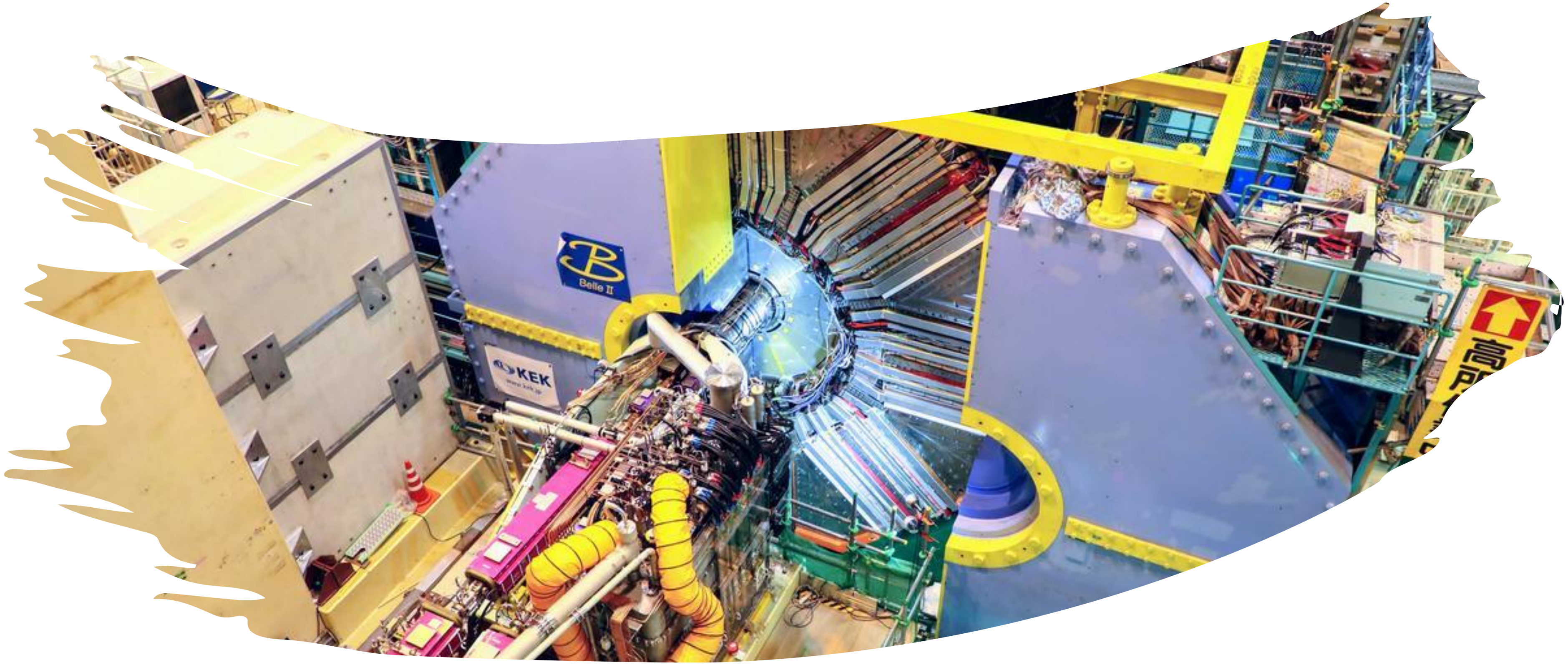


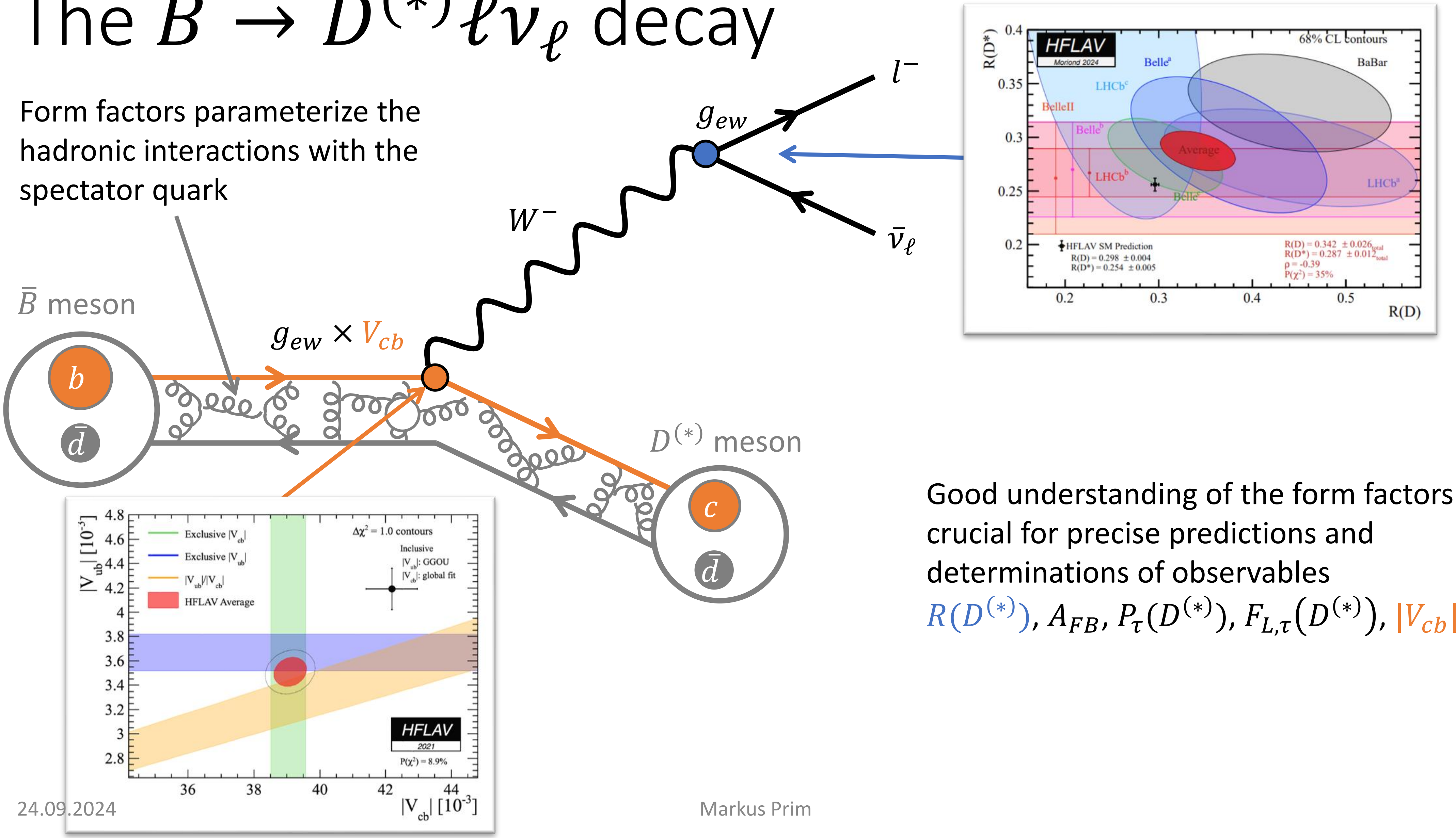
Measurement of the $B \to D^{*}\ell\nu_{\ell}$ Form Factors at Belle

Markus Prim on behalf of the Belle II Collaboration
markus.prim@uni-bonn.de

The $\bar{B} \to D^{(*)} \ell \bar{\nu}_\ell$ decay

$$R(D^{(*)}) = \frac{\mathcal{B}(\bar{B} \to D^{(*)} \tau \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^{(*)} \ell \bar{\nu}_\ell)}$$

Form factors parameterize the hadronic interactions with the spectator quark

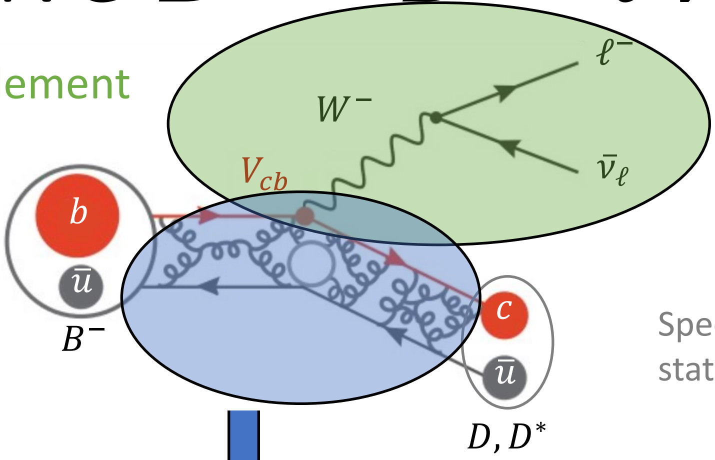


Good understanding of the form factors is crucial for precise predictions and determinations of observables

$R(D^{(*)})$, A_{FB}, $P_\tau(D^{(*)})$, $F_{L,\tau}(D^{(*)})$, $|V_{cb}|$

Exclusive $\bar{B} \to D^{(*)}\ell\bar{\nu}_\ell$

Leptonic Matrix Element



Specific final state meson

$$\Gamma(\bar{B} \to D\ell\bar{\nu}_\ell) \propto |V_{cb}|^2\mathcal{G}(1) \qquad \mathcal{G}(1) = h_+(1)$$

$$\Gamma(\bar{B} \to D^*\ell\bar{\nu}_\ell) \propto |V_{cb}|^2\mathcal{F}(1) \qquad \mathcal{F}(1) = h_{A_1}(1)$$

Hadronic Matrix Elements can not be calculated from first principles

→ Can be parameterized with form factors $h_X = h_X(w)$ and **extracted from data**

→ Theory must provide (at least) inputs on their normalization

$$\frac{\langle D(p')|\bar{c}\gamma^\mu b|B(p)\rangle}{\sqrt{m_B m_D}} = h_+(v+v')^\mu + h_-(v-v')^\mu$$

$$\frac{\langle D^*(p')|\bar{c}\gamma^\mu b|B(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_V \epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\frac{\langle D^*(p')|\bar{c}\gamma^\mu\gamma^5 b|B(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_{A_1}(w+1)\epsilon^{*\mu} - h_{A_2}(\epsilon^*\cdot v)v^\mu - h_{A_3}(\epsilon^*\cdot v)v'^\mu$$

