \rightarrow \bar{D}^0 \ell^+ \nu_\ell) = (2.31 \pm 0.10)\% \quad *$$

$$\mathcal{B}(B^0 \rightarrow D^- \ell^+ \nu_\ell) = (2.06 \pm 0.12)\% \quad *$$

$|V_{cb}|$ from $B \rightarrow D\ell\nu$



To be submitted to Phys. Rev. D



STEP III: $|V_{cb}|$ extraction

$|V_{cb}|$ is extracted using χ^2 fits to the measured w spectra

$$|V_{cb}| = (42.2 \pm 0.5) \times 10^{-3} \quad (\text{inclusive})$$

$$|V_{cb}| = (39.8 \pm 0.6) \times 10^{-3} \quad (\text{exclusive})$$

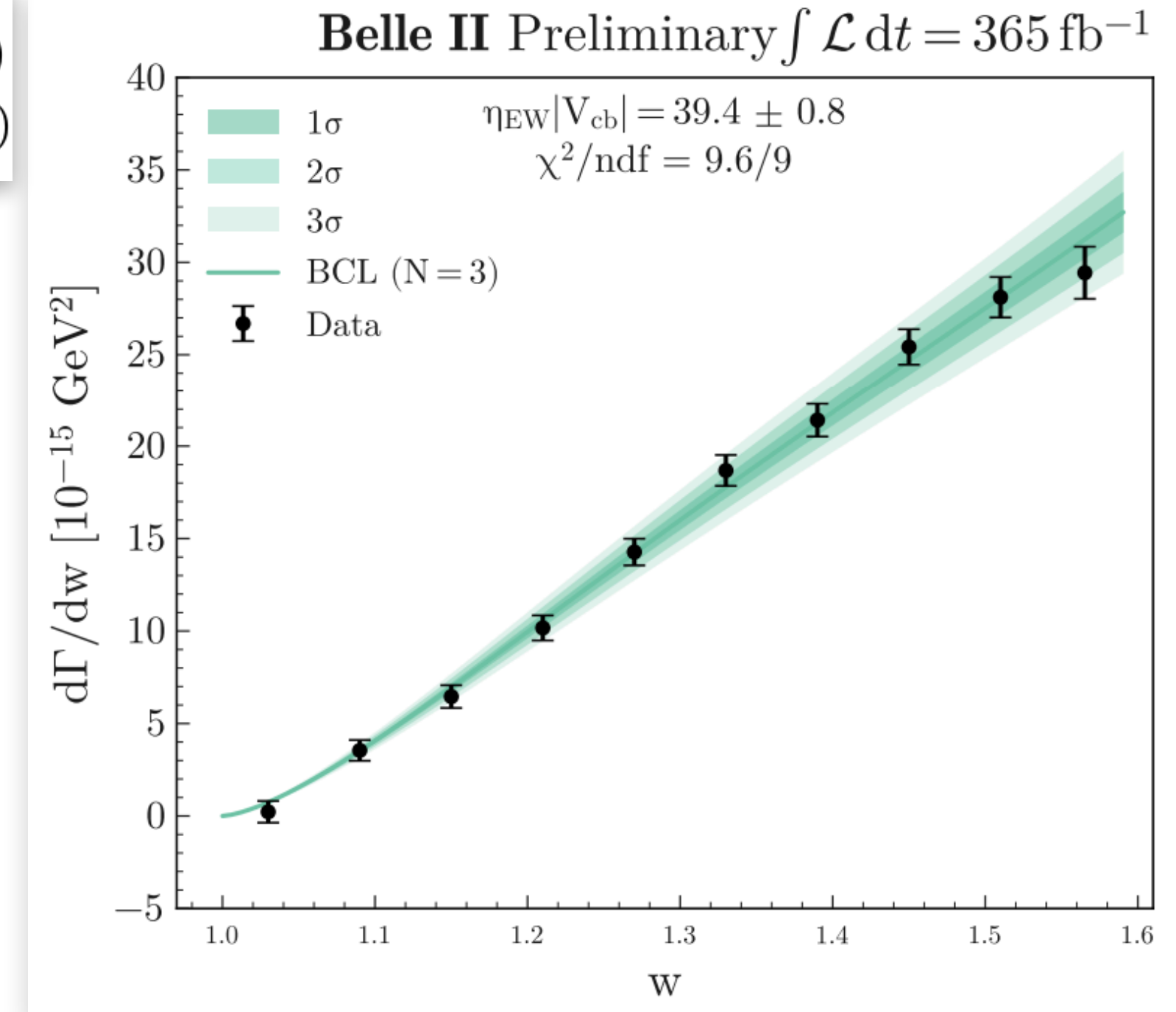
$$\chi^2 = \sum_{i,j}^{10} \left(\frac{\Delta\Gamma_i}{\Delta w} - \frac{\Delta\Gamma_{i,\text{BCL}}}{\Delta w} \right) C_{ij}^{-1} \left(\frac{\Delta\Gamma_j}{\Delta w} - \frac{\Delta\Gamma_{j,\text{BCL}}}{\Delta w} \right) + \chi_{\text{theory}}^2$$

Expt. obs. (points to $\frac{\Delta\Gamma_i}{\Delta w}$)

Theoretical prediction (points to $\frac{\Delta\Gamma_{i,\text{BCL}}}{\Delta w}$)

Inverse covariance matrix (points to C_{ij}^{-1})

BCL parametrisation of FF (5 free parameters) (points to $\frac{\Delta\Gamma_{j,\text{BCL}}}{\Delta w}$)



Parameters (5) of BCL parametrisation from fit

	Values	Correlation coefficients				
a_0^+	0.8959(92)	1	0.26	-0.38	0.95	0.51
a_1^+	-8.03(15)		1	0.17	0.33	0.86
a_2^+	49.3(31)			1	-0.31	0.16
a_0^0	0.7813(73)				1	0.47
a_1^0	-3.38(15)					1

Electroweak correction: 1.0066 ± 0.0002
[Nucl. Phys. B 196, 83 (1982)]

$$+ \eta_{\text{EW}} |V_{cb}| = 39.4 \pm 0.8$$

Result (Preliminary)

$$|V_{cb}|_{\text{BCL}} = (39.2 \pm 0.4_{\text{stat.}} \pm 0.6_{\text{sys.}} \pm 0.5_{\text{th.}}) \times 10^{-3}$$

Among the dominant systematics include: limited simulation stats.,

estimation of $N_{bb'}$, vertex fit corrections, background w modelling [\[backup\]](#)

Most precise till date using $B \rightarrow D\ell\nu$



Hadronic decays of *B* mesons

[arXiv:2412.19624](#) | Submitted to Phys. Rev. D

$$\begin{aligned}\mathcal{B}(B^0 \rightarrow \rho^+ \rho^-) &= (2.88^{+0.23+0.29}_{-0.22-0.27}) \times 10^{-5} \\ f_L &= 0.921^{+0.024+0.017}_{-0.025-0.015}, \\ S &= -0.26 \pm 0.19 \pm 0.08, \\ C &= -0.02 \pm 0.12^{+0.06}_{-0.05},\end{aligned}$$

Belle II result improves ϕ_2 precision by ~8% ! Consistent with WA values!

[Phys. Rev. D 110, L031102 \(2024\)](#)

Quantity ($\times \mathcal{B}(\Omega_c^0 \rightarrow \Omega^- \pi^+)$)	Upper limit (at 95% CL)
$\mathcal{B}(B \rightarrow \bar{\Lambda}^0 \Omega_c^0)$	9.7×10^{-8}
$\mathcal{B}(B \rightarrow \bar{\Lambda}^0 \Omega_c(2770)^0)$	31.2×10^{-8}
$\mathcal{B}(B \rightarrow \bar{\Lambda}^0 \bar{\Omega}_c^0)$	9.5×10^{-8}
$\mathcal{B}(B \rightarrow \bar{\Lambda}^0 \bar{\Omega}_c(2770)^0)$	10.0×10^{-8}

First upper limit set for BNV decays from Belle

Missing energies leptonic decays of *B* mesons

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau) = [1.24 \pm 0.41(\text{stat.}) \pm 0.19(\text{syst.})] \times 10^{-4}$$

Consistent with WA values! First leptonic results from Belle II with hadronic tagging approach

[arXiv: 2502.04885](#) | Submitted to Phys. Rev. D

$$|V_{ub}|_{B^+ \rightarrow \tau^+ \nu_\tau} = [4.41^{+0.74}_{-0.89}] \times 10^{-3}$$

Missing energies semi-leptonic decays of *B* mesons

To be submitted to Phys. Rev. D

$$\begin{aligned}\mathcal{R}(D^+) &= 0.418 \pm 0.074 \text{ (stat)} \pm 0.051 \text{ (syst)} \\ \mathcal{R}(D^{*+}) &= 0.306 \pm 0.034 \text{ (stat)} \pm 0.018 \text{ (syst)}\end{aligned}$$

Compatible with SM within 1.7σ !

First results from Belle II with semi-leptonic tagging approach

To be submitted to Phys. Rev. D

$$|V_{cb}|_{\text{BCL}} = (39.2 \pm 0.4_{\text{stat.}} \pm 0.6_{\text{syst.}} \pm 0.5_{\text{th.}}) \times 10^{-3}$$

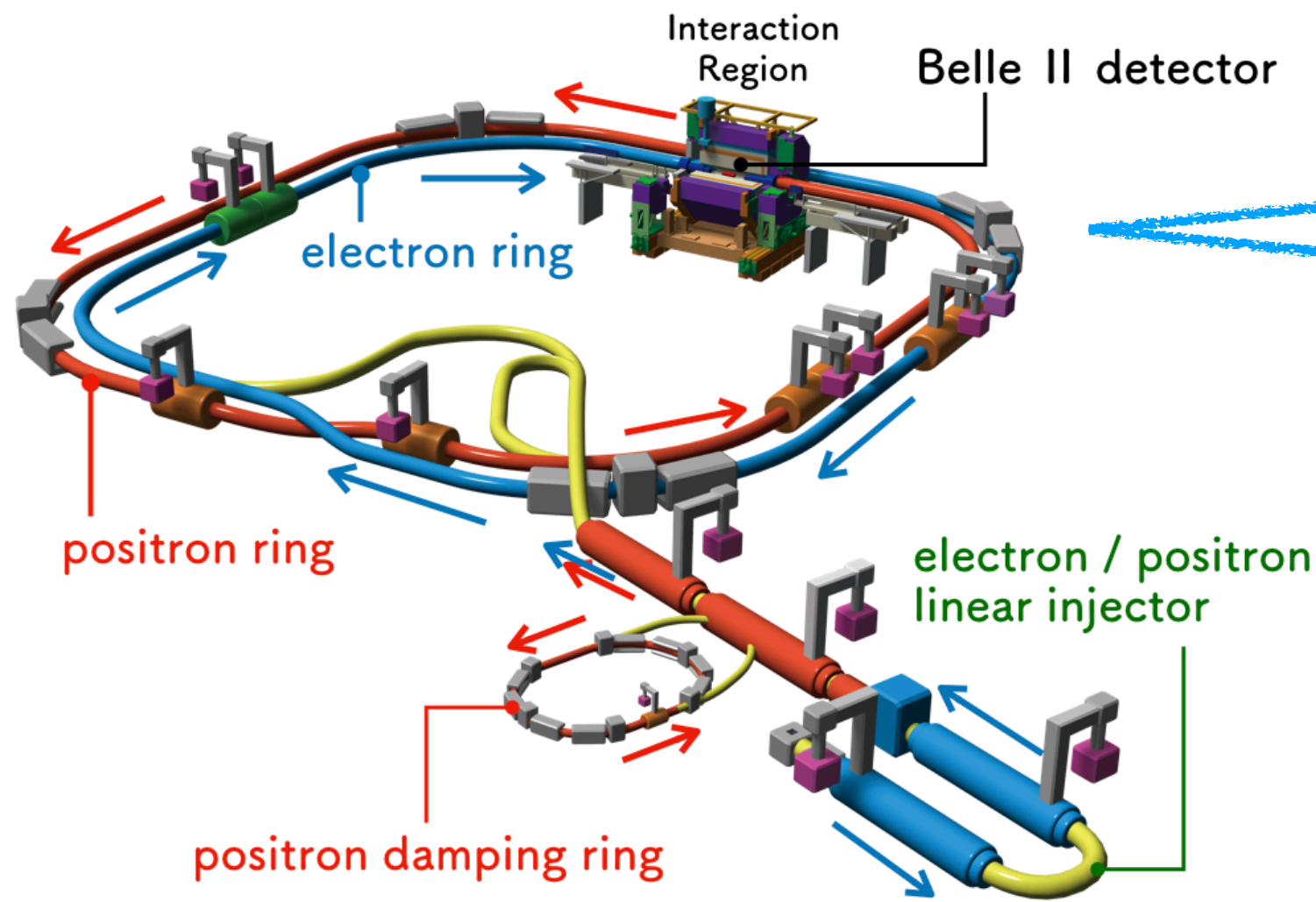
Most precise $|V_{cb}|$ value from $B \rightarrow D\ell\nu$

