Cabibbo-Kobayashi-Maskawa Matrix Measurements at Belle and Belle II

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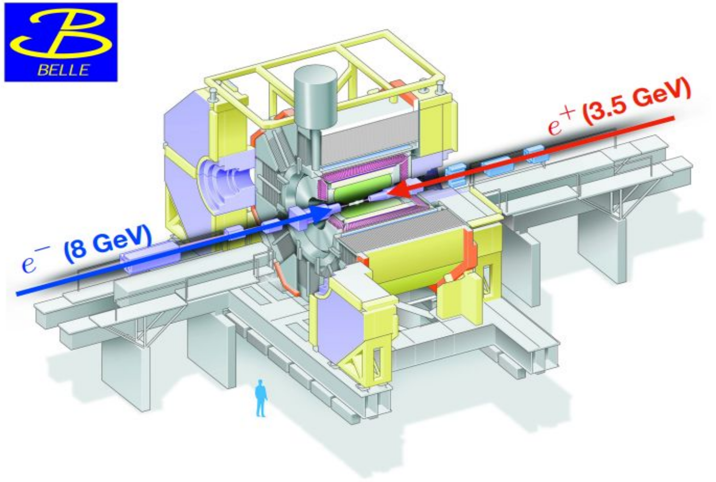
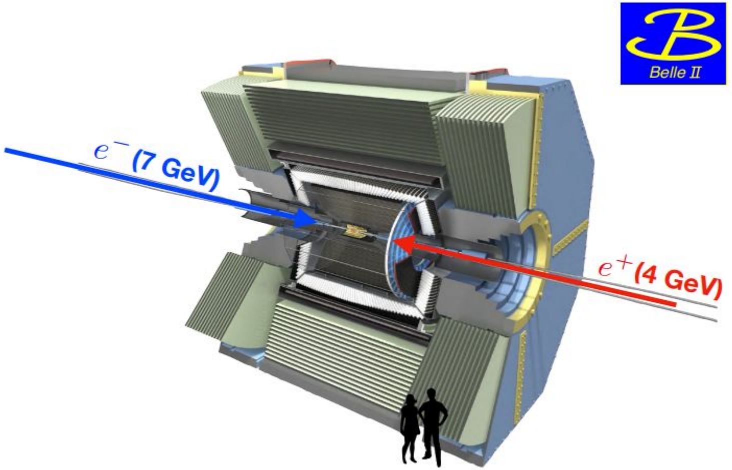
On behalf of the Belle II collaboration

Phenomenology 2025 Symposium

The Belle and Belle II Detectors



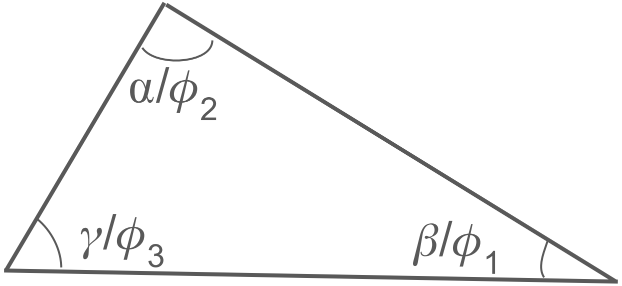
- Operated from 1999 to 2010.
- Commissioned at KEKB accelerator.
- Asymmetric e^+ (3.5 GeV) e^- (8 GeV) collider.
- Collected a total of 1 ab^{-1} of data.



- Operating since 2019.
- Commissioned at SuperKEKB accelerator.
- Asymmetric e^+ (4 GeV) e^- (7GeV) collider.
- From 2019 - 2022, 400 fb^{-1} of data was collected.

CKM Matrix and the Unitary Triangle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



CKM Matrix

- Describes flavor-changing weak interaction of the quarks.
- Each matrix element determines the strength of quark-level transition.
- 3 mixing angles and 1 CP-violating phase.