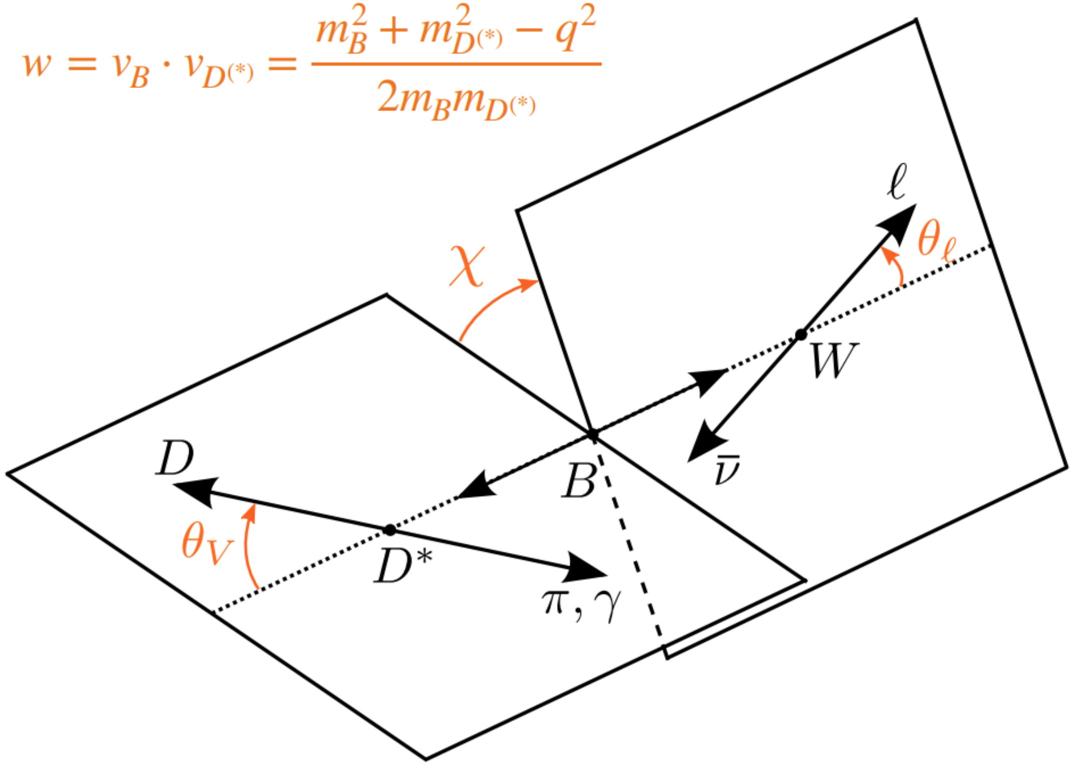
Heavy Quark Symmetry Basis

BELLE

Differential distributions
arXiv:2301.07529 (Published in PRD)
Angular coefficients
arXiv:2310.20286 (Accepted by PRL)

Exclusive $\bar{B} \to D^* \ell \bar{\nu}_\ell$

$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$



- Form factors are a function of w only
- Angles provide information on, e.g.
  - Forward-backward asymmetry
  - Longitudinal polarization fraction
  - “S” observables sensitive to new physics

$$\frac{\mathrm{d}\Gamma(B \to D^* \ell \nu_\ell)}{\mathrm{d}w \,\mathrm{d}\cos\theta_\ell \,\mathrm{d}\cos\theta_\mathrm{V} \,\mathrm{d}\chi} = \frac{6 m_\mathrm{B} m_{\mathrm{D}^*}^2}{8(4\pi)^4} \sqrt{w^2 - 1}(1 - 2wr + r^2) G_\mathrm{F}^2 \eta_\mathrm{EW}^2 |V_\mathrm{cb}|^2$$
$$\times \Big((1 - \cos\theta_\ell)^2 \sin^2\theta_\mathrm{V} H_+^2 + (1 + \cos\theta_\ell)^2 \sin^2\theta_\mathrm{V} H_-^2$$
$$+ 4 \sin^2\theta_\ell \cos^2\theta_\mathrm{V} H_0^2 - 2 \sin^2\theta_\ell \sin^2\theta_\mathrm{V} \cos 2\chi H_+ H_-$$
$$- 4 \sin\theta_\ell (1 - \cos\theta_\ell) \sin\theta_\mathrm{V} \cos\theta_\mathrm{V} \cos\chi H_+ H_0$$
$$+ 4 \sin\theta_\ell (1 + \cos\theta_\ell) \sin\theta_\mathrm{V} \cos\theta_\mathrm{V} \cos\chi H_- H_0 \Big) ,$$

- Measuring the 4D rate is not feasible
- **So, what do we do?**

Measurement Strategy

- Measure the marginal distributions of the 4D differential decay rate
- Measure the angular coefficients $J(w)$ in bins of w

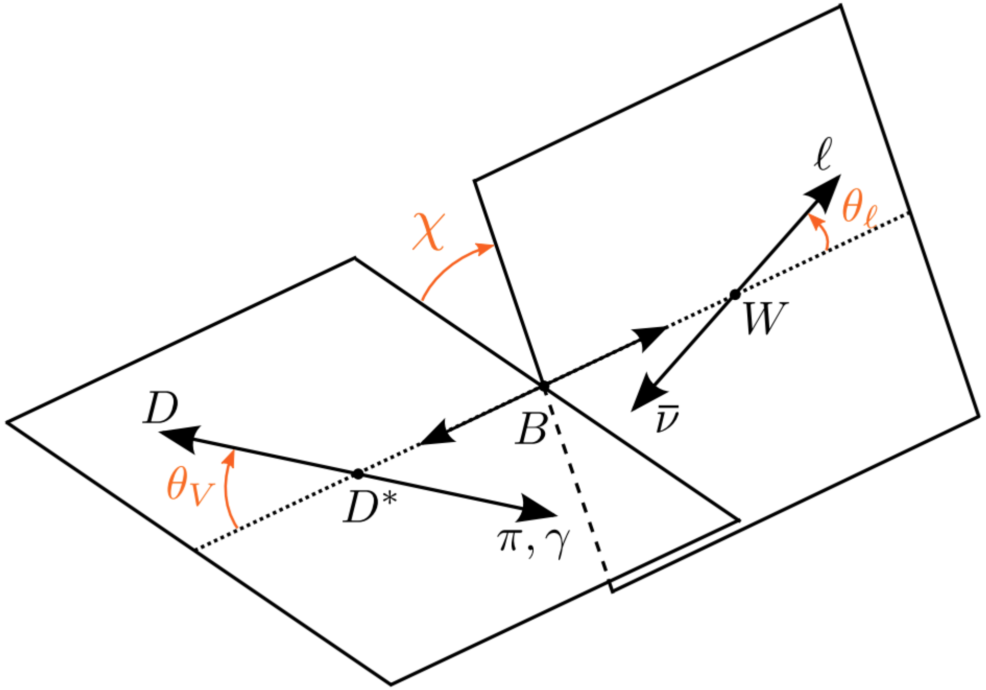
Conceptually both analyses are very similar:

- Signal extraction via a model independent variable M_{miss}^2
- Correction for migration and acceptance

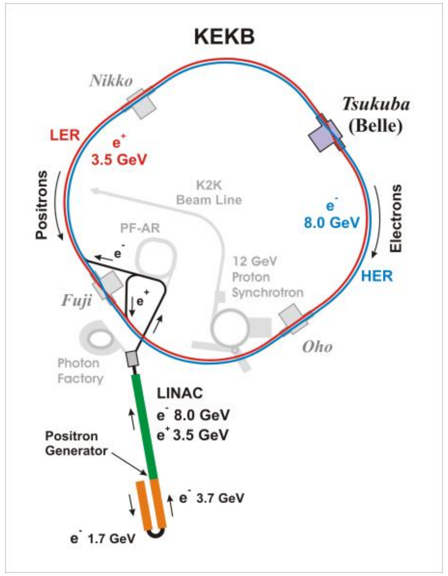
Belle, Prim, et al
arXiv:2301.07529
(Published in PRD)

Belle, Prim, et al
arXiv:2310.20286
(Accepted by PRL)

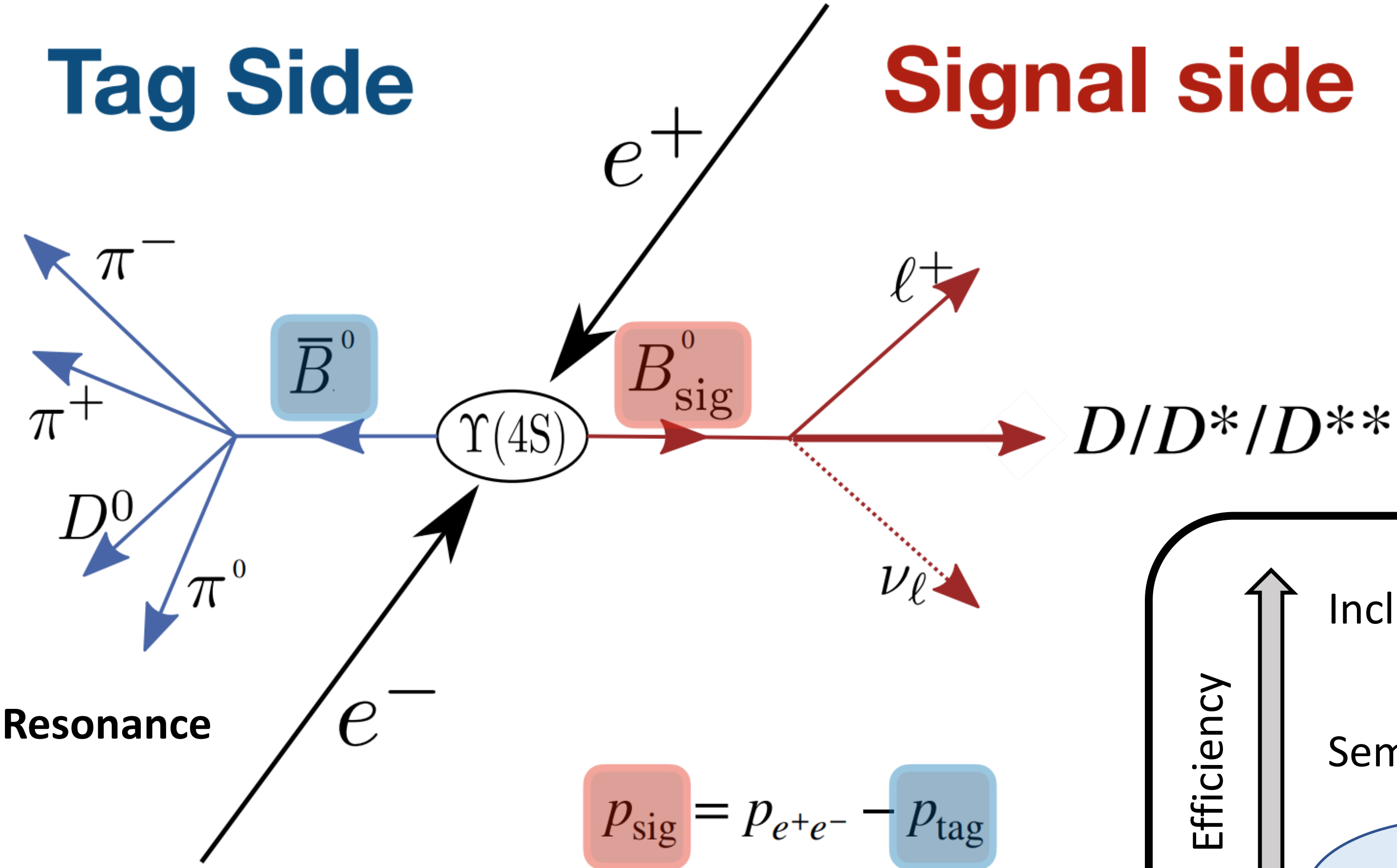
$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$