First results from Belle II with untagged approach

Thank You

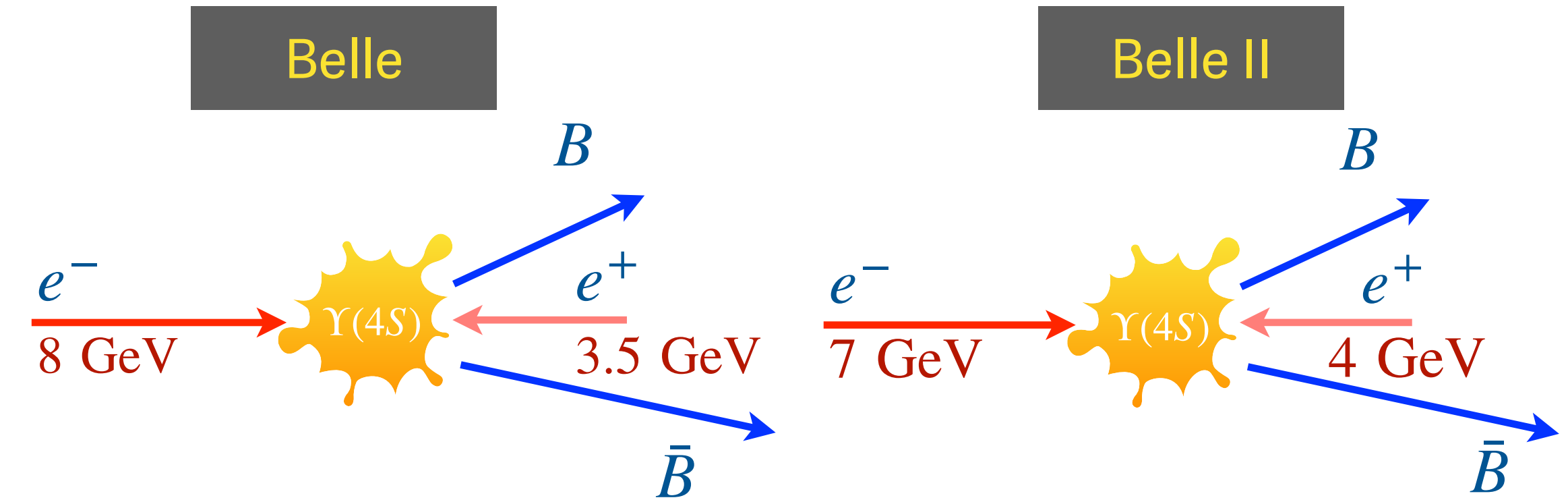
Additional slides

The SuperKEKB and the Belle II detector



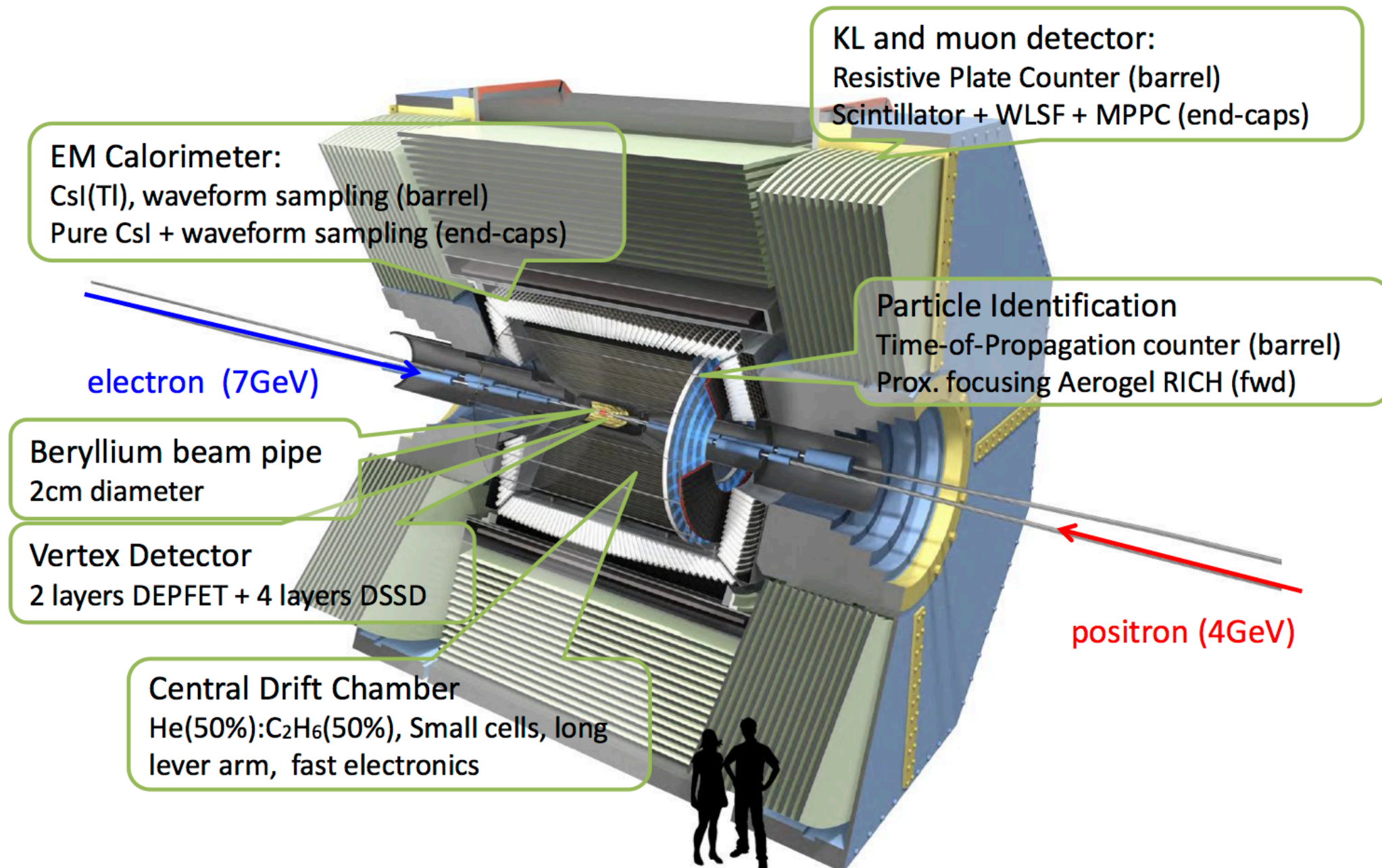
In comparison to KEKB

1/20 reduction in beam size
(more effective collisions)
Lesser asymmetry in beam energies (to reduce bkg.)

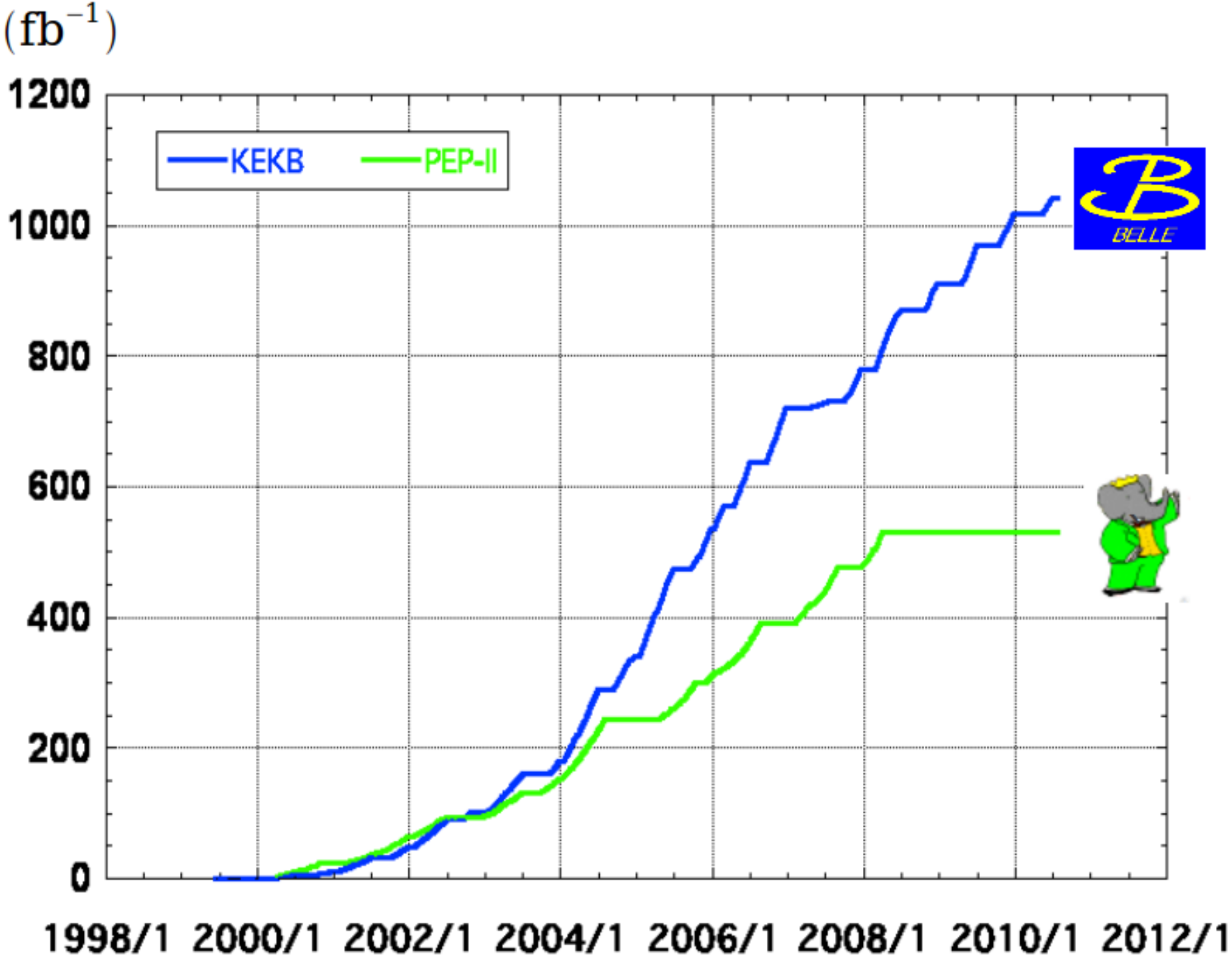


Belle II Performances

- VXD, $\sigma \sim 15 \mu\text{m}$
- CDC, $\sigma(p_T)/p_T \sim 0.4 \%$
- ECL, $\sigma(E)/E \sim 5 \%$
- PID (SVD, ARICH, TOP, KLM)
 - K eff. $\sim 90 \%$ (π mis – ID $\sim 5\%$)
 - μ ID eff. $\sim 90 \%$ (π mis – ID $\sim 5\%$)



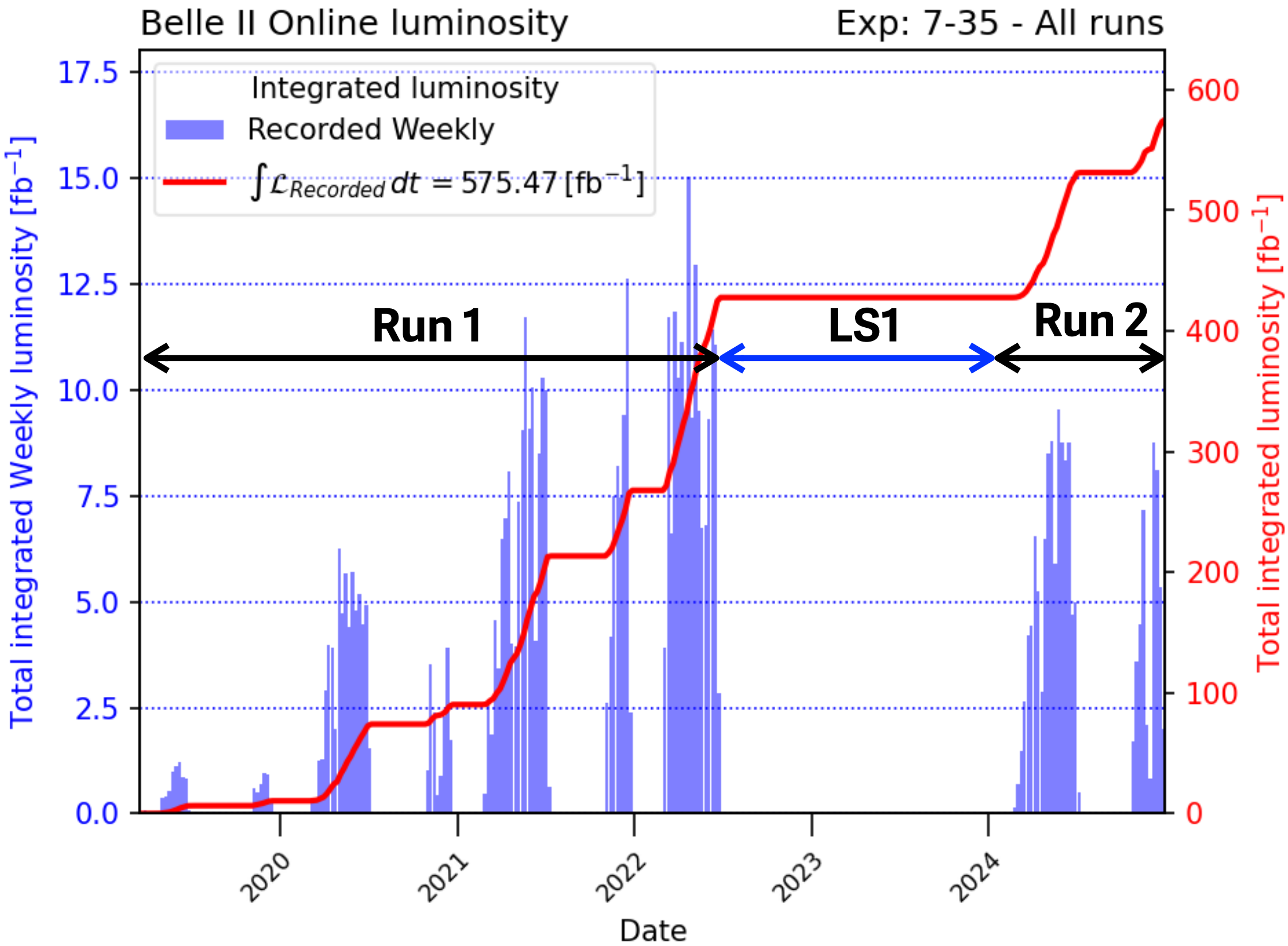
Luminosity records



> 1 ab⁻¹
On resonance:
Y(5S): 121 fb⁻¹
Y(4S): 711 fb⁻¹
Y(3S): 3 fb⁻¹
Y(2S): 25 fb⁻¹
Y(1S): 6 fb⁻¹
Off reson./scan:
~ 100 fb⁻¹

~ 550 fb⁻¹
On resonance:
Y(4S): 433 fb⁻¹
Y(3S): 30 fb⁻¹
Y(2S): 14 fb⁻¹
Off resonance:
~ 54 fb⁻¹

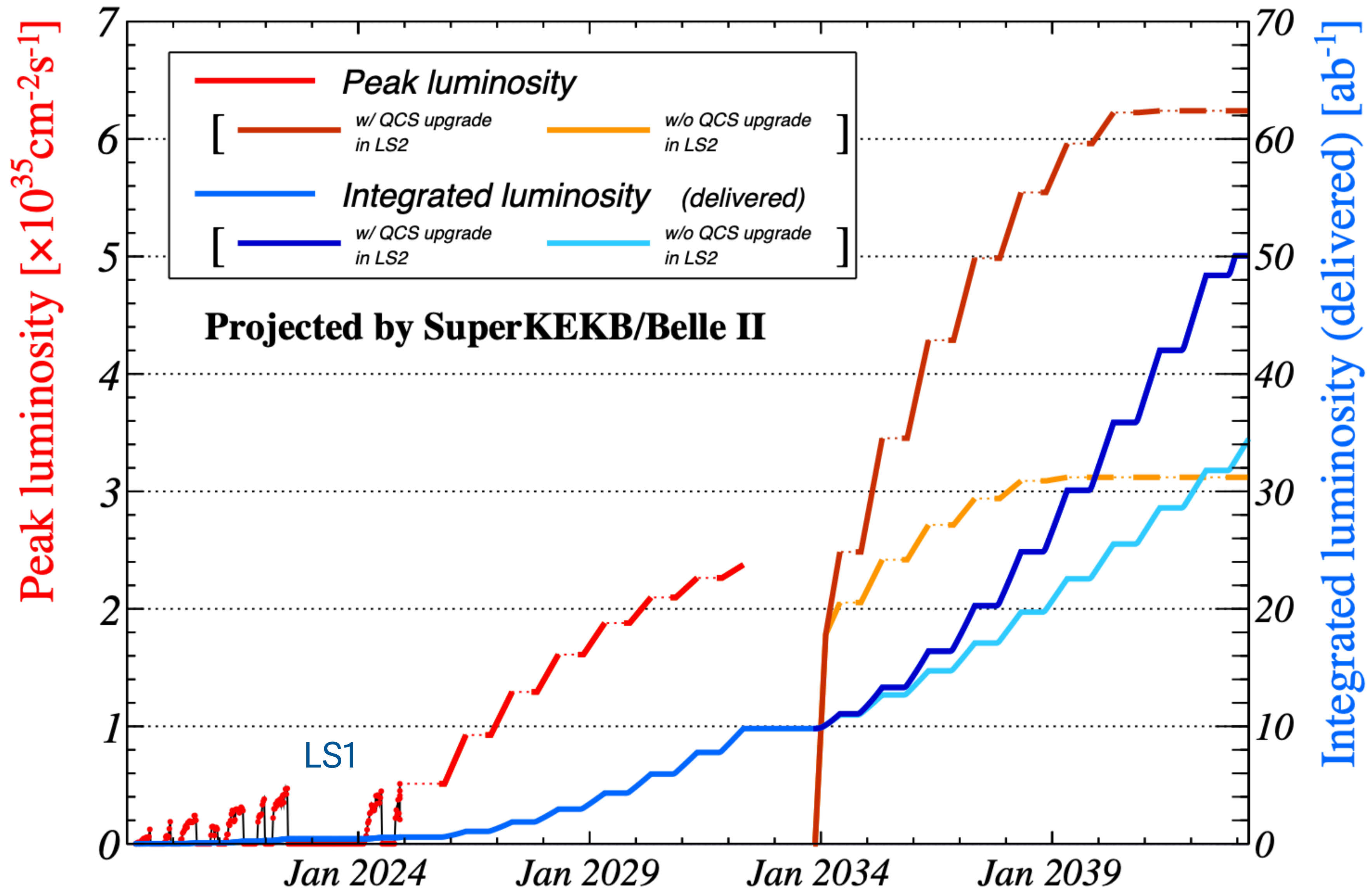
World Record Luminosity of $2.11 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ achieved (June 2009) with crab cavities at KEBB