Unitary Triangle

- Unitarity of the CKM matrix leads to 6 unitary triangles.

$$V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0$$

Measuring $|V_{cb}|$ from $B \to D\ell\nu_\ell$

- CKM matrix element $|V_{cb}|$ governs $b \to c$ quark transitions.
- Probe $|V_{cb}|$ via exclusive semileptonic decays:
  - ℓ is an electron or a muon
    - $B \to D^*\ell\nu_\ell$
    - $B \to D\ell\nu_\ell$

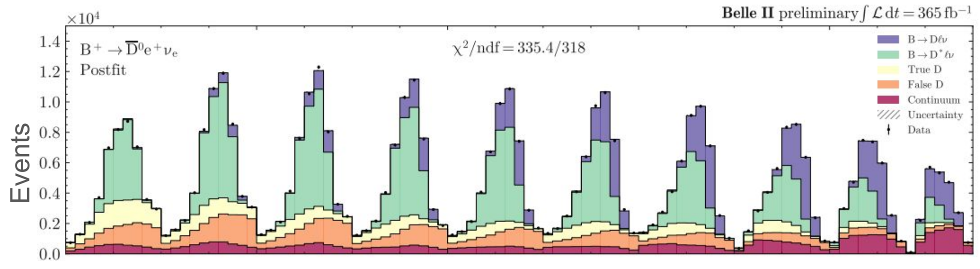
      Neutral mode: $B^0 \to D^-\ell^+\nu_\ell$

      Charged mode: $B^+ \to \overline{D}^0\ell^+\nu_\ell$

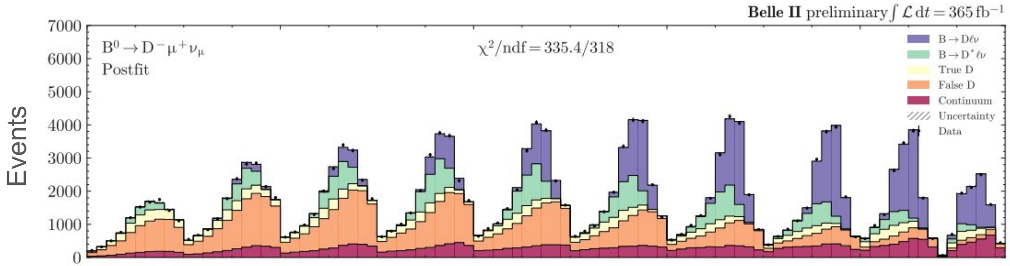
- Two important variables:

$$\cos\theta_{BY} = \frac{2E_B^*E_Y^* - M_B^2 - M_Y^2}{2|p_B^*||p_Y^*|}$$

$$w = \frac{M_B^2 + M_D^2 - q^2}{2M_BM_D}$$



(cosθ_{BY}, w) bin



(cosθ_{BY}, w) bin

Results

- The branching fractions of each of the two modes were measured:

$$\mathcal{B}(B^0 \to D^- \ell^+ \nu_\ell) = (2.06 \pm 0.12)\%$$
$$\mathcal{B}(B^+ \to \bar{D}^0 \ell^+ \nu_\ell) = (2.31 \pm 0.10)\%$$

- Combining the charged and neutral modes, lepton flavor universality was tested:

$$R^{e/\mu} = 1.02 \pm 0.03$$

- $|V_{cb}|$ was determined in two ways:

$|V_{cb}|_{CLN}$ = (38.5 ± 1.3) x 10^{-3}

$|V_{cb}|_{BCL}$ = (39.2 ± 0.88) x 10^{-3}

The uncertainties on the branching fractions and on $R^{e/\mu}$ are the quadratic sums of the statistical and systematic uncertainties.