Measurement Strategy at Belle



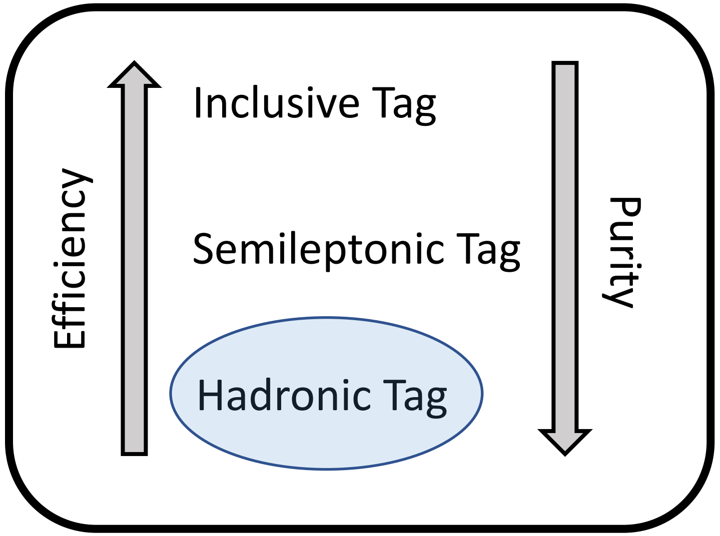
$\mathcal{L} = 711\ \mathrm{fb}^{-1}$ **@** $\Upsilon(4S)$ **Resonance**



Tag Side

Signal side

$$p_{\mathrm{sig}} = p_{e^+e^-} - p_{\mathrm{tag}}$$



Inclusive Tag

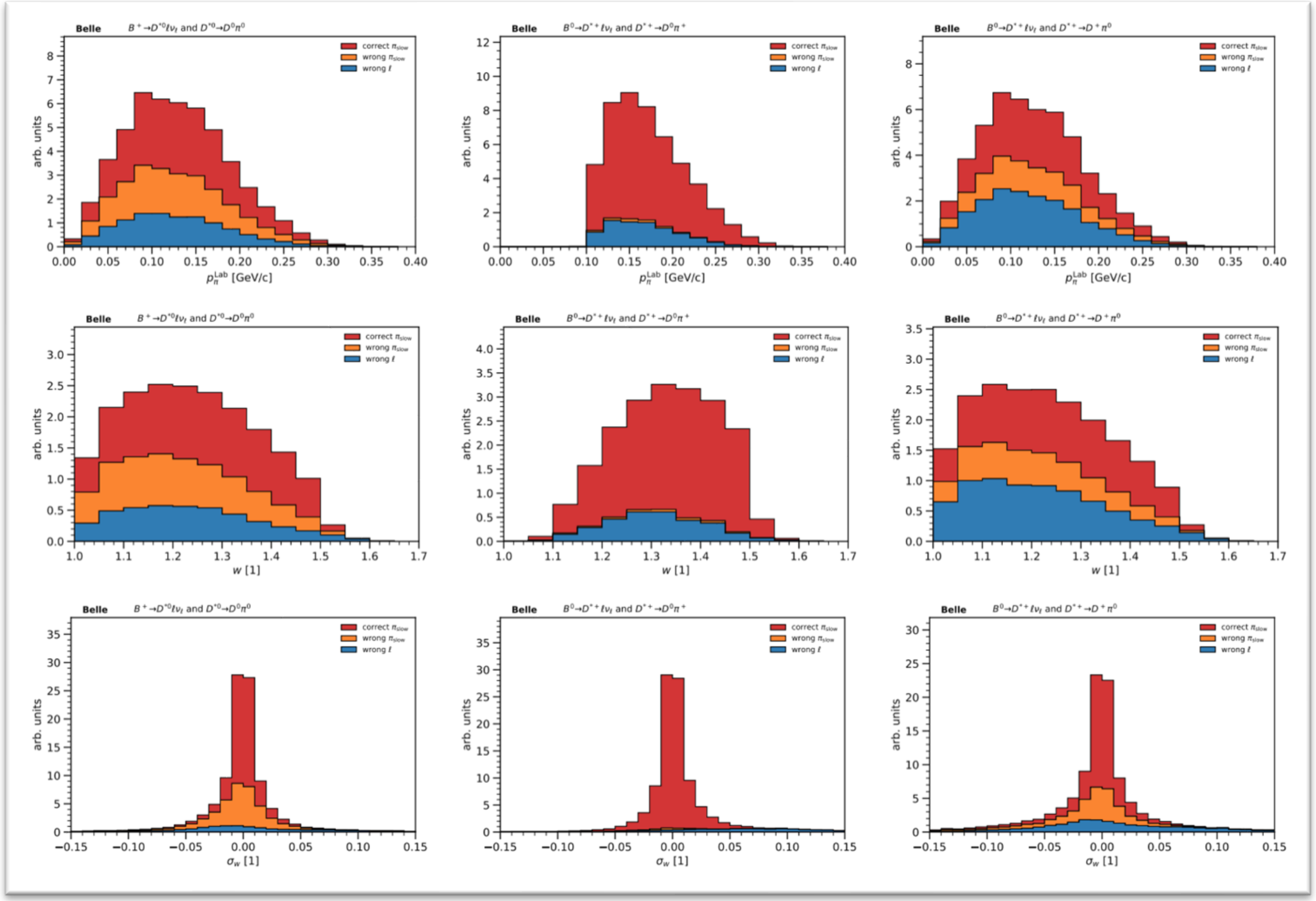
Semileptonic Tag

Hadronic Tag

$\bar{B} \to D^* \ell \bar{\nu}_\ell$ Channels

$$\bar{B}^0 \to D^{*+}(\to D^0 \pi_s^+, D^+ \pi_s^0)\ell\bar{\nu}_\ell$$

$$B^- \to D^{*0}(\to D^0 \pi_s^0)\ell\bar{\nu}_\ell$$

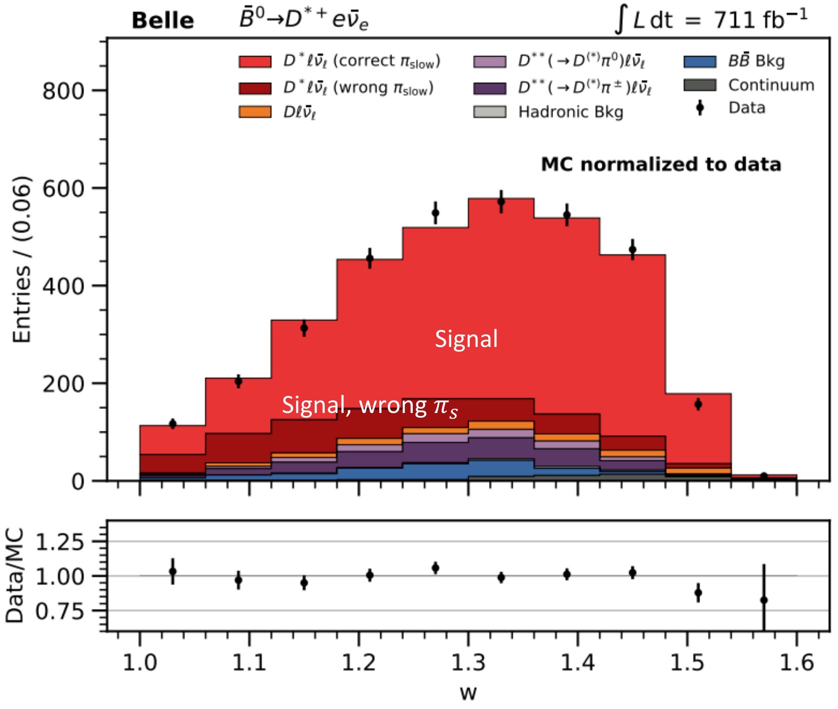


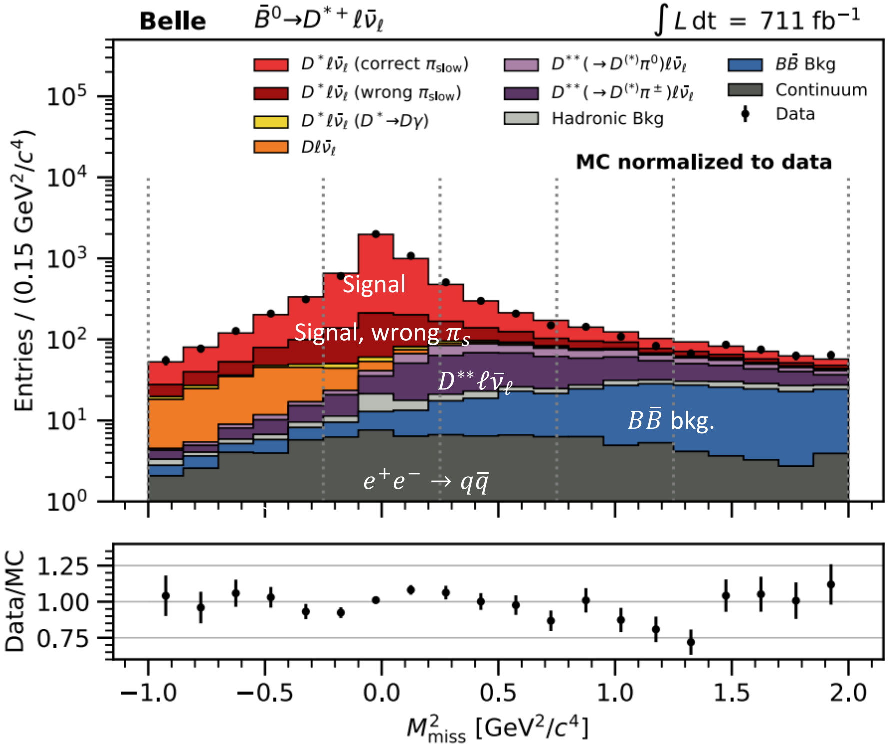
First time we consider neutral slow pions

→ larger kinematic coverage

→ but more mis-identified pions and worse resolution

Background Subtraction $\bar{B} \to D^* \ell \bar{\nu}_\ell$



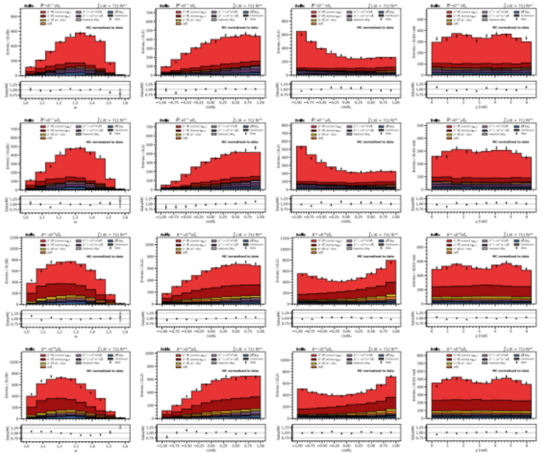


Background subtraction in independent variable to reduce model dependency.