New record of peak luminosity
 $5.105 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

27-12-2024 at 13:40 hrs JST

Luminosity projection plot



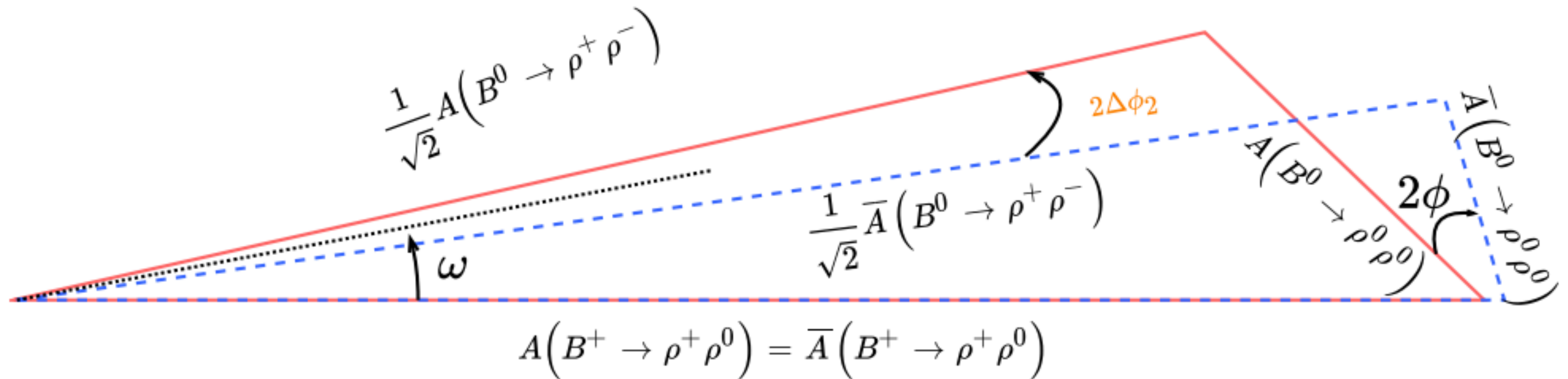
$$B^0 \rightarrow \rho^+ \rho^-$$



[arXiv:2412.19624](https://arxiv.org/abs/2412.19624) | Submitted to Phys. Rev. D



Isospin triangle for $B \rightarrow \rho\rho$



$$B^0 \rightarrow \rho^+ \rho^-$$



Table of systematics on \mathcal{B} and f_L

Source	\mathcal{B} [%]	$f_L [10^{-2}]$
Tracking	± 0.54	—
π^0 efficiency	± 7.67	—
PID	± 0.08	—
\mathcal{T}_C	± 2.87	—
MC sample size	± 0.24	± 0.2
Single candidate selection	± 0.55	± 0.3
SCF ratio	$+2.97$ -2.45	$+0.2$ -0.3
\mathcal{B} 's of peaking backgrounds	$+0.94$ -0.98	± 0.1
$\tau^+ \tau^-$ background yield	$+0.65$ -0.69	± 0.0
Signal model	$+1.14$ -2.02	± 0.2
$q\bar{q}$ model	$+0.49$ -0.51	$+0.1$ -0.2
$B\bar{B}$ model	$+1.00$ -0.40	$+0.3$ -0.1
$\tau^+ \tau^-$ model	$+0.17$ -0.26	$+0.0$ -0.1
Peaking model	$+1.37$ -1.01	$+0.3$ -0.5
Interference	± 1.20	± 0.5
Data-MC mis-modeling	$+3.51$ -1.70	$+0.8$ -0.3
Fit bias	± 1.03	± 1.2
f_{+-}/f_{00}	± 1.51	—
N_{BB}	± 1.45	—
Total systematic uncertainty	$+10.07$ -9.51	$+1.7$ -1.5
Statistical uncertainty	$+7.93$ -7.58	$+2.4$ -2.5

Table of systematics on S and C

Source	$S [10^{-2}]$	$C [10^{-2}]$
\mathcal{B} 's of peaking backgrounds	$+0.6$ -0.5	± 0.1
$\tau\tau$ background yield	± 0.9	$+0.0$ -0.1
Data-MC mis-modeling	$+0.6$ -1.1	$+1.5$ -0.6
Single candidate selection	± 1.3	± 1.9
SCF ratio	$+0.5$ -0.4	$+0.7$ -0.0
Signal model	$+1.1$ -1.4	$+0.3$ -0.4
$q\bar{q}$ model	$+2.2$ -1.0	± 0.2
$B\bar{B}$ model	± 0.9	$+0.7$ -0.5
$\tau^+ \tau^-$ model	± 0.1	± 0.0
Peaking model	$+0.8$ -0.4	$+0.2$ -0.4
Fit bias	± 2.0	± 0.6
Interference	± 2.8	± 1.7
Resolution	$+3.4$ -4.4	$+1.9$ -1.4
Δt PDF for $q\bar{q}$ and $B\bar{B}$	$+3.8$ -1.8	$+0.7$ -0.1
Tag side interference	± 0.5	± 2.1
Wrong tag fraction	$+0.2$ -0.3	± 0.5
Background CP violation	$+3.8$ -3.6	$+4.2$ -3.7
CP violation in TP signal	$+0.8$ -0.2	$+0.2$ -0.4
Tracking detector misalignment	± 1.4	± 0.5
τ_{B^0} and Δm_d	$+1.4$ -1.6	± 0.3
Total systematic uncertainty	$+8.2$ -7.8	$+6.1$ -5.3
Statistical uncertainty	± 18.8	± 12.1

$$B^+ \rightarrow \tau^+ \nu$$

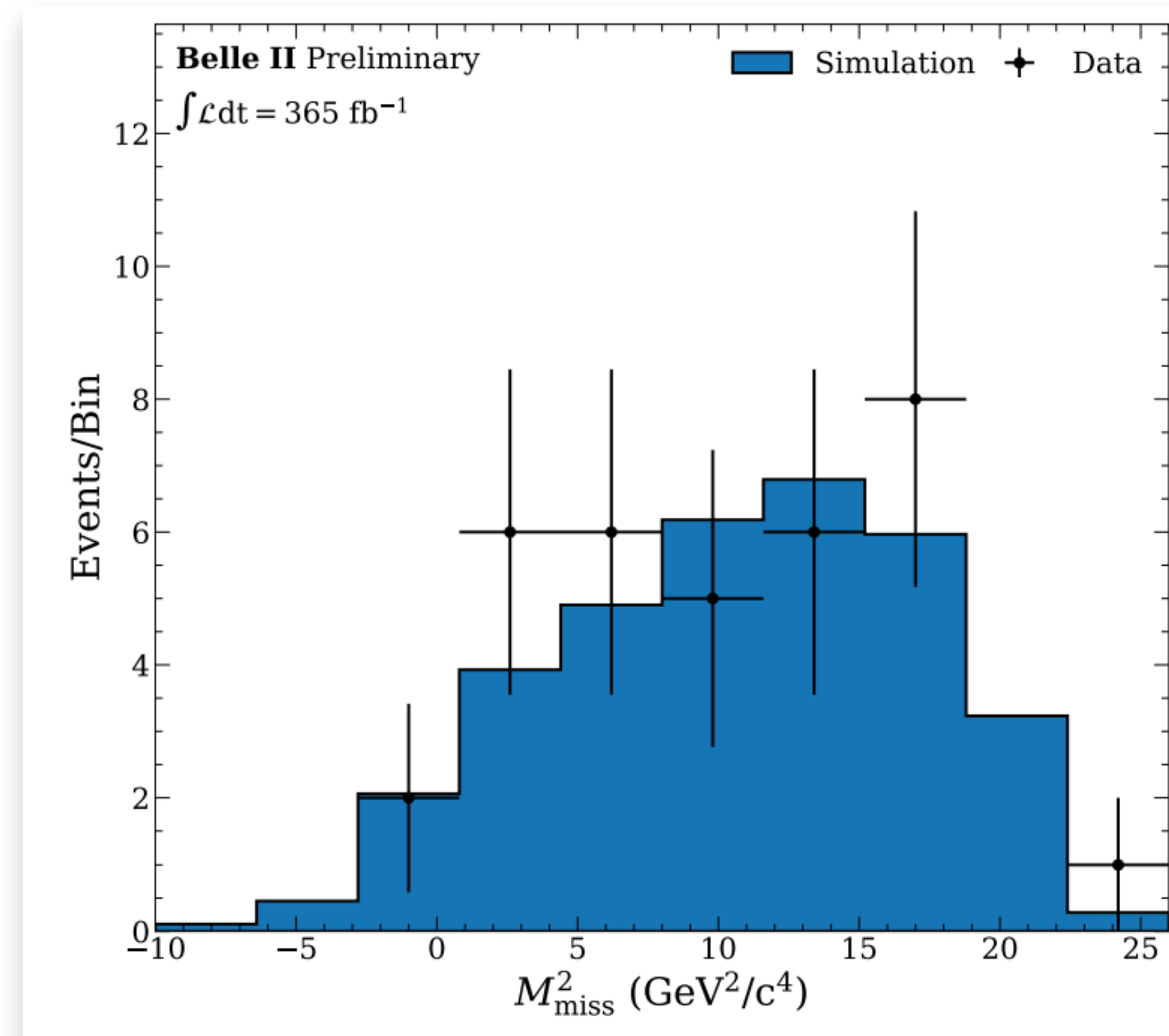
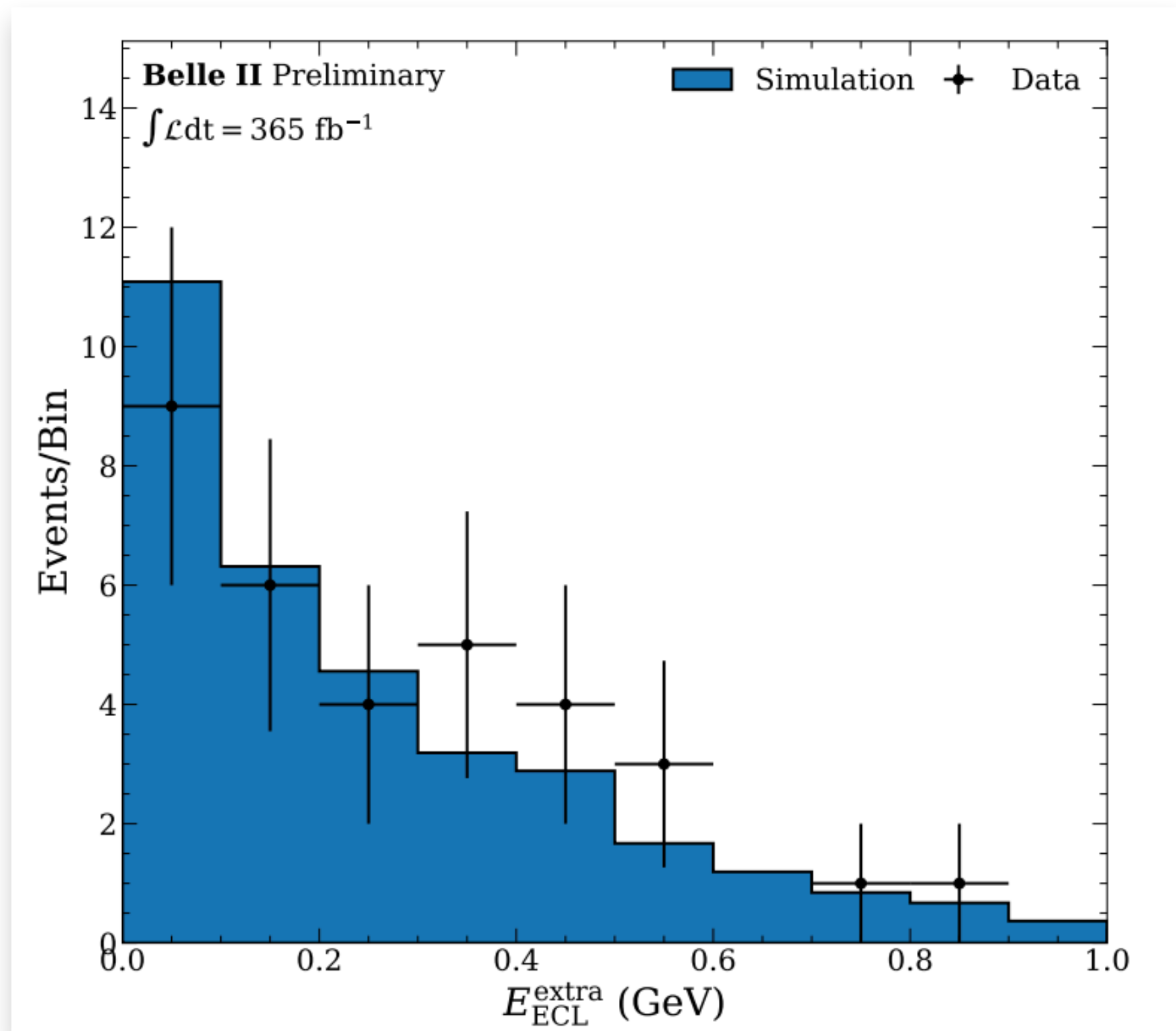


arXiv: 2502.04885 | Submitted to Phys. Rev. D



- Validation:
 - Signal embedding procedure using $B^+ \rightarrow K^+ J/\psi$ sample

Signal embedded control channels



TAB. V. Observed values of the signal yields and branching fractions, obtained from single fits for each τ^+ decay mode and the simultaneous fit.