	Signal Yield	$\mathcal{B}$ [%]
$B^+ \to \bar{D}^0 e^+ \nu_e$	75,186	2.34 ± 0.11
$B^+ \to \bar{D}^0 \mu^+ \nu_\mu$	61,259	2.27 ± 0.11
$B^0 \to D^- e^+ \nu_e$	47,617	2.07 ± 0.13
$B^0 \to D^- \mu^+ \nu_\mu$	39,648	2.05 ± 0.13
$B^+ \to \bar{D}^0 \ell^+ \nu_\ell$		2.31 ± 0.10
$B^0 \to D^- \ell^+ \nu_\ell$		2.06 ± 0.12
$B \to D\ell\nu$		2.10 ± 0.08

Caprini-Lellouch-Neubert (CLN)
Bourrely-Caprini-Lellouch (BCL)

Measuring $|V_{ub}|$ using the $B^+ \to \tau^+\nu_\tau$ decay

Belle II

arXiv:2502.04885

- **Goal**:

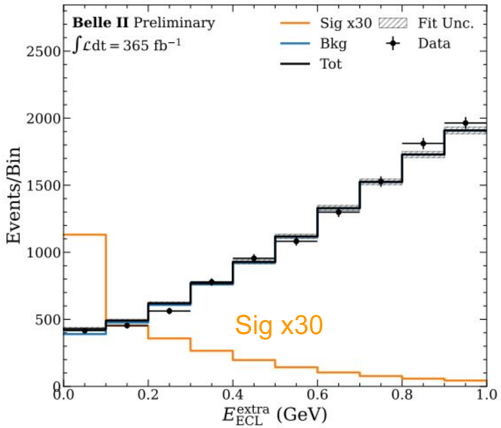
$$\mathcal{B}(B^+ \to \tau^+\nu_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi}\left[1 - \frac{m_\tau^2}{m_B^2}\right]^2 f_B^2 |V_{ub}|^2 \tau_B$$

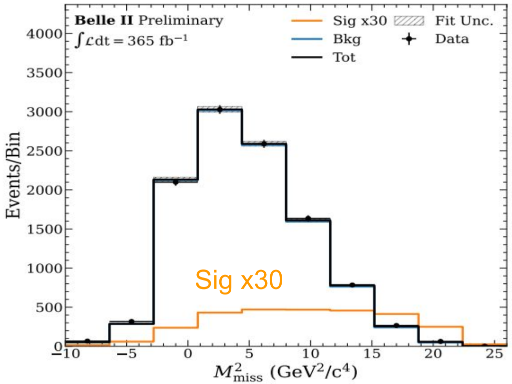
- $B^+ \to \tau^+\nu_\tau$ decay, then look at four τ^+ decay modes:
  - $\tau^+ \to e^+\nu_e\mu_\tau$
  - $\tau^+ \to \mu^+\nu_\mu\nu_\tau$
  - $\tau^+ \to \pi^+\nu_\tau$
  - $\tau^+ \to \rho^+\nu_\tau$
  - These decays make up 72% of all τ decays.

- **Two key variables:**

$E_{\mathrm{ECL}}^{\mathrm{extra}}$: Extra energy in the calorimeter not associated with the B^+ decay.

M_{miss}^2 : Missing squared mass -- due to undetected particles ie., neutrino.





Results

Belle II

arXiv:2502.04885

- A 2D maximum likelihood fit to data is performed using $E_{\mathrm{ECL}}^{\mathrm{extra}}$ and M_{miss}^2 as discriminating variables:
  - The fit determines five parameters: **one common branching fraction** + **background yield for each of the four decay modes**.
- The branching fraction was extracted:

$$\mathcal{B}(B^+ \to \tau^+\nu_\tau) = (1.24 \pm 0.41) \times 10^{-4}$$

  - The uncertainty is statistical only.

TAB. I. Published results for $\mathcal{B}(B^+ \to \tau^+\nu_\tau)$ by Belle, *BABAR* and the PDG average.