Extraction Method: Missing Mass Squared

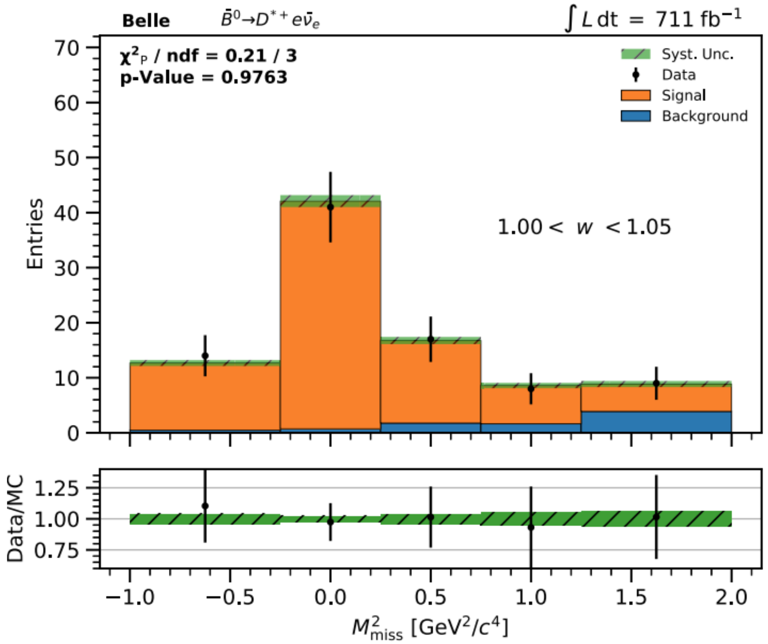
$$0 = m_\nu^2 = \mathrm{M}_{\mathrm{miss}}^2 = (p_{e^+e^-} - \mathrm{p_B} - \mathrm{p_{D^*}} - \mathrm{p}_\ell)^2$$

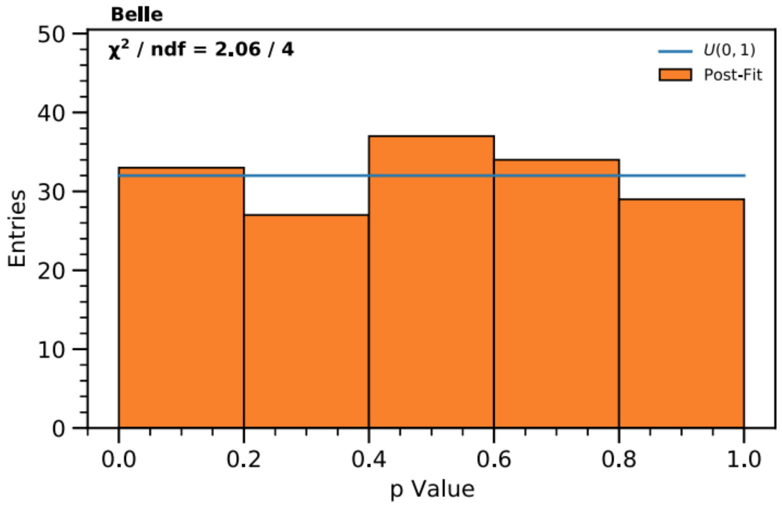
Background Subtraction $\bar{B} \rightarrow D^{*}\ell\bar{\nu}_{\ell}$



Repeat in 4 channels, 4 variables, 10 bins each

→ 160 fits $\mathrm{M}^2_{\mathrm{miss}}$



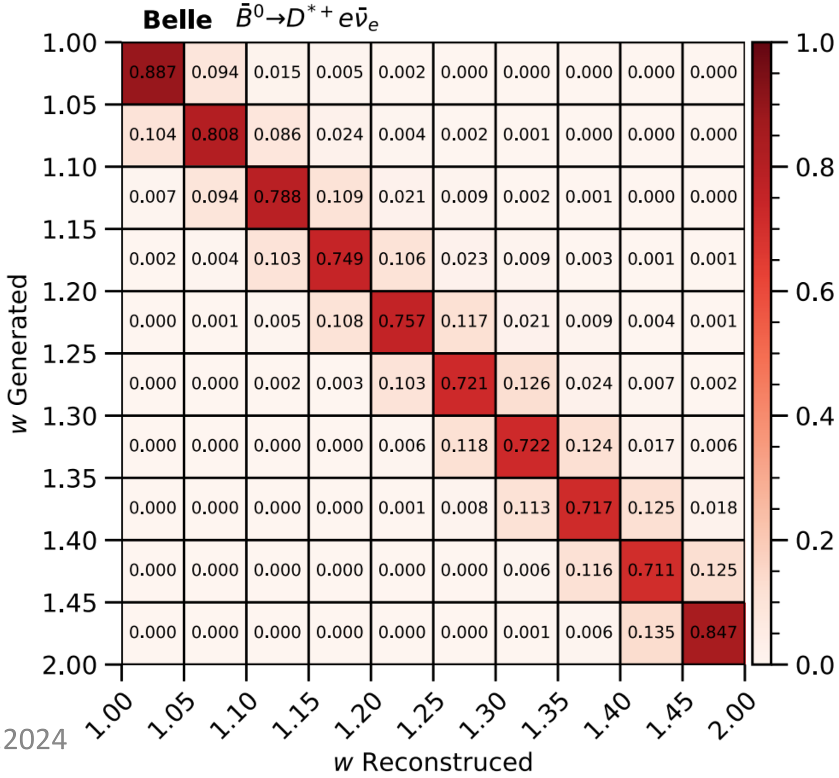


The p-value distribution for the 160 fits

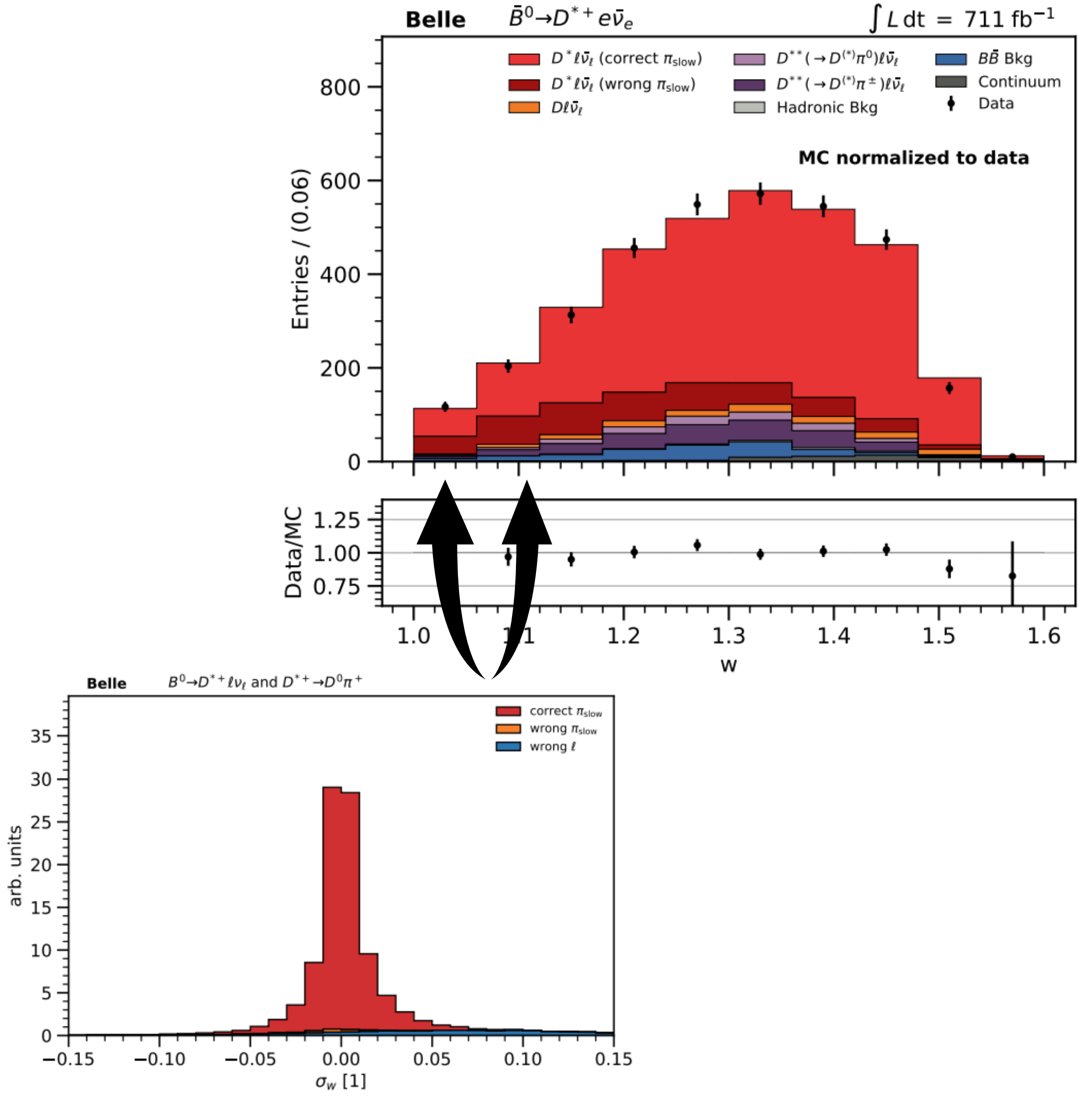
Unfolding & Acceptance

- We measure the e.g., *w* distribution smeared by the detector resolution, and impacted by acceptance effects
- We are interested in the true underlying distribution

→ Correct for migration effects and efficiencies



w Generated \ w Reconstructed	1.00–1.05	1.05–1.10	1.10–1.15	1.15–1.20	1.20–1.25	1.25–1.30	1.30–1.35	1.35–1.40	1.40–1.45	1.45–2.00
1.00–1.05	0.887	0.094	0.015	0.005	0.002	0.000	0.000	0.000	0.000	0.000
1.05–1.10	0.104	0.808	0.086	0.024	0.004	0.002	0.001	0.000	0.000	0.000
1.10–1.15	0.007	0.094	0.788	0.109	0.021	0.009	0.002	0.001	0.000	0.000
1.15–1.20	0.002	0.004	0.103	0.749	0.106	0.023	0.009	0.003	0.001	0.001
1.20–1.25	0.000	0.001	0.005	0.108	0.757	0.117	0.021	0.009	0.004	0.001
1.25–1.30	0.000	0.000	0.002	0.003	0.103	0.721	0.126	0.024	0.007	0.002
1.30–1.35	0.000	0.000	0.000	0.000	0.006	0.118	0.722	0.124	0.017	0.006
1.35–1.40	0.000	0.000	0.000	0.000	0.001	0.008	0.113	0.717	0.125	0.018
1.40–1.45	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.116	0.711	0.125
1.45–2.00	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.135	0.847

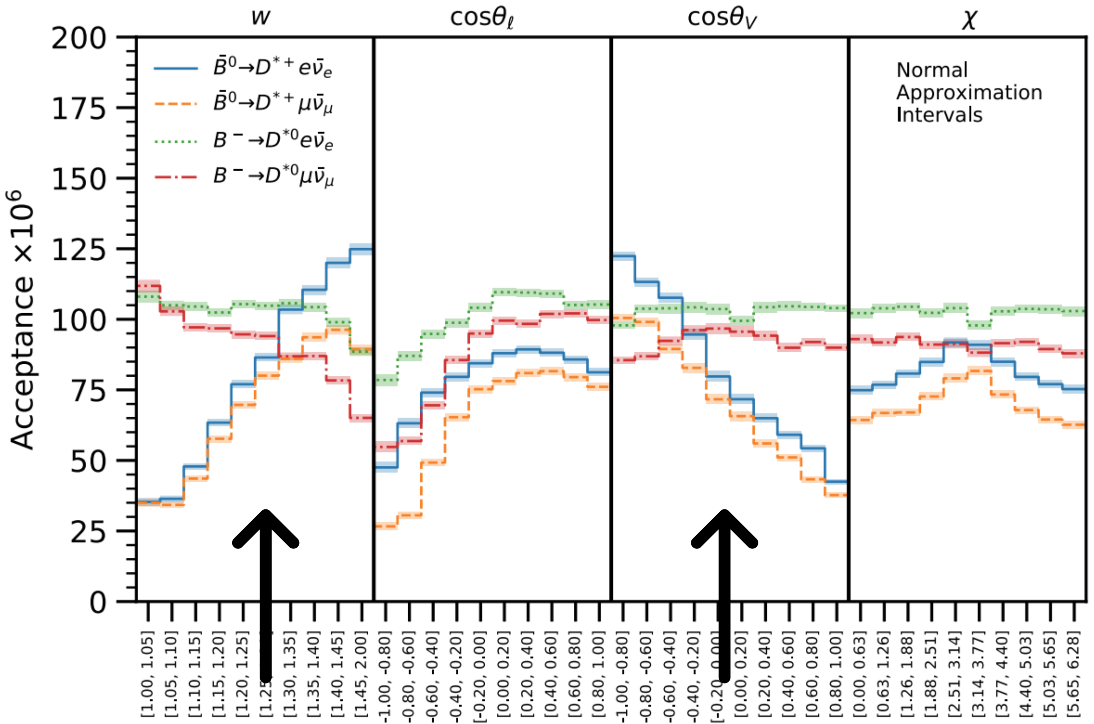


Resolution effect encoded in the migration matrix, extracted from simulation. Simulation assumptions are accounted for in the systematics budget.