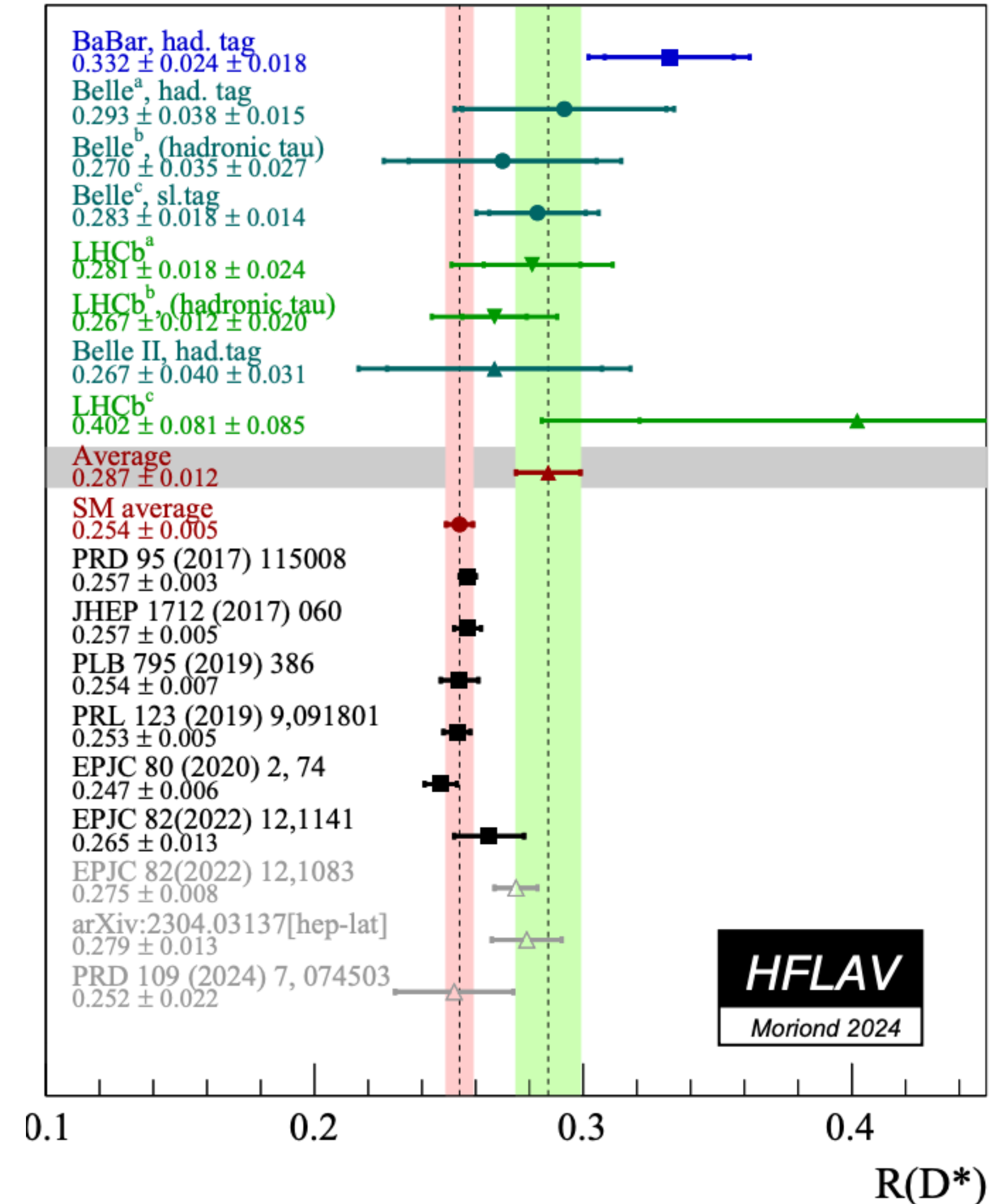
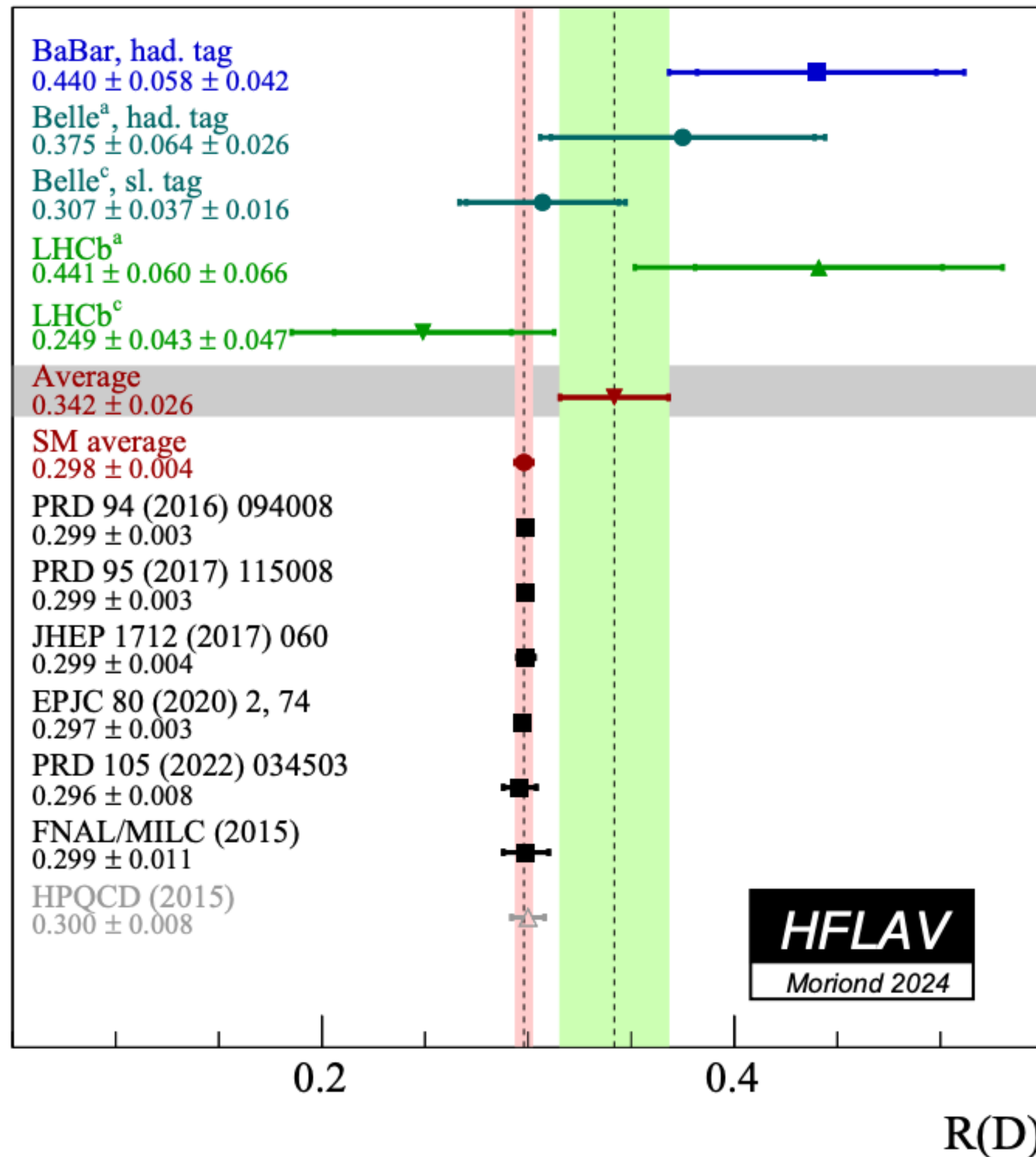
Decay mode	n_s	$\mathcal{B}(10^{-4})$
Simultaneous	94 ± 31	1.24 ± 0.41
$e^+ \nu_e \bar{\nu}_\tau$	13 ± 16	0.51 ± 0.63
$\mu^+ \nu_\mu \bar{\nu}_\tau$	40 ± 20	1.67 ± 0.83
$\pi^+ \bar{\nu}_\tau$	31 ± 13	2.28 ± 0.93
$\rho^+ \bar{\nu}_\tau$	6 ± 25	0.42 ± 1.82

Reco \ True					
	e^+ (%)	μ^+ (%)	π^+ (%)	ρ^+ (%)	other(%)
e^+	97	0.1	0.1	0	2.8
μ^+	0	87	0.9	0.1	12
π^+	0.1	3.3	55.7	16	24.9
ρ^+	0.4	4.5	27.8	61.2	6.1

Table of systematics

Source	Syst.
Simulation statistics	13.3%
Fit variables PDF corrections	5.5%
Decays branching fractions in MC	4.1%
Tag B^- reconstruction efficiency	2.2%
Continuum reweighting	1.9%
π^0 reconstruction efficiency	0.9%
Continuum normalization	0.7%
Particle identification	0.6%
Number of produced $\Upsilon(4S)$	1.5%
Fraction of $B^+ B^-$ pairs	2.1%
Tracking efficiency	0.2%
Total	15.5%

$R(D^+)$ and $R(D^{*+})$



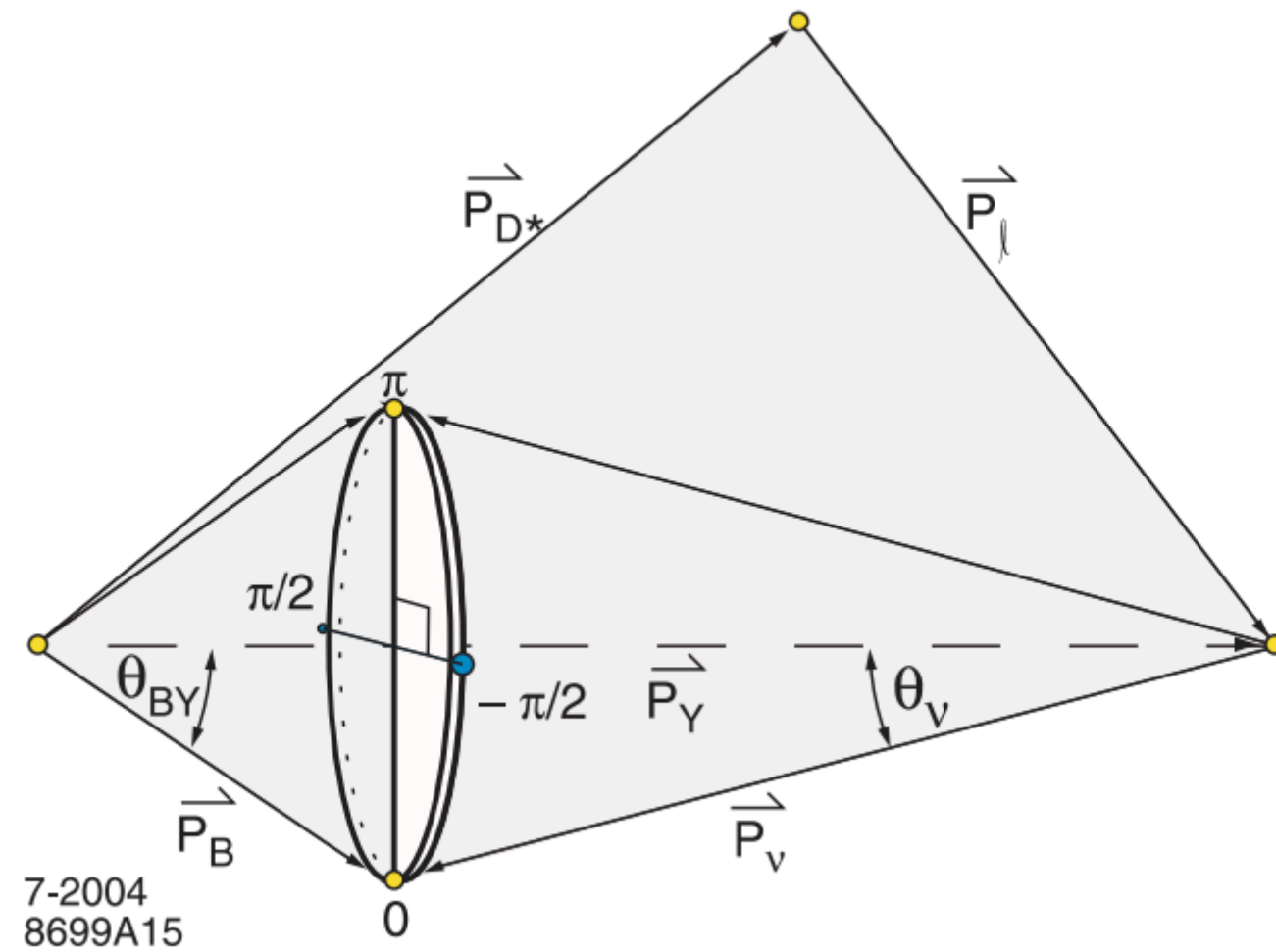
$R(D^+)$ and $R(D^{*+})$



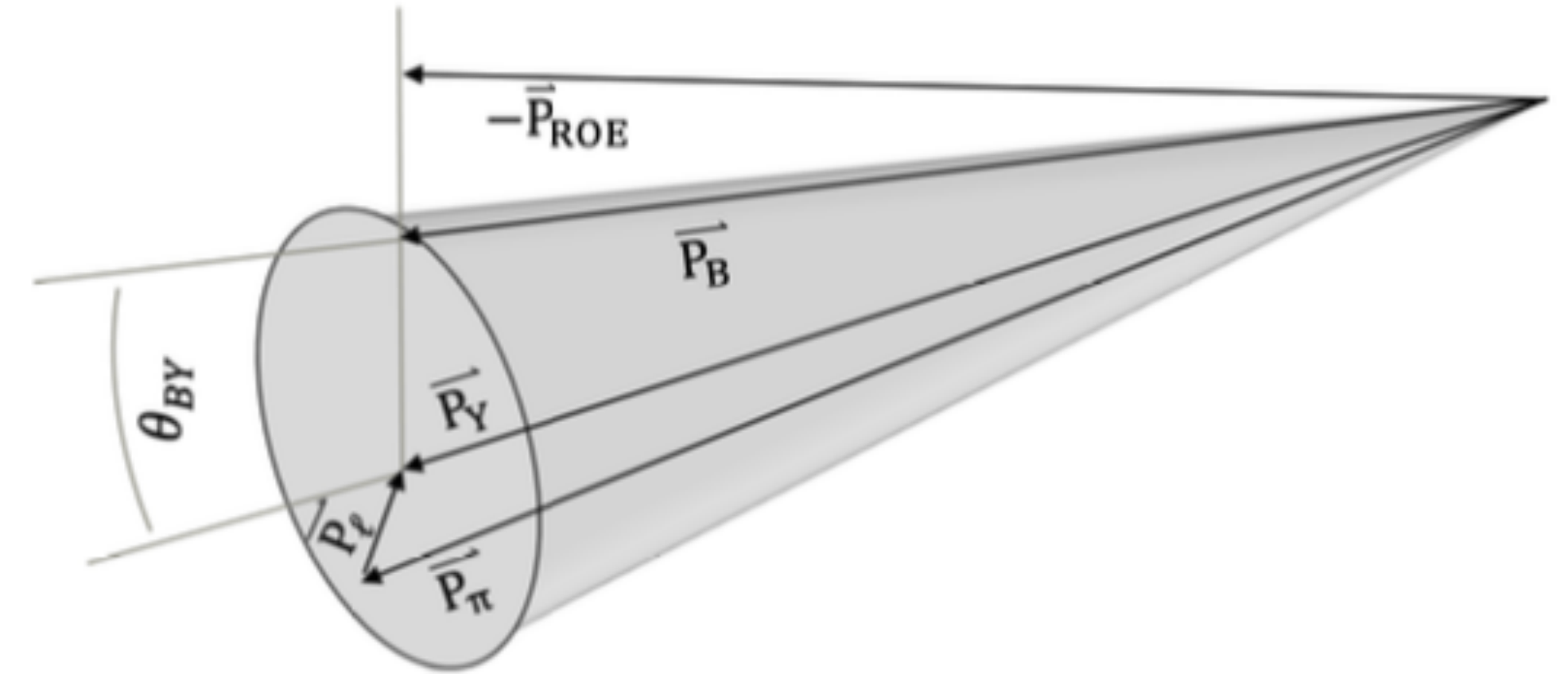
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Diamond Frame (BaBar), [PRD 74 (Nov, 2006) 092004.]