Experiment	Tag	$\mathcal{B}(10^{-4})$
Belle	Hadronic	$0.72^{+0.27}_{-0.25} \pm 0.11$
BABAR	Hadronic	$1.83^{+0.53}_{-0.49} \pm 0.24$
Belle	Semileptonic	$1.25 \pm 0.28 \pm 0.27$
BABAR	Semileptonic	$1.8 \pm 0.8 \pm 0.2$
PDG		1.09 ± 0.24

Assuming the Standard Model and using the measured value of the branching fraction, the value of $|V_{ub}|$ was determined:

$$|V_{ub}|_{B^+\to\tau^+\nu_\tau} = [4.41^{+0.74}_{-0.89}] \times 10^{-3}$$

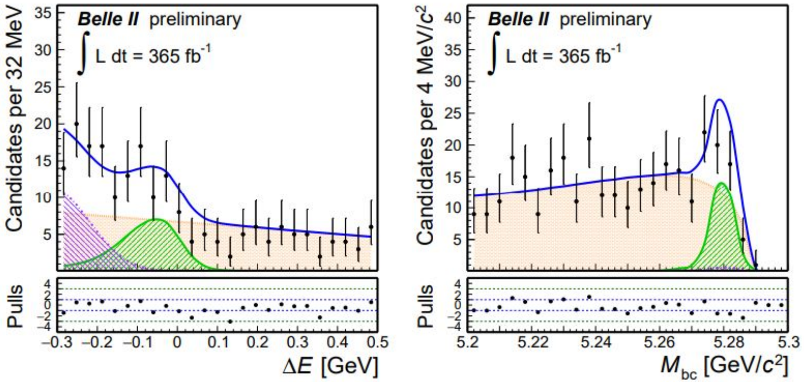
Measuring branching fraction from $B^0 \to \pi^0 \pi^0$

arXiv:2412:14260

- **Goal:** Measure branching fraction and CP-violating flavor-dependent decay rate of $B^0 \to \pi^0 \pi^0$.

$$\mathcal{A}_{CP}(B^0 \to \pi^0\pi^0) = \frac{\Gamma(\overline{B}^0 \to \pi^0\pi^0) - \Gamma(B^0 \to \pi^0\pi^0)}{\Gamma(\overline{B}^0 \to \pi^0\pi^0) + \Gamma(B^0 \to \pi^0\pi^0)}$$

- **Signal Events:**
  - For $B^0 \to \pi^0 \pi^0$, the dominant decay of the π^0 meson is $\pi^0 \to \gamma\gamma$.
  - Separate $B^0 \to \pi^0 \pi^0$ from background using four variables in a multi-dimensional fit:
    - M_{bc}, ΔE, C_t, and ω_t



Number of Signal Events: 126 ± 20

C_t: Continuum suppression using classifier variable C — rejects non-B background.
ω_t: transformed from w using the probability integral transform to simplify the fit.

Results

arXiv:2412:14260

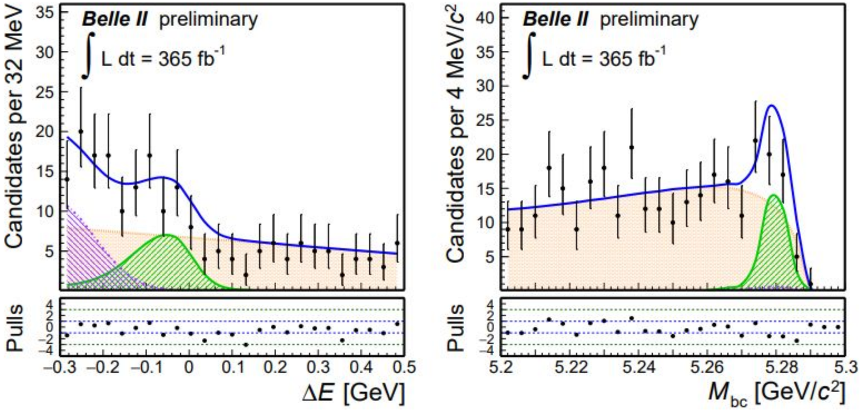
- The branching fraction was extracted:

$$\mathcal{B}(B^0 \to \pi^0\pi^0) = \frac{N f_s (1 + f^{+-}/f^{00})}{2\,\varepsilon\, N_{B\overline{B}}\, \mathcal{B}(\pi^0 \to \gamma\gamma)^2}$$

$$\mathcal{B}(B^0 \to \pi^0\pi^0) = (1.25 \pm 0.20 \pm 0.11) \times 10^{-6}$$

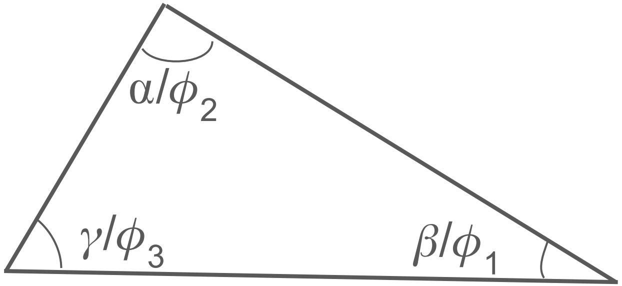
- CP-violating asymmetry was extracted:

$$\mathcal{A}_{CP}(B^0 \to \pi^0\pi^0) = 0.03 \pm 0.30 \pm 0.04$$

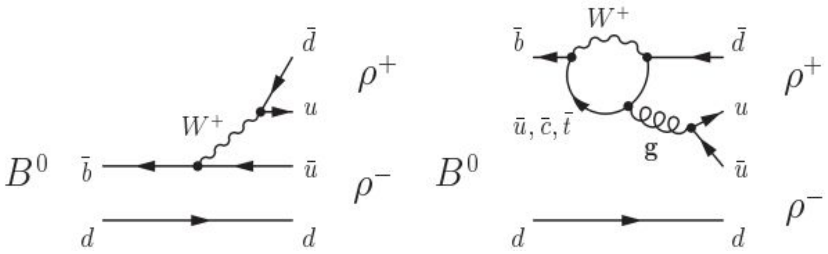


Measuring ϕ_2 from $B^0 \rightarrow \varrho^+\varrho^-$ Decays

PRD 111, 092001 (2025)



Unitary Triangle



Tree level and penguin transitions

- **Goal**: Constrain CKM angle ϕ_2 via $B^0 \rightarrow \varrho^+\varrho^-$
- **CP-violation** from direct CP violation (C) and mixing induced CP violation (S).
- Decay to proceed through **tree level** $b \rightarrow u\bar{u}d$ transition and $b \rightarrow d$ **penguin transitions.**
- There are 3 helicity states of the ϱ meson pair in this decay:
  - 1 Longitudinal polarization (LP)
  - 2 Transverse polarization (TP)
- **Measurements made:** branching fraction of decay, fraction of LP, and CP-violation parameters (C, S).

Results

PRD 111, 092001 (2025)

- The branching fraction of the decay:

$$\mathcal{B}(B^0 \to \rho^+\rho^-) = (2.89^{+0.23}_{-0.22}{}^{+0.29}_{-0.27}) \times 10^{-5}$$

- The fraction of longitudinal polarization:

$$f_L = 0.921^{+0.024}_{-0.025}{}^{+0.017}_{-0.015}$$

- CP-violation parameters:

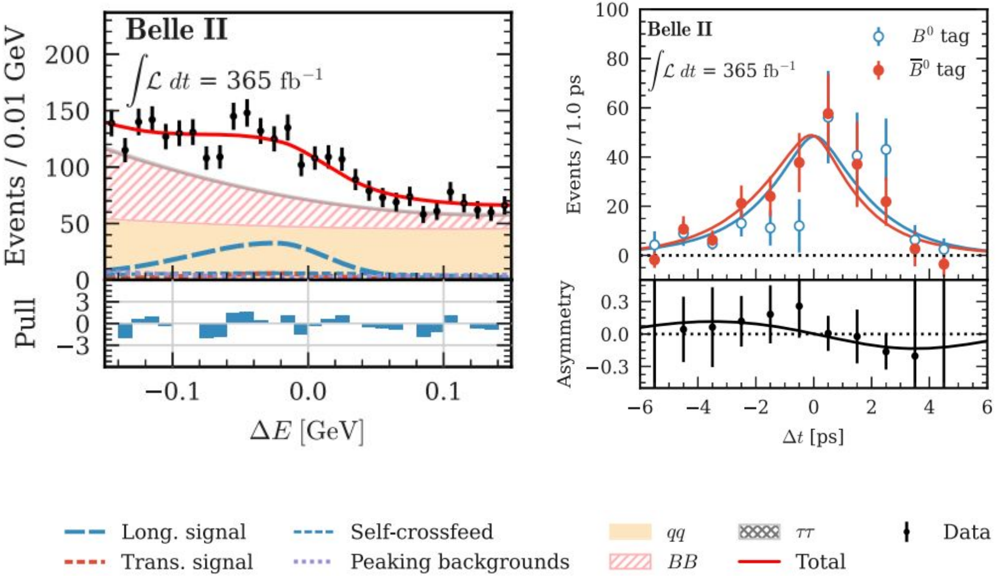
$$C = -0.02 \pm 0.12^{+0.06}_{-0.05}$$

$$S = -0.26 \pm 0.19 \pm 0.08$$

- Measurements of $\Delta\phi_2$ and ϕ_2:

$$\phi_2 = (92.6^{+4.5}_{-4.7})^\circ$$

$$\Delta\phi_2 = (2.4^{+3.8}_{-3.7})^\circ$$



Summary of Today's Results

Belle II

$|V_{cb}|$ from $B \to D\ell\nu_\ell$

$|V_{cb}|_{CLN}$ = (38.5 ± 1.3) x 10^{-3}

$|V_{cb}|_{BCL}$ = (39.2 ± 0.88) x 10^{-3}

Belle II

$|V_{ub}|$ using $B^+ \to \tau^+\nu$

$$|V_{ub}|_{B^+\to\tau^+\nu_\tau} = [4.41^{+0.74}_{-0.89}] \times 10^{-3}$$

arXiv:2502.04885

Summary of Today's Results

$B^0 \to \pi^0\pi^0$ results

Belle II

$$\mathcal{B}(B^0 \to \pi^0\pi^0) = (1.25 \pm 0.20 \pm 0.11) \times 10^{-6}$$

$$\mathcal{A}_{CP}(B^0 \to \pi^0\pi^0) = 0.03 \pm 0.30 \pm 0.04$$

arXiv:2412:14260

ϕ_2 from $B^0 \to \varrho^+\varrho^-$

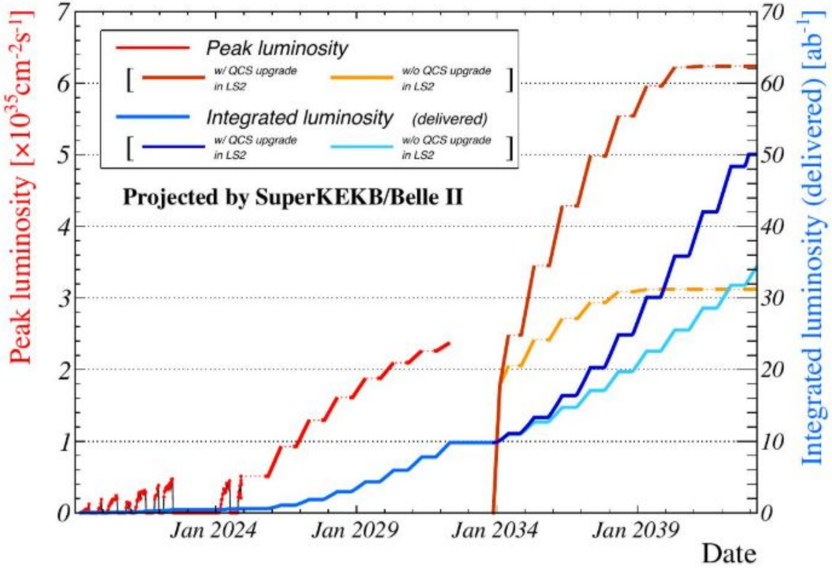
Belle II

$$\phi_2 = (92.6^{+4.5}_{-4.7})^\circ$$

PRD 111, 092001 (2025)