Unfolding & Acceptance

- We measure the e.g., *w* distribution smeared by the detector resolution, and impacted by acceptance effect
- We are interested in the true underlying distribution

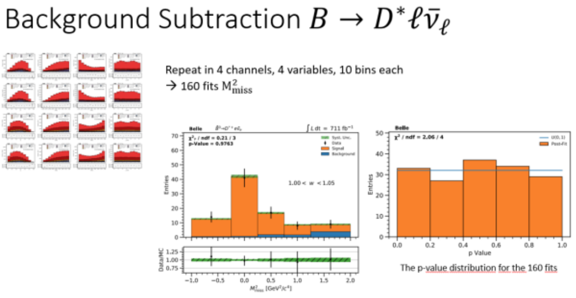
→ Correct for migration effects and efficiencies



Acceptance extracted from simulation. Simulation assumptions are accounted for in the systematics budget

Difference in the differential efficiency is caused by the slow pion efficiency: charged vs neutral

Systematics

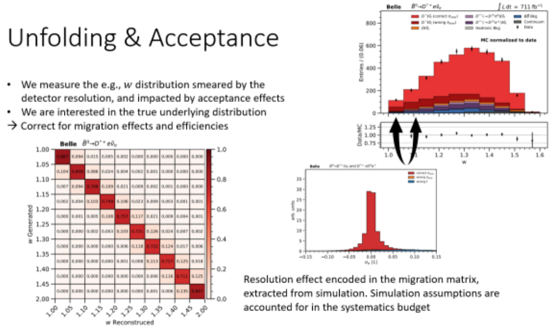

Background Subtraction $B \to D^*\ell\bar{\nu}_\ell$

Repeat in 4 channels, 4 variables, 10 bins each
→ 160 fits M^2_{miss}

The p-value distribution for the 160 fits

M^2_{miss} almost model-independent
→ No significant systematic effects here

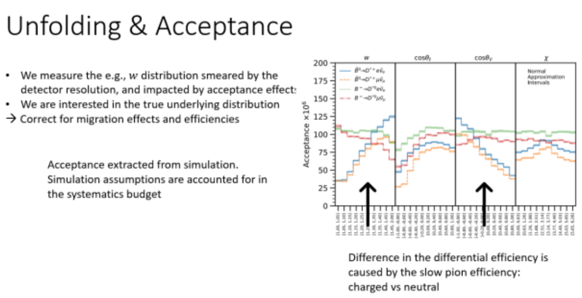
Systematic effects enter in the unfolding procedure:
Vary the MC simulation according to the size of the systematic effects, and repeat unfolding and acceptance correction (simultaneously)


Unfolding & Acceptance

- We measure the e.g., w distribution smeared by the detector resolution, and impacted by acceptance effects
- We are interested in the true underlying distribution

→ Correct for migration effects and efficiencies

Resolution effect encoded in the migration matrix, extracted from simulation. Simulation assumptions are accounted for in the systematics budget


Unfolding & Acceptance

- We measure the e.g., w distribution smeared by the detector resolution, and impacted by acceptance effect:
- We are interested in the true underlying distribution

→ Correct for migration effects and efficiencies

Acceptance extracted from simulation. Simulation assumptions are accounted for in the systematics budget

Difference in the differential efficiency is caused by the slow pion efficiency: charged vs neutral

We can check the slow pion & lepton identification efficiency by testing the compatibility of different decay modes

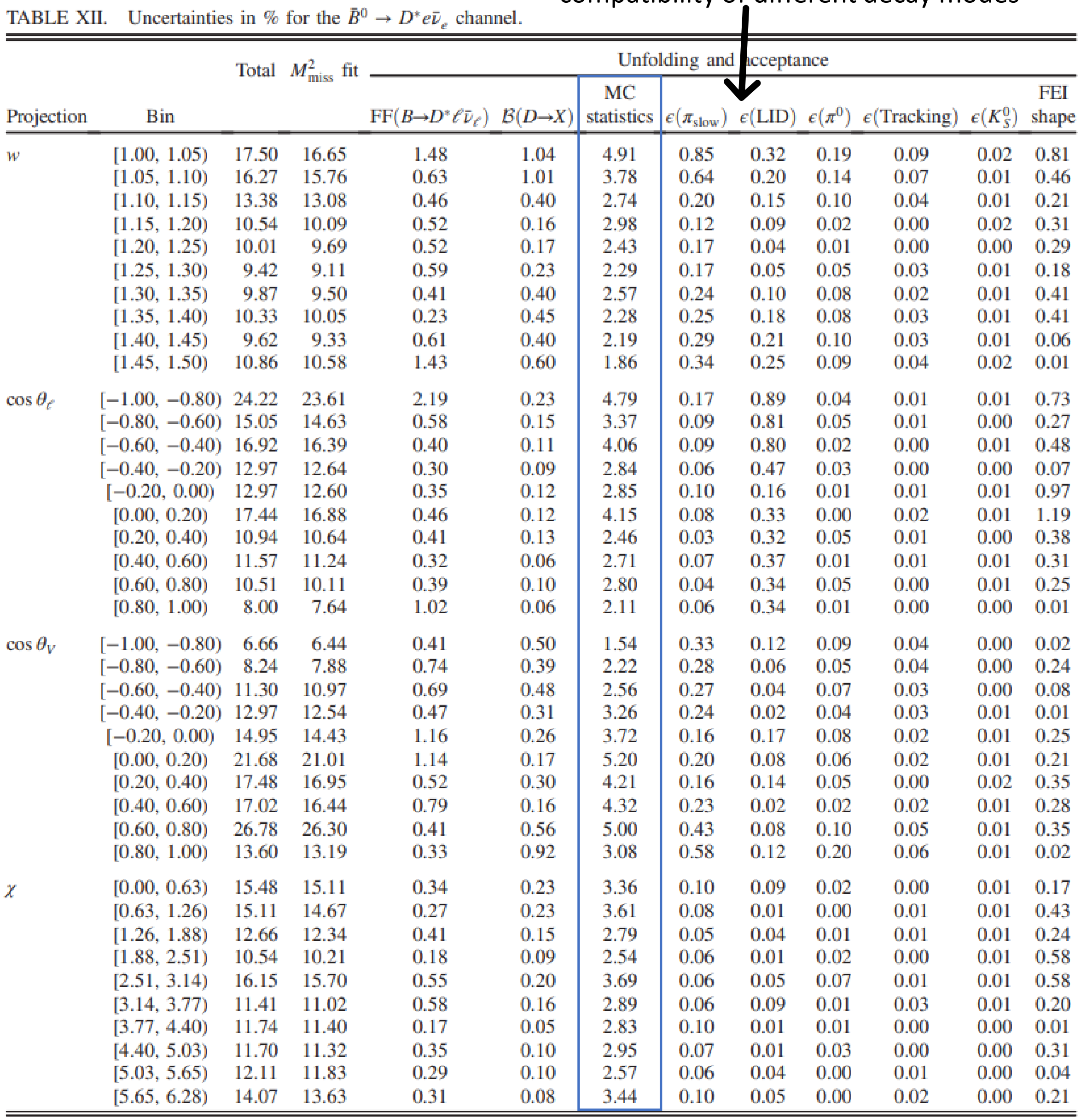
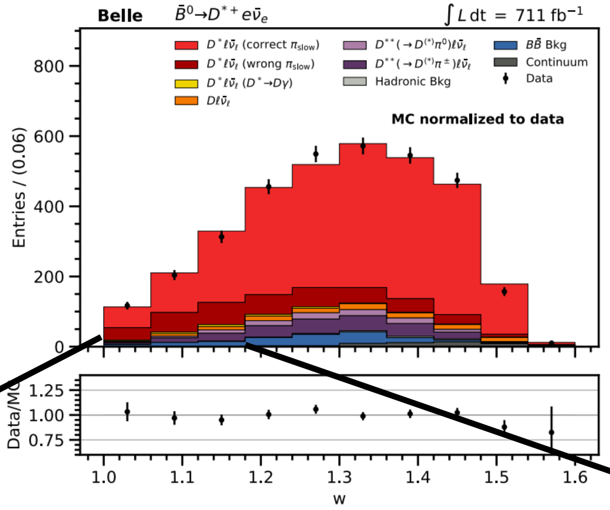


TABLE XII. Uncertainties in % for the $\bar{B}^0 \to D^* e \bar{\nu}_e$ channel.