ROE method (Belle)

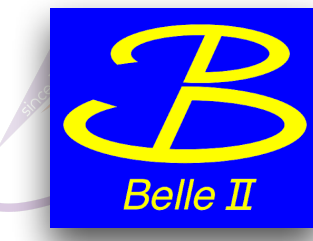


- $\cos \theta_{BY}$ determined from energy-momentum conservation principle and assuming the missing particle is ν ($m_\nu \sim 0$)
- The azimuthal angle (ϕ) is unknown
- ϕ can be determined with the constraint that B lies on the cone with an opening angle, θ_{BY} and :
 - Weighted average of 4 possible B -directions about the cone (BaBar)

$R(D^+)$ and $R(D^{*+})$

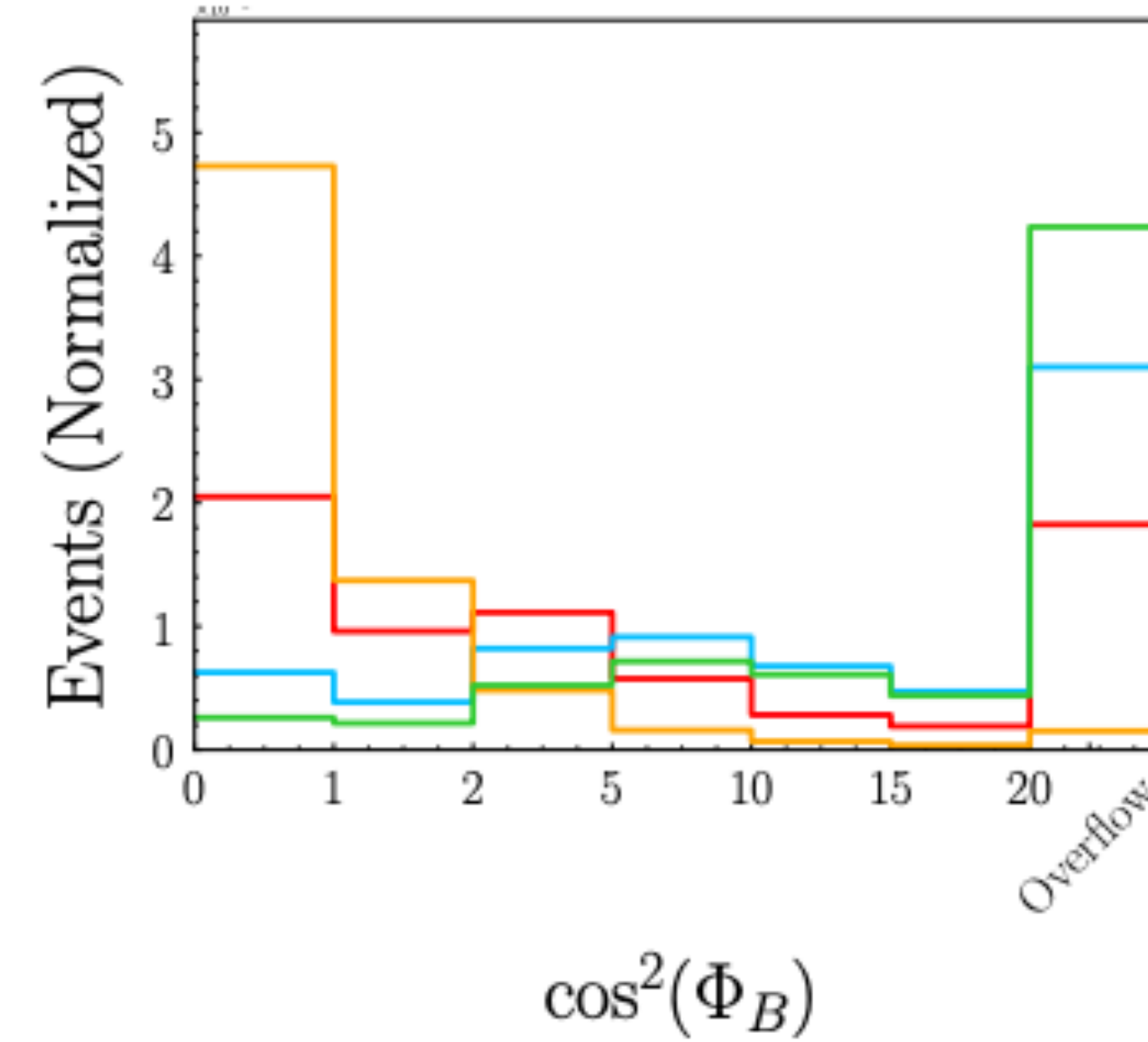
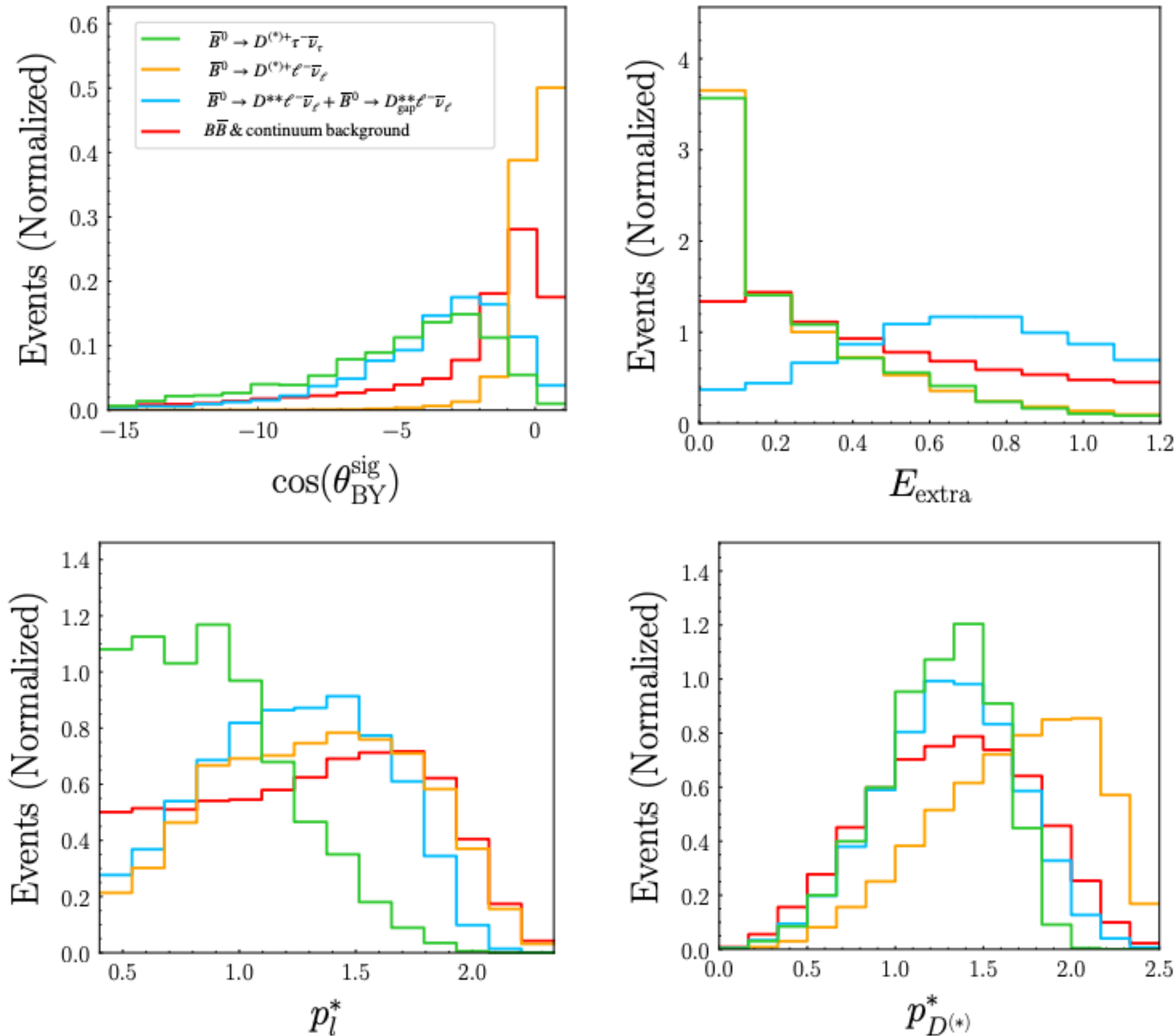