Conclusion and Outlook

- Belle II offers a great environment for precision tests of multiple CKM matrix elements and angles from the unitary triangle.
- Very active field with innovative techniques to measure those quantities:
  - With current collected data set, Belle II provides world-leading results!
  - With increased luminosity in each run period, more interesting results are underway!



Thank you for your attention!

Extra slides

Source of systematic uncertainties $B^+ \to \tau^+ \nu_\tau$

TAB. VI. Summary of systematic uncertainties (syst.) on the fitted branching fraction presented as relative uncertainties. The effect of each source is evaluated in the simultaneous fit of the four signal modes. The last three sources do not affect the signal yields.

Source	Syst.
Simulation statistics	13.3%
Fit variables PDF corrections	5.5%
Decays branching fractions in MC	4.1%
Tag B^- reconstruction efficiency	2.2%
Continuum reweighting	1.9%
π^0 reconstruction efficiency	0.9%
Continuum normalization	0.7%
Particle identification	0.6%
Number of produced $\Upsilon(4S)$	1.5%
Fraction of B^+B^- pairs	2.1%
Tracking efficiency	0.2%
Total	15.5%

Source of systematic uncertainties $B \to D\ell\nu_\ell$

Table V. Fractional contributions to the total uncertainty on the extracted value of $|V_{cb}|$. The sizes of the contributions are given relative to the central value.

Source		Uncertainty [%]
Statistical		0.9
Systematic		1.5
	Simulation sample size	0.5
	N_{bb}	0.5
	f_{00}/f_{+-}	0.1
	$f_{\not{B}}$	0.3
	$\mathcal{B}(D \to K\pi(\pi))$	0.3
	Vertex fit χ^2 correction	0.5
	$\mathcal{B}(B \to X_c\ell\nu_\ell)$	0.3
	Lepton identification	0.2
	Kaon identification	0.5
	Tracking efficiency	0.3
	Signal PDF	0.4
	$B \to D^*\ell\nu_\ell$ form factor	0.1
	Background w modelling	0.5
	$E^*_Y - m_Y$ reweighing	0.3
	$B^{0/-}$ lifetime	0.1
Theoretical		1.3
	Lattice QCD inputs	1.2
	Long-distance QED	0.4
Total		2.1

Source of systematic uncertainties $B^0 \to \pi^0 \pi^0$

TABLE I. Fractional systematic uncertainties on the branching fraction and absolute systematic uncertainties on the CP asymmetry. Total systematic uncertainties, resulting from their sums in quadrature, are also given and compared with statistical uncertainties.

Source	$\mathcal{B}$	$\mathcal{A}_{CP}$
π^0 efficiency	8.1%	n/a
Continuum-suppression efficiency	1.9%	n/a
$B\overline{B}$-background model	1.7%	0.01
Signal model	1.2%	0.02
Continuum-background model	0.9%	0.03
$\Upsilon(4S)$ branching fractions $(1 + f^{+-}/f^{00})$	1.5 %	n/a
Sample size $N_{B\bar{B}}$	1.5%	n/a
$B^0\overline{B}^0$-oscillation probability	n/a	< 0.01
Wrong-tag probability calibration	n/a	0.01
Total systematic uncertainty	8.9%	0.04
Statistical uncertainty	15.9%	0.30

Source of systematic uncertainties $B^0 \to \rho^+\rho^-$

TABLE VI. Systematic uncertainties for $\mathcal{B}$ and f_L. Relative uncertainties are shown for $\mathcal{B}$.