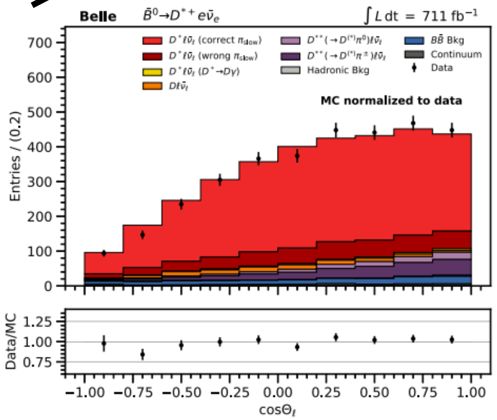
		Total	M^2_{miss} fit	Unfolding and acceptance								
Projection	Bin			$\mathrm{FF}(B \to D^*\ell\bar{\nu}_\ell)$	$\mathcal{B}(D \to X)$	MC statistics	$\epsilon(\pi_{\mathrm{slow}})$	$\epsilon(\mathrm{LID})$	$\epsilon(\pi^0)$	$\epsilon(\mathrm{Tracking})$	$\epsilon(K^0_S)$	FEI shape
w	[1.00, 1.05)	17.50	16.65	1.48	1.04	4.91	0.85	0.32	0.19	0.09	0.02	0.81
	[1.05, 1.10)	16.27	15.76	0.63	1.01	3.78	0.64	0.20	0.14	0.07	0.01	0.46
	[1.10, 1.15)	13.38	13.08	0.46	0.40	2.74	0.20	0.15	0.10	0.04	0.01	0.21
	[1.15, 1.20)	10.54	10.09	0.52	0.16	2.98	0.12	0.09	0.02	0.00	0.02	0.31
	[1.20, 1.25)	10.01	9.69	0.52	0.17	2.43	0.17	0.04	0.01	0.00	0.00	0.29
	[1.25, 1.30)	9.42	9.11	0.59	0.23	2.29	0.17	0.05	0.05	0.03	0.01	0.18
	[1.30, 1.35)	9.87	9.50	0.41	0.40	2.57	0.24	0.10	0.08	0.02	0.01	0.41
	[1.35, 1.40)	10.33	10.05	0.23	0.45	2.28	0.25	0.18	0.08	0.03	0.01	0.41
	[1.40, 1.45)	9.62	9.33	0.61	0.40	2.19	0.29	0.21	0.10	0.03	0.01	0.06
	[1.45, 1.50)	10.86	10.58	1.43	0.60	1.86	0.34	0.25	0.09	0.04	0.02	0.01
$\cos\theta_\ell$	[−1.00, −0.80)	24.22	23.61	2.19	0.23	4.79	0.17	0.89	0.04	0.01	0.01	0.73
	[−0.80, −0.60)	15.05	14.63	0.58	0.15	3.37	0.09	0.81	0.05	0.01	0.00	0.27
	[−0.60, −0.40)	16.92	16.39	0.40	0.11	4.06	0.09	0.80	0.02	0.00	0.01	0.48
	[−0.40, −0.20)	12.97	12.64	0.30	0.09	2.84	0.06	0.47	0.03	0.00	0.00	0.07
	[−0.20, 0.00)	12.97	12.60	0.35	0.12	2.85	0.10	0.16	0.01	0.01	0.01	0.97
	[0.00, 0.20)	17.44	16.88	0.46	0.12	4.15	0.08	0.33	0.00	0.02	0.01	1.19
	[0.20, 0.40)	10.94	10.64	0.41	0.13	2.46	0.03	0.32	0.05	0.01	0.00	0.38
	[0.40, 0.60)	11.57	11.24	0.32	0.06	2.71	0.07	0.37	0.01	0.01	0.01	0.31
	[0.60, 0.80)	10.51	10.11	0.39	0.10	2.80	0.04	0.34	0.05	0.00	0.01	0.25
	[0.80, 1.00)	8.00	7.64	1.02	0.06	2.11	0.06	0.34	0.01	0.00	0.00	0.01
$\cos\theta_V$	[−1.00, −0.80)	6.66	6.44	0.41	0.50	1.54	0.33	0.12	0.09	0.04	0.00	0.02
	[−0.80, −0.60)	8.24	7.88	0.74	0.39	2.22	0.28	0.06	0.05	0.04	0.00	0.24
	[−0.60, −0.40)	11.30	10.97	0.69	0.48	2.56	0.27	0.04	0.07	0.03	0.00	0.08
	[−0.40, −0.20)	12.97	12.54	0.47	0.31	3.26	0.24	0.02	0.04	0.03	0.01	0.01
	[−0.20, 0.00)	14.95	14.43	1.16	0.26	3.72	0.16	0.17	0.08	0.02	0.01	0.25
	[0.00, 0.20)	21.68	21.01	1.14	0.17	5.20	0.20	0.08	0.06	0.02	0.01	0.21
	[0.20, 0.40)	17.48	16.95	0.52	0.30	4.21	0.16	0.14	0.05	0.00	0.02	0.35
	[0.40, 0.60)	17.02	16.44	0.79	0.16	4.32	0.23	0.02	0.02	0.02	0.01	0.28
	[0.60, 0.80)	26.78	26.30	0.41	0.56	5.00	0.43	0.08	0.10	0.05	0.01	0.35
	[0.80, 1.00)	13.60	13.19	0.33	0.92	3.08	0.58	0.12	0.20	0.06	0.01	0.02
χ	[0.00, 0.63)	15.48	15.11	0.34	0.23	3.36	0.10	0.09	0.02	0.00	0.01	0.17
	[0.63, 1.26)	15.11	14.67	0.27	0.23	3.61	0.08	0.01	0.00	0.01	0.01	0.43
	[1.26, 1.88)	12.66	12.34	0.41	0.15	2.79	0.05	0.04	0.01	0.01	0.01	0.24
	[1.88, 2.51)	10.54	10.21	0.18	0.09	2.54	0.06	0.01	0.02	0.00	0.01	0.58
	[2.51, 3.14)	16.15	15.70	0.55	0.20	3.69	0.06	0.05	0.07	0.01	0.01	0.58
	[3.14, 3.77)	11.41	11.02	0.58	0.16	2.89	0.06	0.09	0.01	0.03	0.01	0.20
	[3.77, 4.40)	11.74	11.40	0.17	0.05	2.83	0.10	0.01	0.01	0.00	0.00	0.01
	[4.40, 5.03)	11.70	11.32	0.35	0.10	2.95	0.07	0.01	0.03	0.00	0.00	0.31
	[5.03, 5.65)	12.11	11.83	0.29	0.10	2.57	0.06	0.04	0.00	0.01	0.00	0.04
	[5.65, 6.28)	14.07	13.63	0.31	0.08	3.44	0.10	0.05	0.00	0.02	0.00	0.21

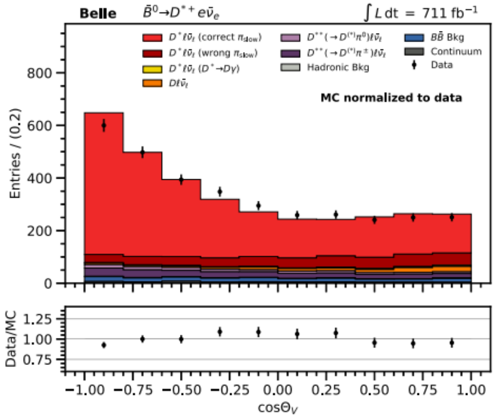
MC statistics dominant systematic effect

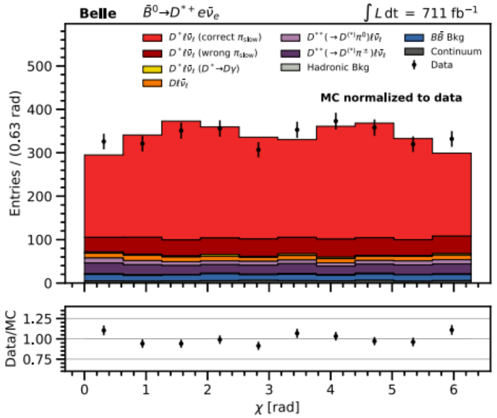
Angular Coefficients of $\bar{B} \to D^* \ell \bar{\nu}_\ell$



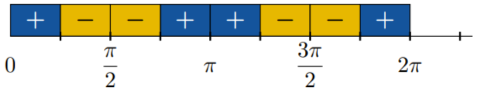
Measure angular information in bins of w instead of “full” marginal distributions



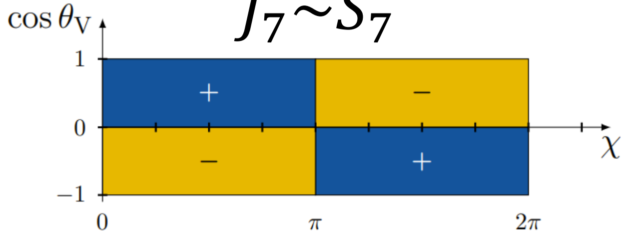




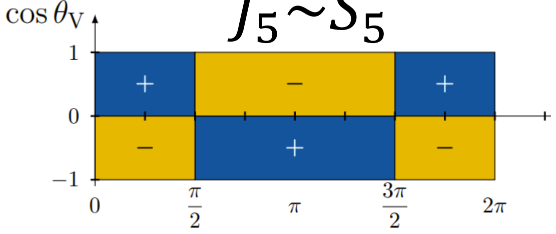
$J_3 \sim S_3$



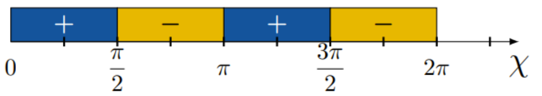
$J_7 \sim S_7$



$J_5 \sim S_5$

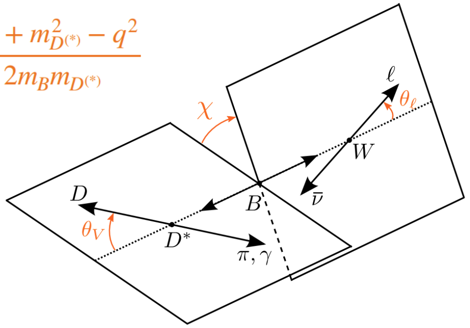


$J_9 \sim S_9$



$$\mathcal{A}_x(w) = \frac{N_x^+(w) - N_x^-(w)}{N_x^+(w) + N_x^-(w)}.$$

$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$



Angular Coefficients of $\bar{B} \to D^* \ell \bar{\nu}_\ell$

Instead of binning in $w, \cos\theta_\ell, \cos\theta_V, \chi$, we now bin the data to determine the angular coefficients in bins of w and:

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$$\bar{J}_i = \frac{1}{N_i} \sum_{j=1}^{8} \sum_{k,l=1}^{4} \eta_{i,j}^{\chi} \eta_{i,k}^{\theta_\ell} \eta_{i,l}^{\theta_V} \left[\chi^{(j)} \otimes \chi^{(k)} \otimes \chi^{(l)} \right]$$

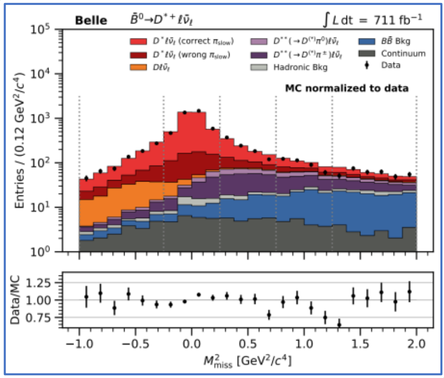
$J_i = J_i(w)$

Normalization

Weights

Unfolded Yields

J_i	η_i^χ	$\eta_i^{\theta_\ell}$	$\eta_i^{\theta_V}$	normalization N_i
J_{1s}	$\{+\}$	$\{+,a,a,+\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{1c}	$\{+\}$	$\{+,a,a,+\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_{2s}	$\{+\}$	$\{-,b,b,-\}$	$\{-,c,c,-\}$	$2\pi(-2/3)2$
J_{2c}	$\{+\}$	$\{-,b,b,-\}$	$\{+,d,d,+\}$	$2\pi(-2/3)(2/5)$
J_3	$\{+,-,-,+,+,-,-,+\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$
J_4	$\{+,+,-,-,-,-,+,+\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_5	$\{+,+,-,-,-,-,+,+\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_{6s}	$\{+\}$	$\{+,+,-,-\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{6c}	$\{+\}$	$\{+,+,-,-\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_7	$\{+,+,+,+,-,-,-,-\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_8	$\{+,+,+,+,-,-,-,-\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_9	$\{+,+,-,-,+,+,-,-\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$



Conceptually same signal extraction, unfolding and acceptance correction strategy as before!