To be submitted to Phys. Rev. D



Input variables to BDT

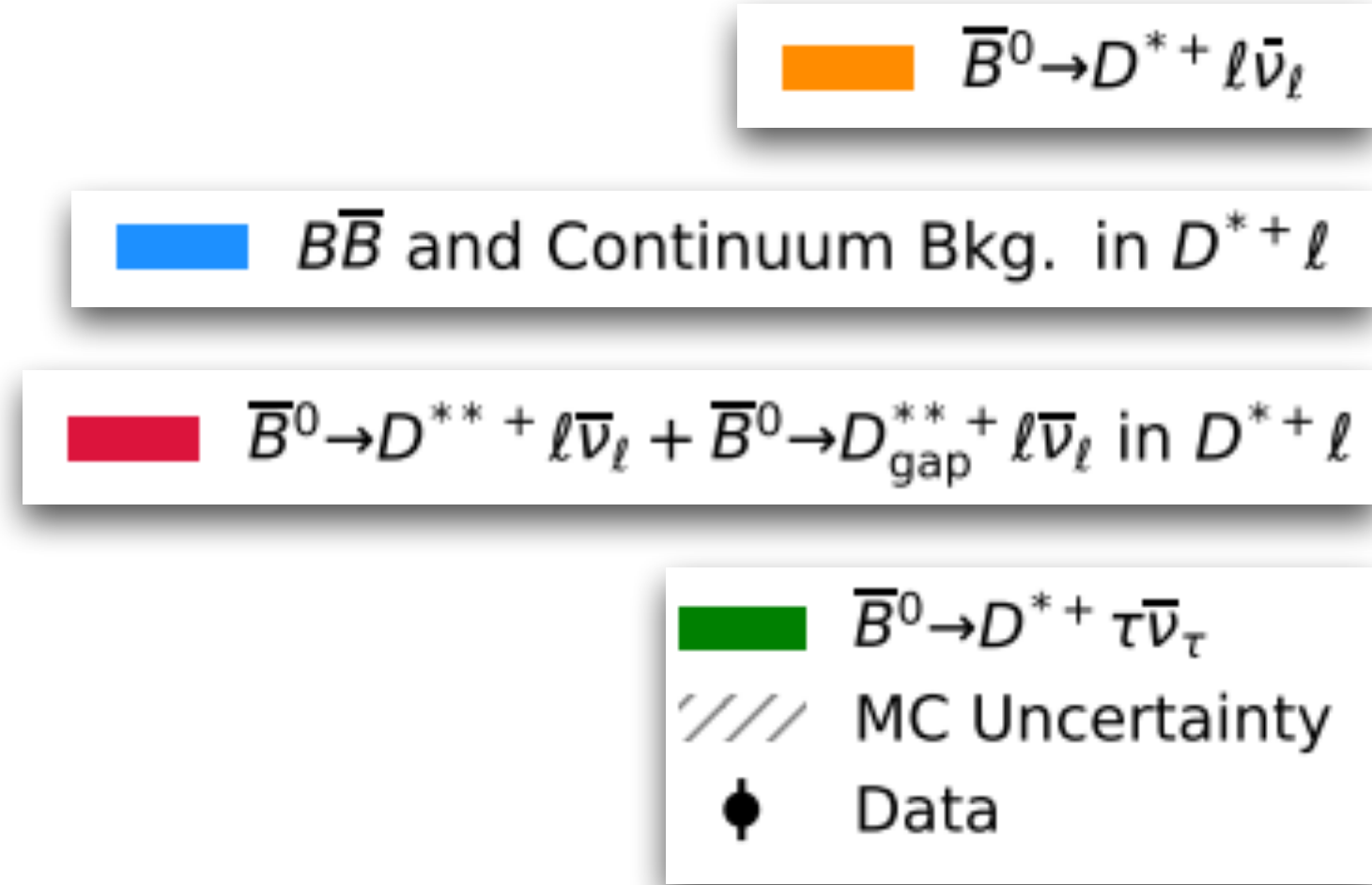
Belle II simulation



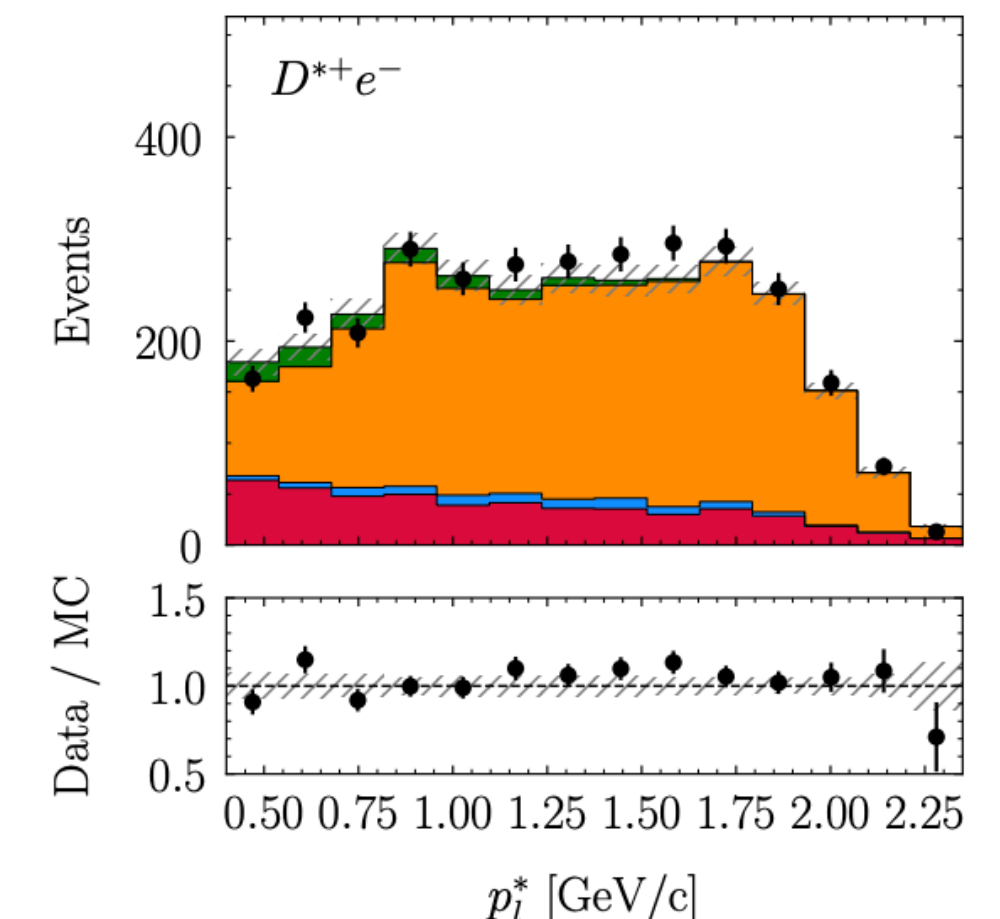
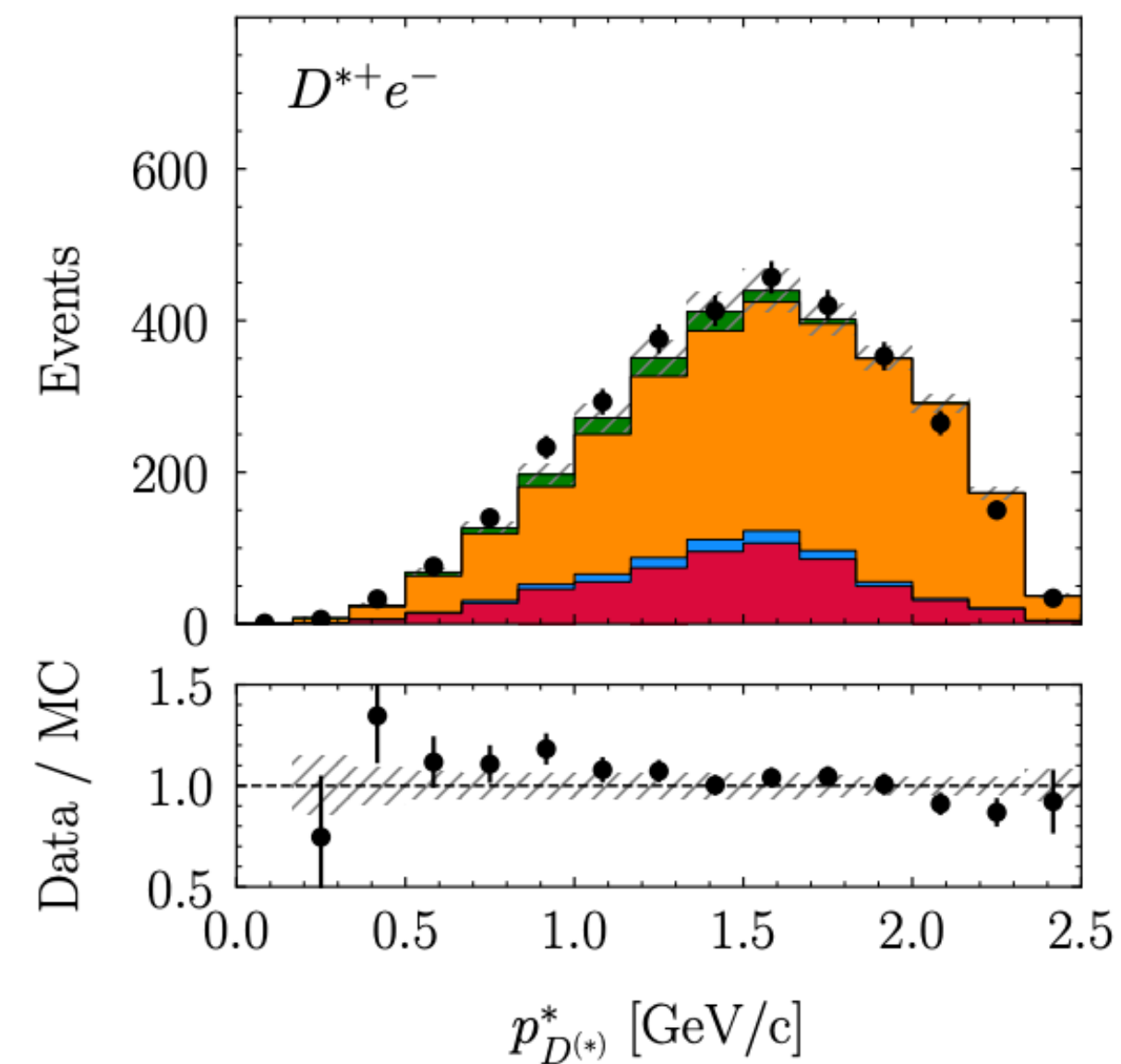
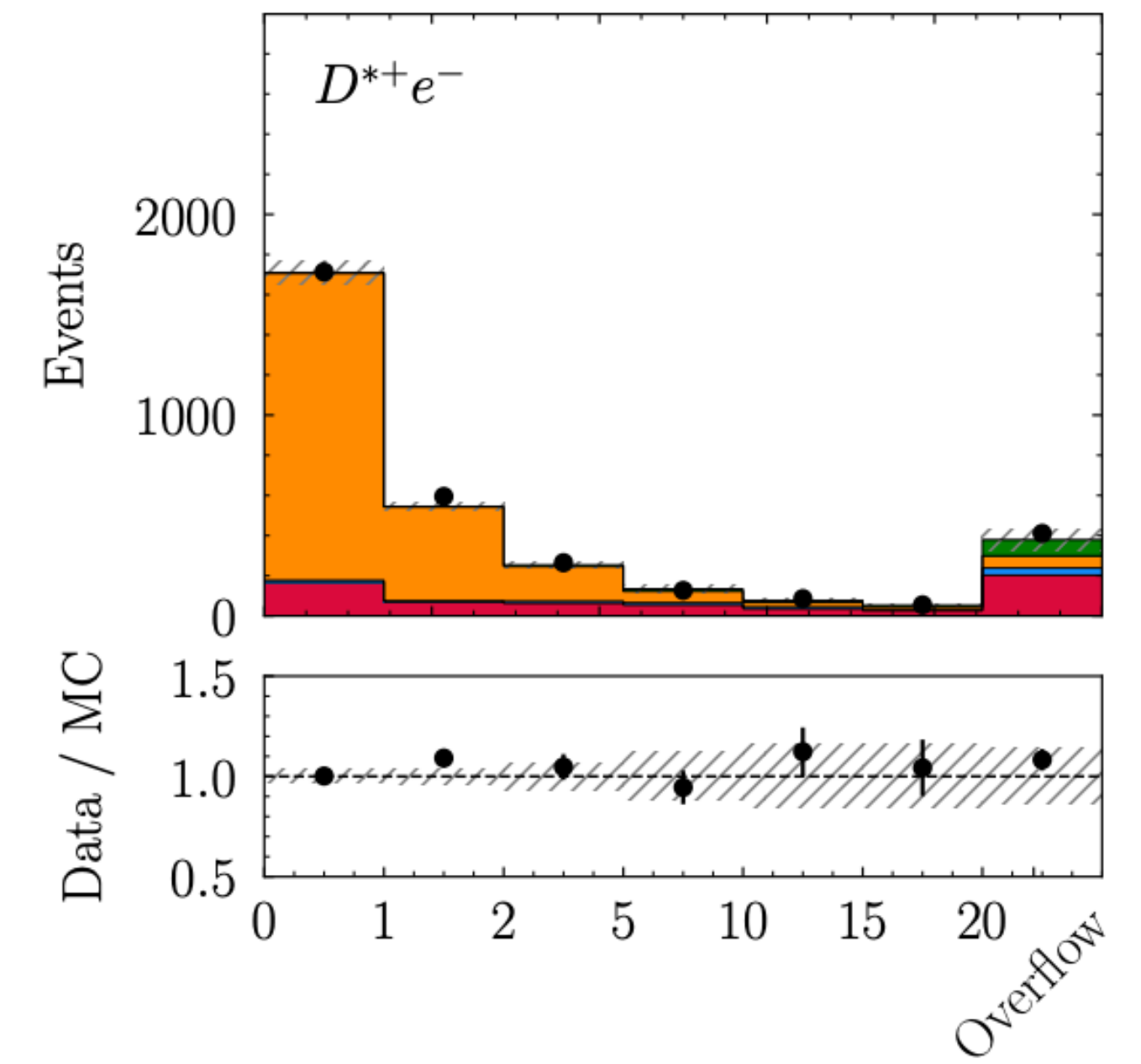
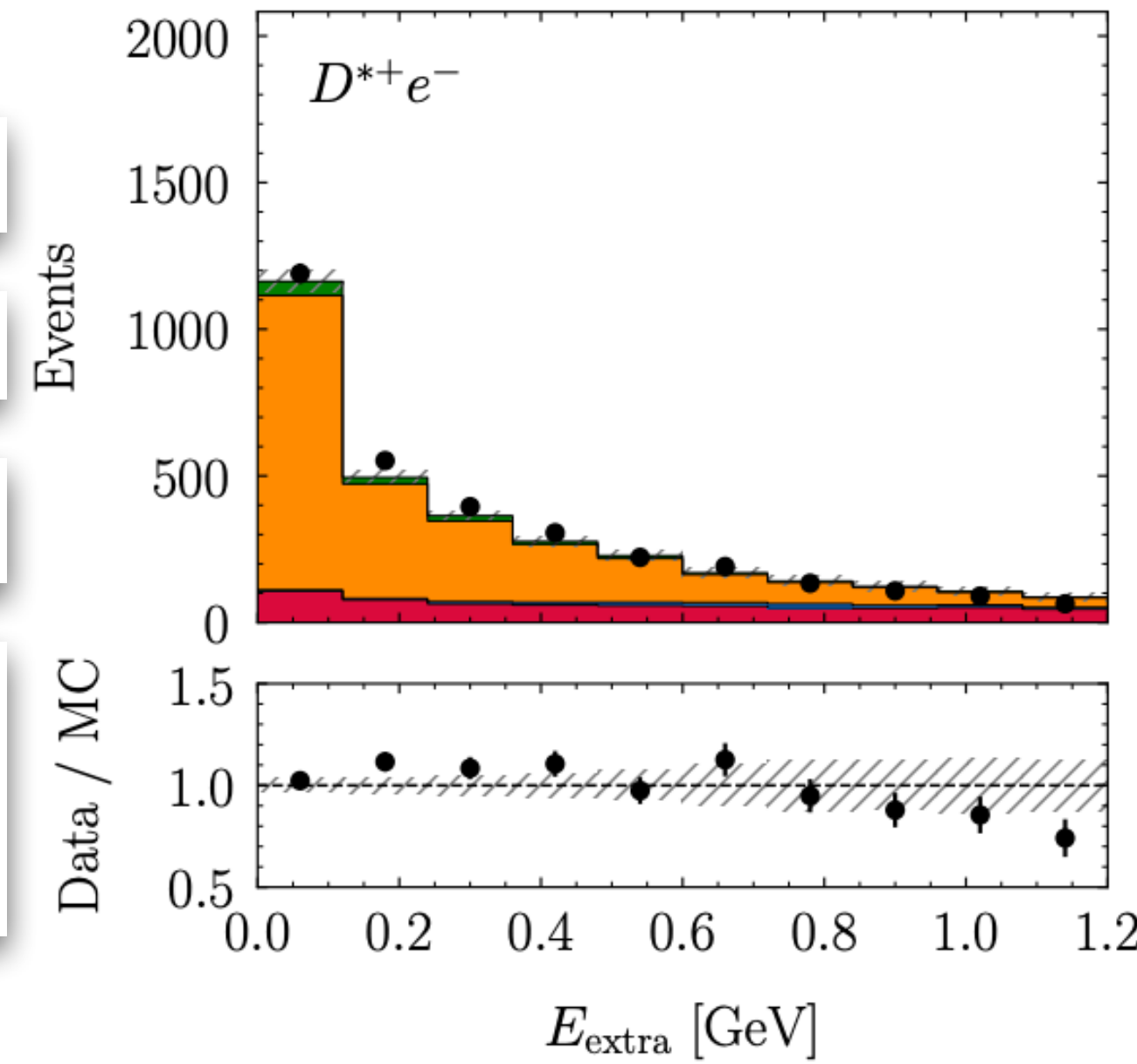
$$\cos^2 \Phi_B = \frac{\cos^2 \theta_{BY}^{\text{sig}} + \cos^2 \theta_{BY}^{\text{tag}} + 2 \cos \theta_{BY}^{\text{sig}} \cos \theta_{BY}^{\text{tag}} \cos \gamma}{\sin^2 \gamma}$$

$R(D^+)$ and $R(D^{*+})$

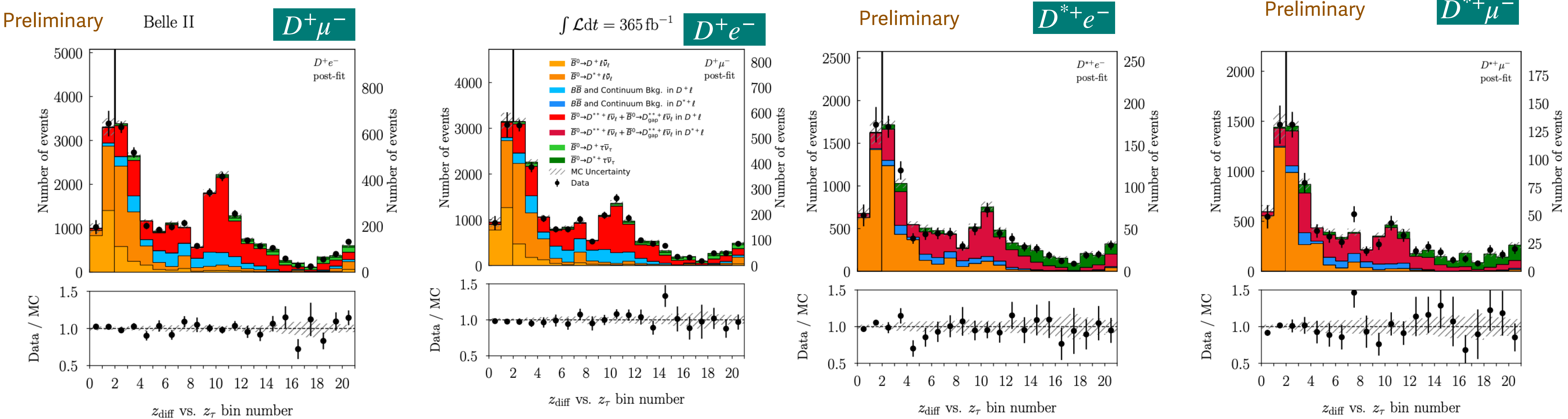
Input variables to BDT (data/MC agreements)



$$\cos^2 \Phi_B = \frac{\cos^2 \theta_{BY}^{\text{sig}} + \cos^2 \theta_{BY}^{\text{tag}} + 2 \cos \theta_{BY}^{\text{sig}} \cos \theta_{BY}^{\text{tag}} \cos \gamma}{\sin^2 \gamma}$$