Source	$\mathcal{B}$ [%]	$f_L[10^{-2}]$
Tracking	±0.54	...
π^0 efficiency	±7.67	...
PID	±0.08	...
$\mathcal{T}_C$	±2.87	...
MC sample size	±0.24	±0.2
Single candidate selection	±0.55	±0.3
SCF ratio	$^{+2.97}_{-2.45}$	$^{+0.2}_{-0.3}$
$\mathcal{B}$'s of peaking backgrounds	$^{+0.94}_{-0.98}$	±0.1
$\tau^+\tau^-$ background yield	$^{+0.65}_{-0.69}$	±0.0
Signal model	$^{+1.14}_{-2.02}$	±0.2
$q\bar{q}$ model	$^{+0.49}_{-0.51}$	$^{+0.1}_{-0.2}$
$B\bar{B}$ model	$^{+1.00}_{-0.40}$	$^{+0.3}_{-0.1}$
$\tau^+\tau^-$ model	$^{+0.17}_{-0.26}$	$^{+0.0}_{-0.1}$
Peaking model	$^{+1.37}_{-1.01}$	$^{+0.3}_{-0.5}$
Interference	±1.20	±0.5
Data-MC mismodeling	$^{+3.51}_{-1.70}$	$^{+0.8}_{-0.3}$
Fit bias	±1.03	±1.2
f_{00}	$^{+1.67}_{-1.50}$	...
$N_{\Upsilon(4S)}$	±1.45	...
Total systematic uncertainty	$^{+10.10}_{-9.51}$	$^{+1.7}_{-1.5}$
Statistical uncertainty	$^{+7.95}_{-7.61}$	$^{+2.4}_{-2.5}$

TABLE VII. Systematic uncertainties for S and C.

Source	$S[10^{-2}]$	$C[10^{-2}]$
$\mathcal{B}$'s of peaking backgrounds	$^{+0.6}_{-0.5}$	±0.1
$\tau\tau$ background yield	±0.9	$^{+0.0}_{-0.1}$
Data-MC mismodeling	$^{+0.6}_{-1.1}$	$^{+1.5}_{-0.6}$
Single candidate selection	±1.3	±1.9
SCF ratio	$^{+0.5}_{-0.4}$	$^{+0.7}_{-0.0}$
Signal model	$^{+1.1}_{-1.4}$	$^{+0.3}_{-0.4}$
$q\bar{q}$ model	$^{+2.2}_{-1.0}$	±0.2
$B\bar{B}$ model	±0.9	$^{+0.7}_{-0.5}$
$\tau^+\tau^-$ model	±0.1	±0.0
Peaking model	$^{+0.8}_{-0.4}$	$^{+0.2}_{-0.4}$
Fit bias	±2.0	±0.6
Interference	±2.8	±1.7
Resolution	$^{+3.4}_{-4.4}$	$^{+1.9}_{-1.4}$
Δt PDF for $q\bar{q}$ and $B\bar{B}$	$^{+3.8}_{-1.8}$	$^{+0.7}_{-0.1}$
Tag side interference	±0.5	±2.1
Wrong tag fraction	$^{+0.2}_{-0.3}$	±0.5
Background CP violation	$^{+3.8}_{-3.6}$	$^{+4.2}_{-3.7}$
CP violation in TP signal	$^{+0.8}_{-0.2}$	$^{+0.2}_{-0.4}$
Tracking detector misalignment	±1.4	±0.5
τ_{B^0} and Δm_d	$^{+1.4}_{-1.6}$	±0.3
Total systematic uncertainty	$^{+8.2}_{-7.8}$	$^{+6.1}_{-5.3}$
Statistical uncertainty	±18.8	±12.1

CLN form factor fit: heavy quark symmetry, fewer parameters → may underestimate uncertainties.

BCL form factor fit: a more model independent parameterization + fewer theoretical assumptions → better suited to high precision data.