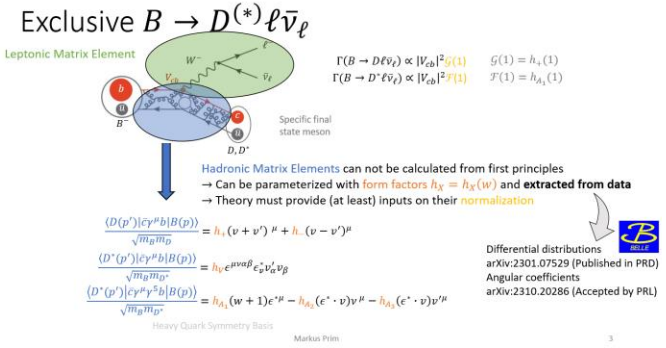
Instead of measuring the signal yield in bins of the marginal distributions:
Measure signal yield in the bins of 36 angles x 4 bins of w x 4 decay modes → 576 fits in $\mathrm{M}_{\mathrm{miss}}^2$

Fitting the data

And a glance at lattice inputs

Lattice Compatibility

As mentioned in the beginning:
We need inputs from LQCD to extract $|V_{cb}|$



Exclusive $B \to D^{(*)}\ell\bar{\nu}_\ell$

$\Gamma(B \to D\ell\bar{\nu}_\ell) \propto |V_{cb}|^2 \mathcal{G}(1)$ $\quad \mathcal{G}(1) = h_+(1)$
$\Gamma(B \to D^*\ell\bar{\nu}_\ell) \propto |V_{cb}|^2 \mathcal{F}(1)$ $\quad \mathcal{F}(1) = h_{A_1}(1)$

Hadronic Matrix Elements can not be calculated from first principles
→ Can be parameterized with form factors $h_X = h_X(w)$ and **extracted from data**
→ Theory must provide (at least) inputs on their normalization

$$\frac{\langle D(p')|\bar{c}\gamma^\mu b|B(p)\rangle}{\sqrt{m_B m_D}} = h_+(v+v')^\mu + h_-(v-v')^\mu$$

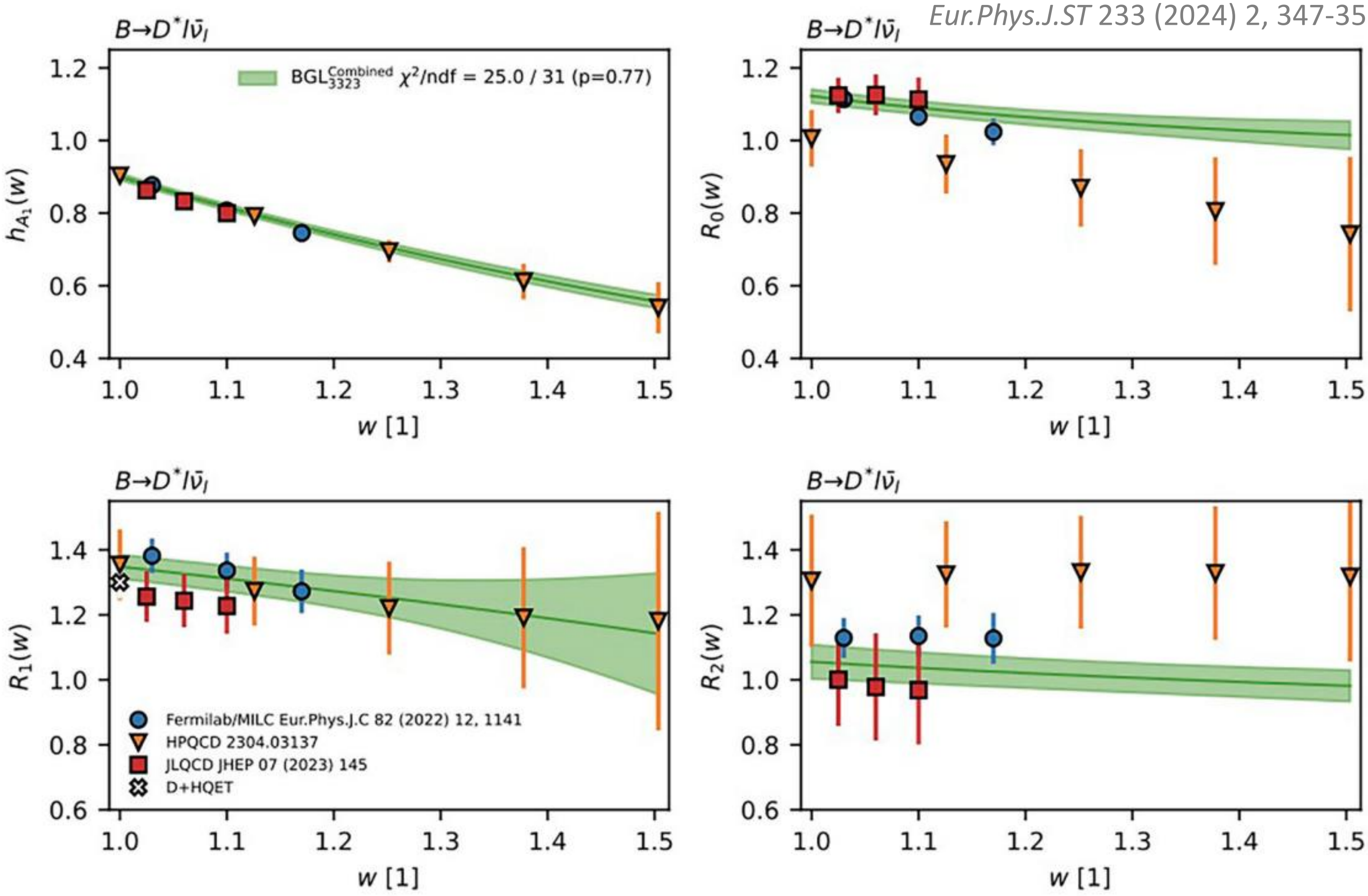
$$\frac{\langle D^*(p')|\bar{c}\gamma^\mu b|B(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_V \epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\frac{\langle D^*(p')|\bar{c}\gamma^\mu\gamma^5 b|B(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_{A_1}(w+1)\epsilon^{*\mu} - h_{A_2}(\epsilon^*\cdot v)v^\mu - h_{A_3}(\epsilon^*\cdot v)v'^\mu$$

Heavy Quark Symmetry Basis

Differential distributions
arXiv:2301.07529 (Published in PRD)
Angular coefficients
arXiv:2310.20286 (Accepted by PRL)

Markus Prim 3



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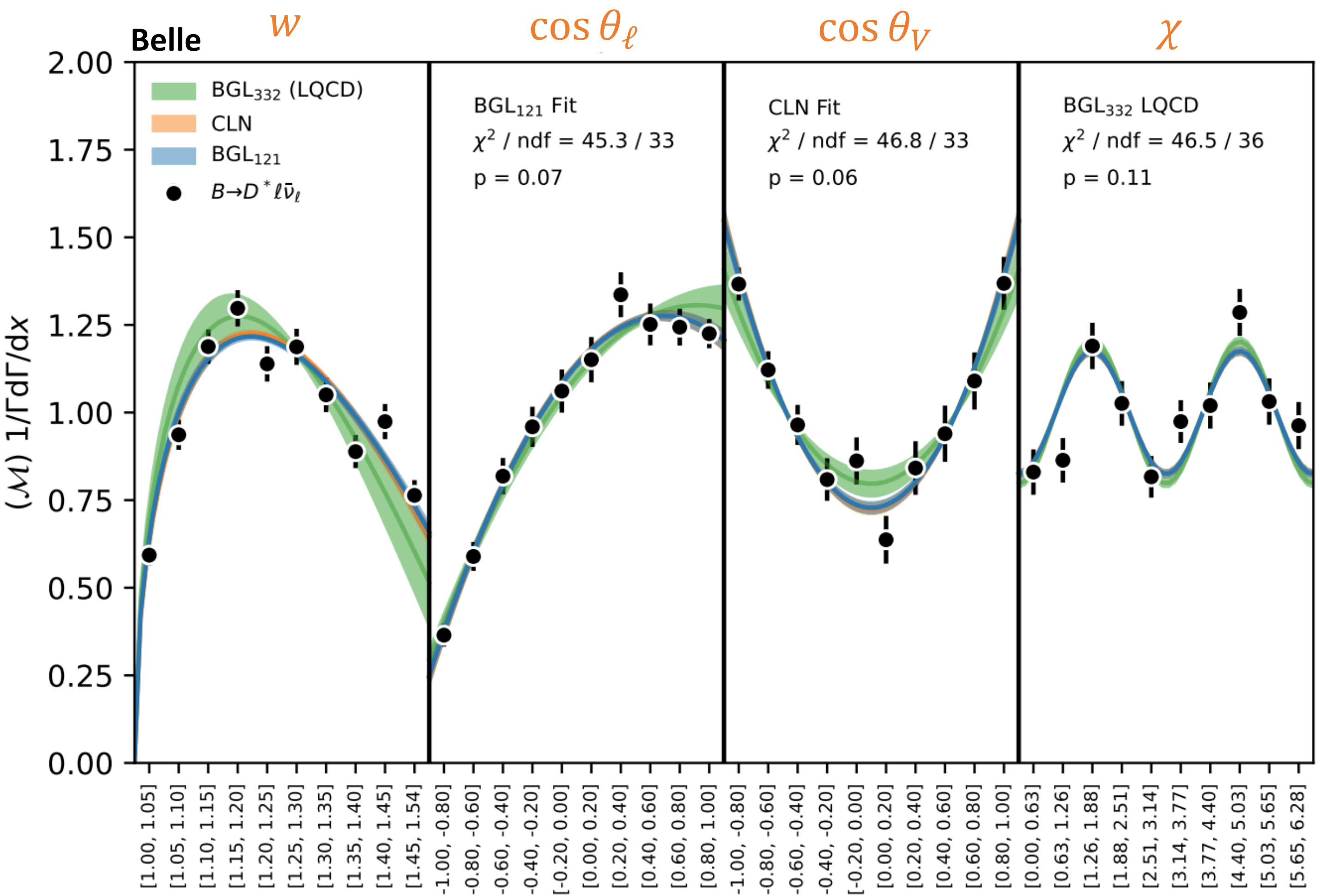
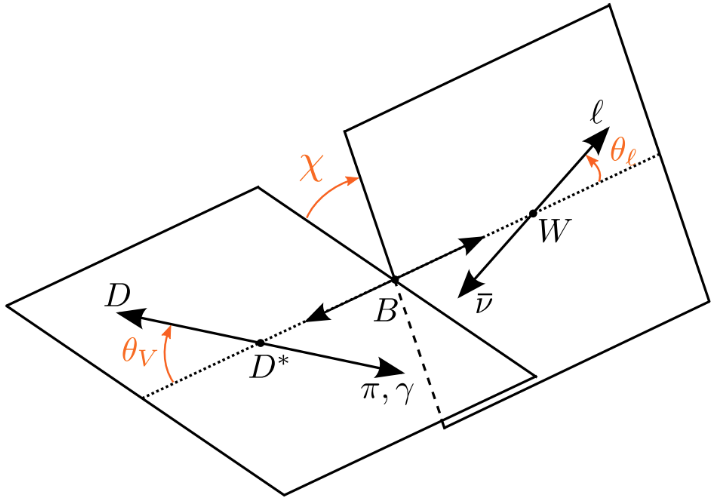
Differential Distributions of $\bar{B} \to D^* \ell \bar{\nu}_\ell$