$R(D^+)$ and $R(D^{*+})$



$\mathcal{R}(D^{(*)})$ with light leptons

$$\mathcal{R}(D_{e/\mu}^+) = 1.07 \pm 0.05(\text{stat}) \pm 0.03(\text{syst})$$

$$\mathcal{R}(D_{e/\mu}^{*+}) = 1.08 \pm 0.04(\text{stat}) \pm 0.03(\text{syst})$$

- Results are consistent with LFU expectations within 1.3σ and 2.0σ , respectively

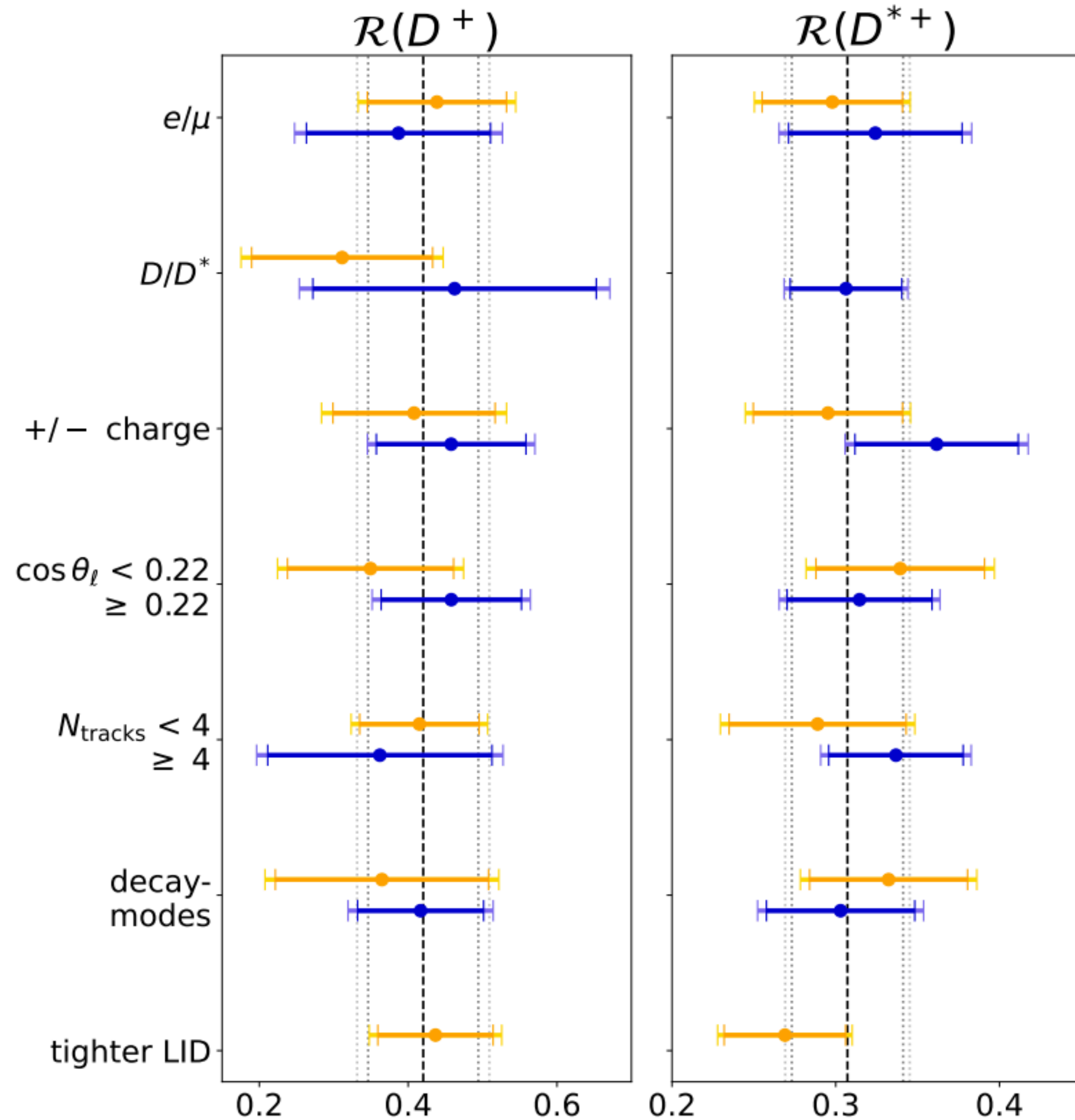
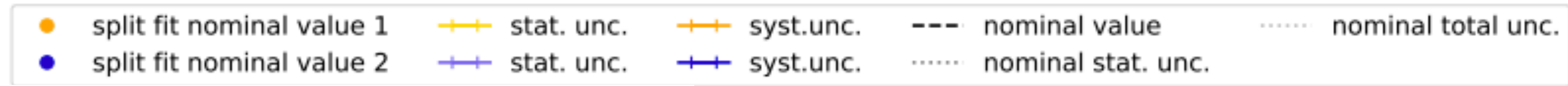
$R(D^+)$ and $R(D^{*+})$



To be submitted to Phys. Rev. D



Stability check



$\mathcal{R}(D^{(*)+})$ determined independently for e/μ

$\mathcal{R}(D^{*+})$ from D^+ are due to limited “feed-downs”, showing large anti-correlation

Split the sample into approx. two equal halves

D^+ has more $N_{\text{tracks}} < 4$, then D^{*+}

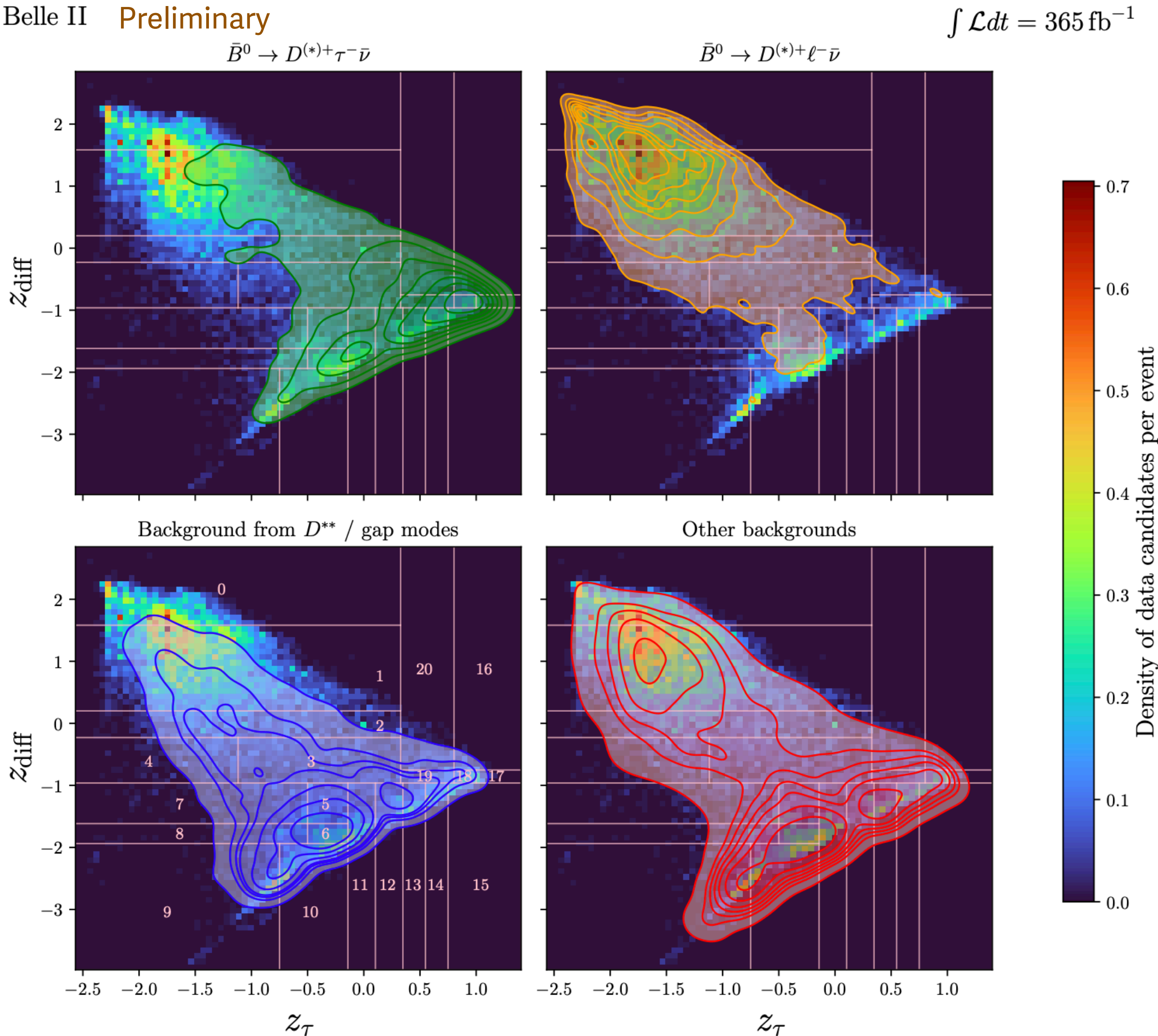
$R(D^+)$ and $R(D^{*+})$



To be submitted to Phys. Rev. D



Belle II Preliminary



Systematic Uncertainty	$\Delta \mathcal{R}(D^+)$	$\Delta \mathcal{R}(D^{*+})$
Add. & Mult. Uncertainties		
MC sample size	0.032 (7.73 %)	0.0155 (5.05 %)
Additive Uncertainties		
LID efficiency (e)	0.0005 (0.13 %)	0.0001 (0.02 %)
LID efficiency (μ)	0.018 (4.26 %)	0.0010 (0.32 %)
Fake rates (e)	0.0198 (4.71 %)	0.0039 (1.26 %)
Fake rates (μ)	0.0004 (0.09 %)	0.0002 (0.06 %)
BDT modeling	0.0140 (3.4 %)	0.0070 (2.2 %)
Gap Branching fractions	0.010 (2.44 %)	0.0005 (0.2 %)
$\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_{\ell}$ FFs	0.0022 (0.55 %)	0.0014 (0.45 %)
$\mathcal{B}(\bar{B} \rightarrow D^{**} \ell \bar{\nu}_{\ell})$	0.0025 (0.60 %)	0.0002 (0.07 %)
$\bar{B} \rightarrow D^{**} \ell \bar{\nu}_{\ell}$ FFs	0.0025 (0.61 %)	0.0007 (0.24 %)
Gap Modeling	0.0022 (0.52 %)	0.0007 (0.22 %)
π^{\pm} from $D^* \rightarrow D\pi$	0.0003 (0.07 %)	0.0003 (0.10 %)
Tracking efficiency	< 0.0001 (< 0.01 %)	< 0.0001 (< 0.01 %)
Multiplicative Uncertainties		
LID efficiency (e)	0.0007 (0.18 %)	0.0004 (0.12 %)
LID efficiency (μ)	0.023 (5.55 %)	0.0002 (0.08 %)
$\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_{\ell}$ FFs	0.0012 (0.29 %)	0.004 (1.3 %)
$\mathcal{B}(\tau^- \rightarrow \ell^- \bar{\nu}_{\ell} \nu_{\tau})$	0.0006 (0.18 %)	0.0006 (0.18 %)
π^{\pm} from $D^* \rightarrow D\pi$	0.0002 (0.04 %)	0.0004 (0.13 %)
Tracking efficiency	< 0.0001 (< 0.01 %)	0.0001 (0.04 %)
Total Syst. Uncertainty	0.051 (12.32 %)	0.018 (5.88 %)

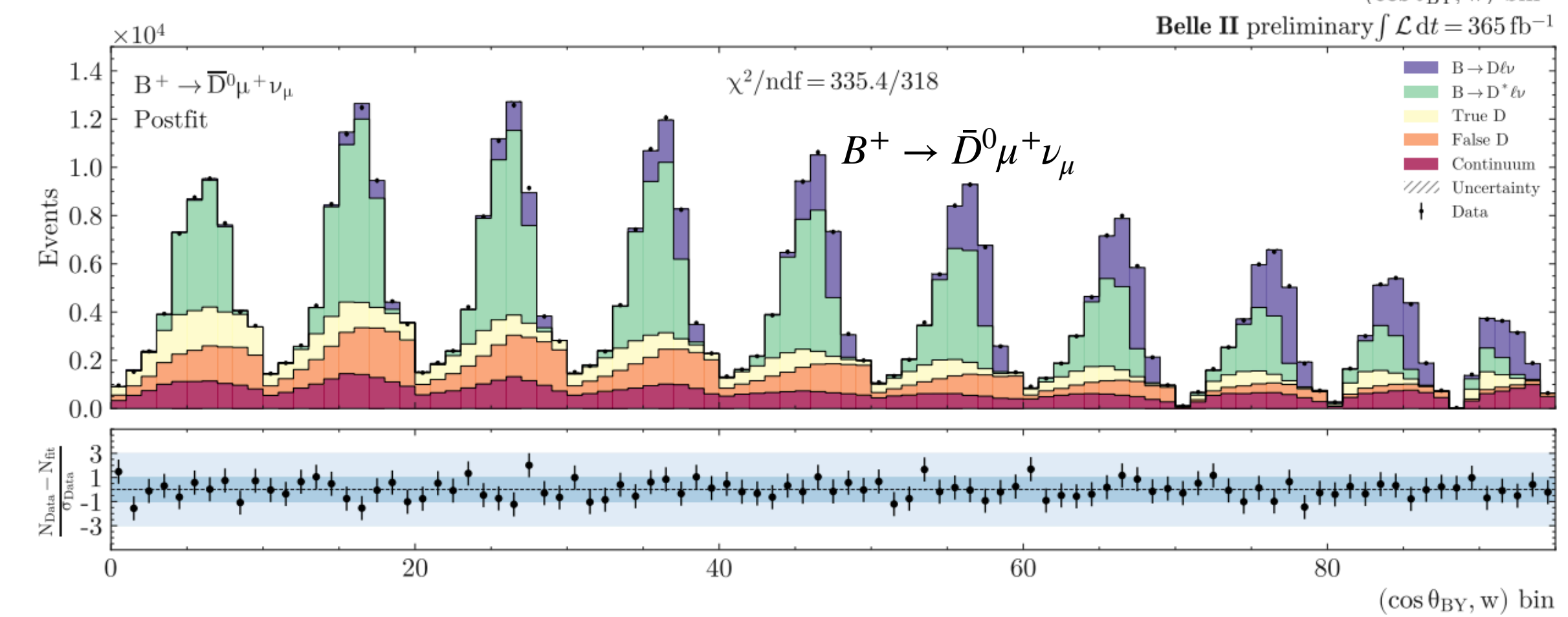
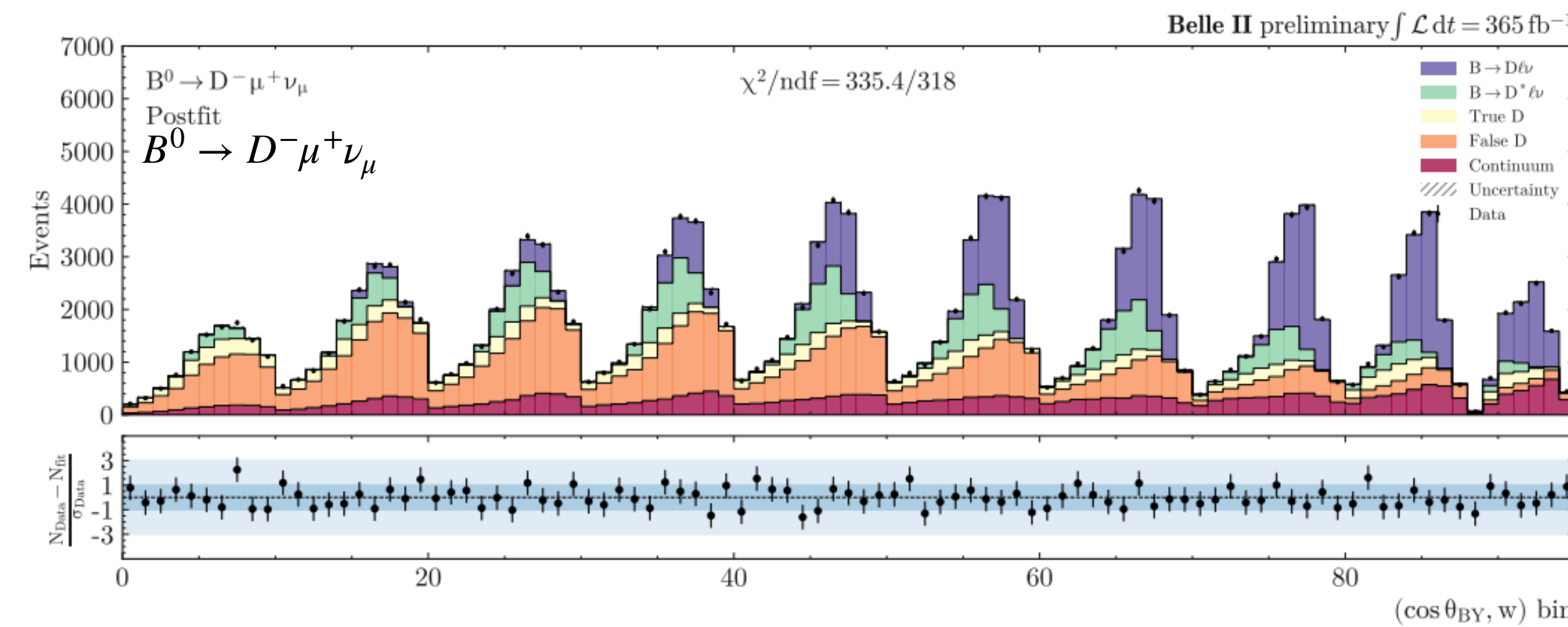
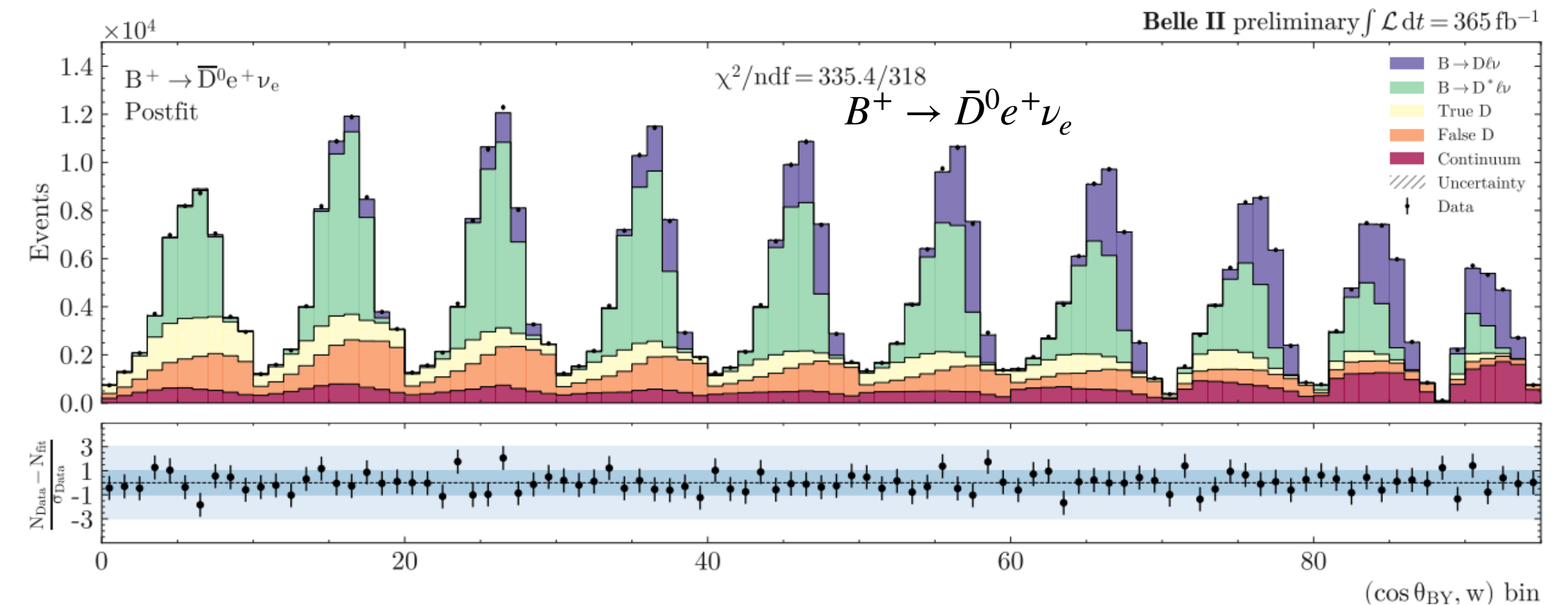
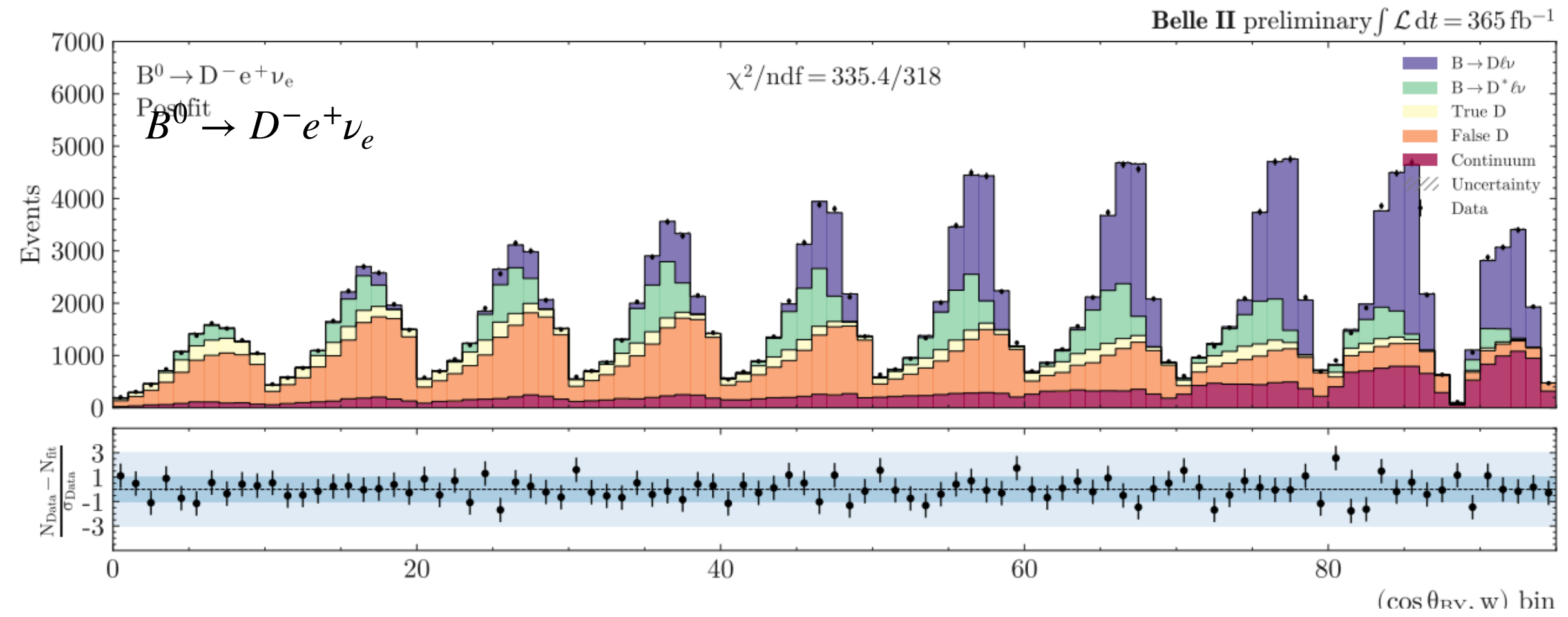
$|V_{cb}|$ from $B \rightarrow D\ell\nu$



To be submitted to Phys. Rev. D



Signal extraction



|V_{cb}| from B → Dℓν



To be submitted to Phys. Rev. D



	Signal Yield	\mathcal{B} [%]
$B^+ \rightarrow \bar{D}^0 e^+ \nu_e$	75,186	2.34 ± 0.11
$B^+ \rightarrow \bar{D}^0 \mu^+ \nu_\mu$	61,259	2.27 ± 0.11
$B^0 \rightarrow D^- e^+ \nu_e$	47,617	2.07 ± 0.13
$B^0 \rightarrow D^- \mu^+ \nu_\mu$	39,648	2.05 ± 0.13
$B^0 \rightarrow D^- \ell^+ \nu_\ell$		2.06 ± 0.12
$B^+ \rightarrow \bar{D}^0 \ell^+ \nu_\ell$		2.31 ± 0.10
$B \rightarrow D \ell \nu$		2.10 ± 0.08

|V_{cb}| is extracted using χ^2 fits to the measured w spectra

CLN parametrisation of the FF

$$\chi^2 = \sum_{i,j} \left(\frac{\Delta\Gamma_i}{\Delta w} - \frac{\Delta\Gamma_{i,\text{CLN}}}{\Delta w} \right) C_{ij}^{-1} \left(\frac{\Delta\Gamma_j}{\Delta w} - \frac{\Delta\Gamma_{j,\text{CLN}}}{\Delta w} \right)$$

Result (Preliminary)

$$\eta_{\text{EW}} \mathcal{G}(1) |V_{cb}| = (40.9 \pm 1.4) \times 10^{-3}$$

$$\rho^2 = 1.09 \pm 0.06 ,$$

$$|V_{cb}|_{\text{CLN}} = (38.5 \pm 1.3) \times 10^{-3}$$

Source	Uncertainty [%]
Statistical	0.9
Systematic	1.5
MC Stat. Error	0.5
N_{bb}	0.5
f_{00}/f_{+-}	0.1
$f_{\not{B}}$	0.3
$\mathcal{B}(D \rightarrow K\pi(\pi))$	0.3
Vertex fit χ^2 correction	0.5
$\mathcal{B}(B \rightarrow X_c \ell \nu_\ell)$	0.3
Lepton identification	0.2
Kaon identification	0.5
Tracking efficiency	0.3
Signal PDF	0.4
$B \rightarrow D^* \ell \nu_\ell$ form factor	0.1
Background w modelling	0.5
$E_Y^* - m_Y$ reweighing	0.3
$B^{0/-}$ lifetime	0.1
Theoretical (FF fits)	1.3
Lattice QCD inputs	1.2
Long-distance QED	0.4
Total	2.1

$\Delta\Gamma_i/\Delta w$ and the correlations across different bins combined for the 4 sub-samples

i	$w_{i,\min}$	$w_{i,\max}$	$\Delta\Gamma_i/\Delta w[10^{-15}\text{GeV}]$	ρ_{ij}										
				1	2	3	4	5	6	7	8	9	10	
1	1.00	1.06	0.22 ± 0.59	1.00	-0.06	0.15	0.08	0.07	0.04	0.03	0.02	0.00	-0.00	
2	1.06	1.12	3.54 ± 0.56		1.00	0.13	0.33	0.26	0.24	0.22	0.19	0.13	0.07	
3	1.12	1.18	6.46 ± 0.61			1.00	0.25	0.44	0.37	0.36	0.31	0.22	0.13	
4	1.18	1.24	10.17 ± 0.68				1.00	0.37	0.59	0.52	0.48	0.34	0.20	
5	1.24	1.30	14.27 ± 0.72					1.00	0.49	0.67	0.55	0.41	0.23	
6	1.30	1.36	18.68 ± 0.84						1.00	0.58	0.71	0.49	0.30	
7	1.36	1.42	21.41 ± 0.89							1.00	0.59	0.60	0.36	
8	1.42	1.48	25.42 ± 0.96								1.00	0.49	0.48	
9	1.48	1.54	28.11 ± 1.09									1.00	0.61	
10	1.54	w_{\max}	29.44 ± 1.41										1.00	

$\Delta\Gamma_i/\Delta w$ for the 4 sub-samples

i	$w_{i,\min}$	$w_{i,\max}$	$\Delta\Gamma_i/\Delta w [10^{-15}\text{GeV}]$			
			$B^0 \rightarrow D^- e^+ \nu_e$	$B^0 \rightarrow D^- \mu^+ \nu_\mu$	$B^+ \rightarrow \bar{D}^0 e^+ \nu_e$	$B^+ \rightarrow \bar{D}^0 \mu^+ \nu_\mu$
1	1.00	1.06	-0.1 ± 0.7	0.6 ± 0.7	-0.5 ± 1.5	1.8 ± 1.7
2	1.06	1.12	3.7 ± 0.8	3.2 ± 0.8	4.1 ± 1.3	3.7 ± 1.4
3	1.12	1.18	6.2 ± 0.8	7.2 ± 0.9	5.5 ± 1.3	6.1 ± 1.2
4	1.18	1.24	9.8 ± 0.9	11.0 ± 0.9	8.9 ± 1.3	10.3 ± 1.4
5	1.24	1.30	13.9 ± 1.0	14.8 ± 1.0	14.0 ± 1.3	13.7 ± 1.3
6	1.30	1.36	18.8 ± 1.2	18.0 ± 1.1	18.6 ± 1.4	18.6 ± 1.5
7	1.36	1.42	21.9 ± 1.2	23.0 ± 1.3	19.6 ± 1.6	19.7 ± 1.6
8	1.42	1.48	25.5 ± 1.4	25.0 ± 1.5	25.5 ± 1.7	25.5 ± 1.8
9	1.48	1.54	29.1 ± 1.7	27.3 ± 1.9	28.1 ± 1.8	26.0 ± 1.9
10	1.54	w_{\max}	32.2 ± 2.5	25.3 ± 2.6	30.5 ± 2.3	27.2 ± 2.5