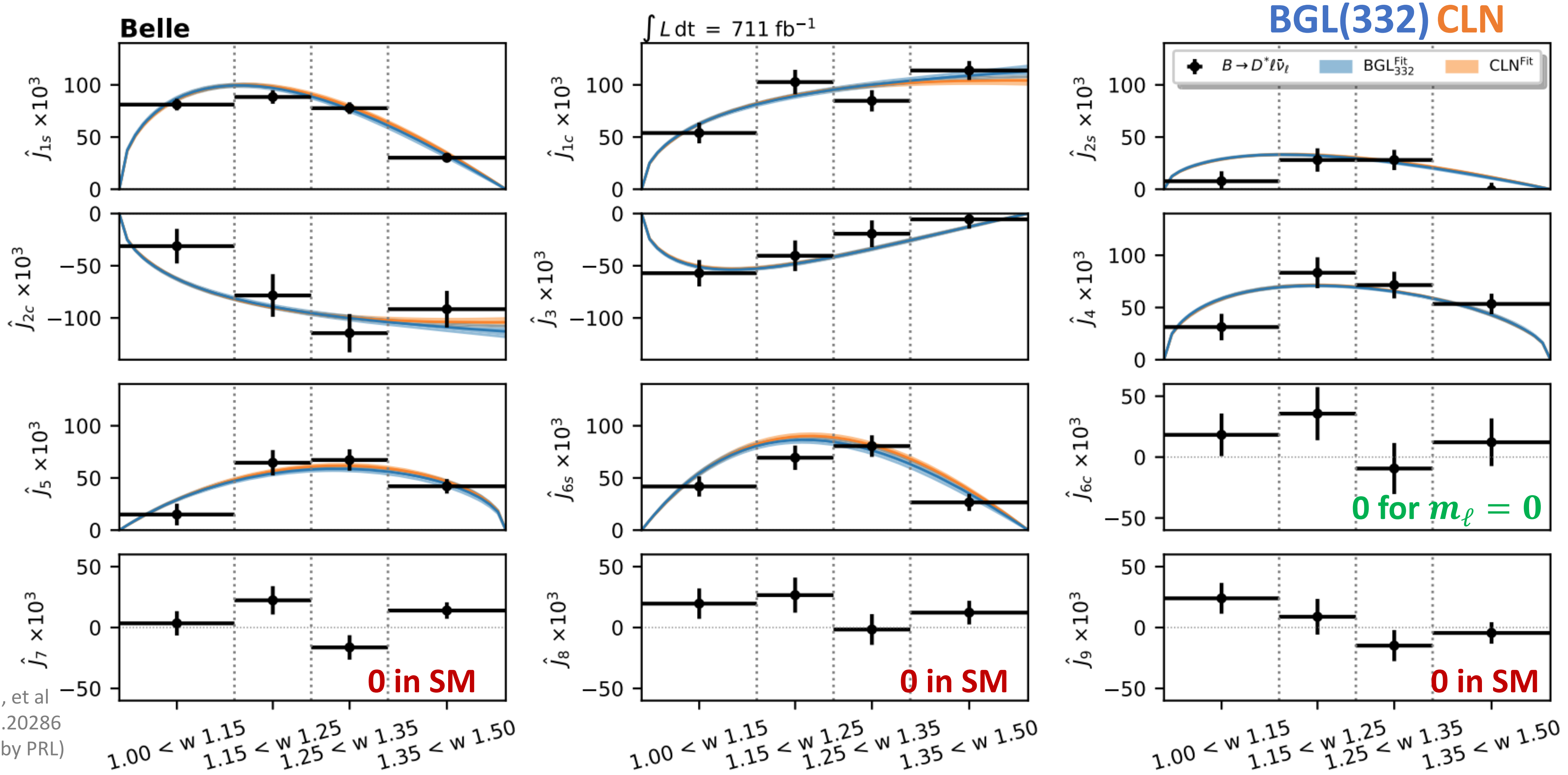


$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$



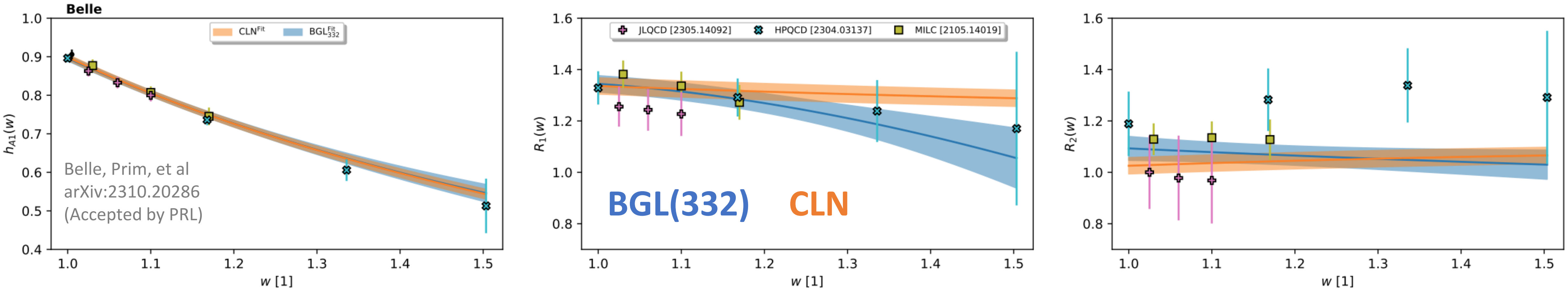
Belle, Prim, et al
arXiv:2301.07529
(Published in PRD)

Angular Coefficients of $\bar{B} \to D^* \ell \bar{\nu}_\ell$



Belle, Prim, et al
arXiv:2310.20286
(Accepted by PRL)

Form Factors of $\bar{B} \rightarrow D^{*} \ell \bar{\nu}_{\ell}$



Belle, Prim, et al
arXiv:2310.20286
(Accepted by PRL)

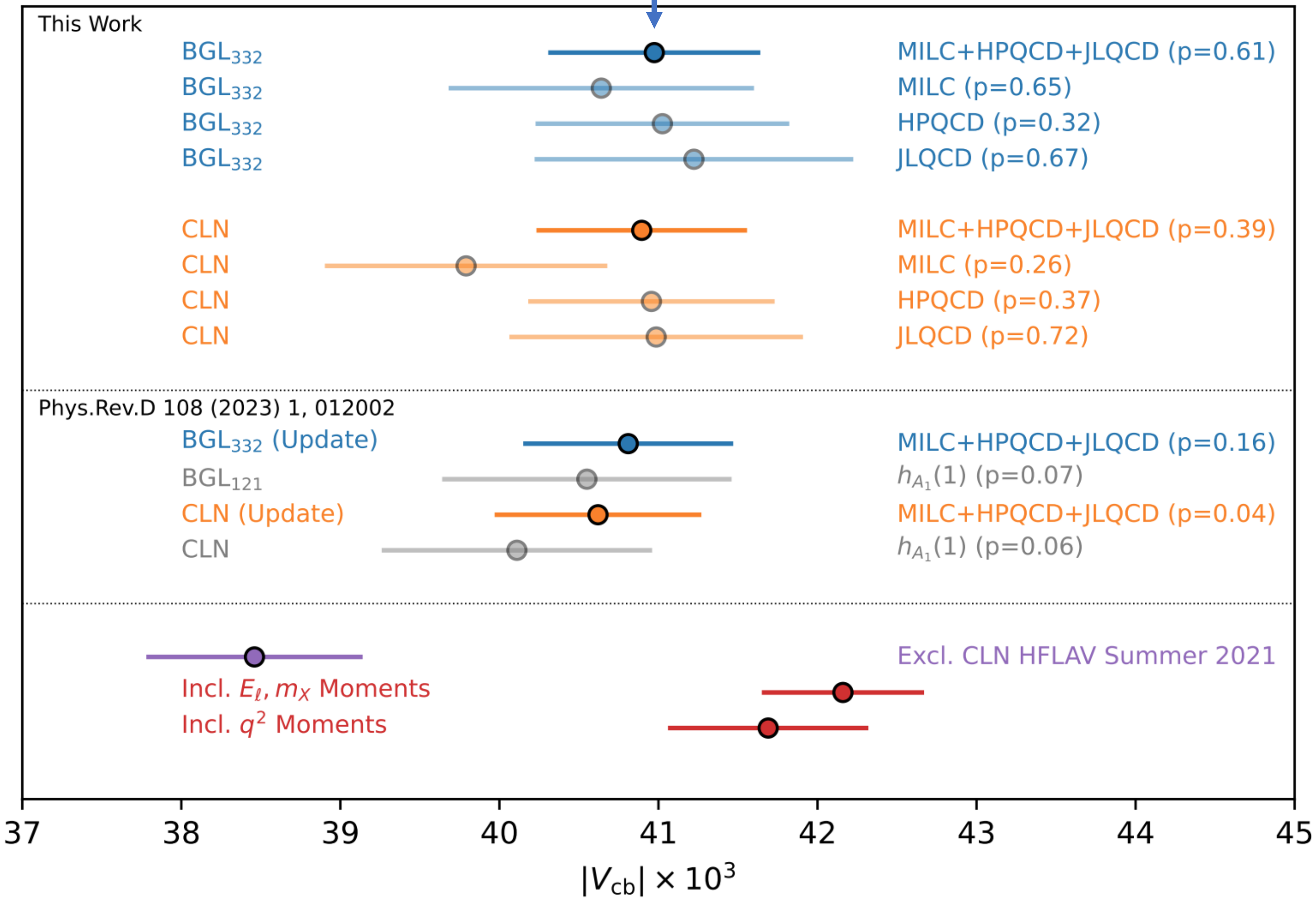
Based on the angular coefficients

Overview on $|V_{cb}|$

$$|V_{\mathrm{cb}}| = (\underset{\text{exp. shape}}{40.7 \pm 0.3} \pm \underset{\mathcal{B}}{0.4} \pm \underset{\text{LQCD}}{0.5}) \times 10^{-3} \qquad (\mathrm{BGL}_{332})$$

Here we use the current world average
$\mathcal{B}(\bar{B} \to D^* \ell \bar{\nu}_\ell)$
$= (4.97 \pm 0.12)\%$

(both measurements only measure shapes!)



LFU Observables of $\bar{B} \to D^* \ell \bar{\nu}_\ell$

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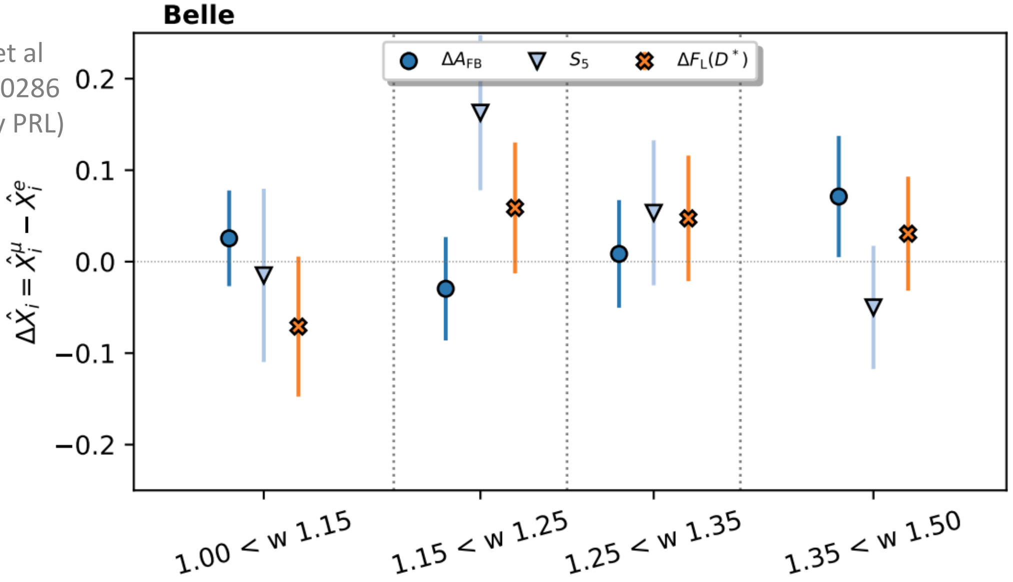
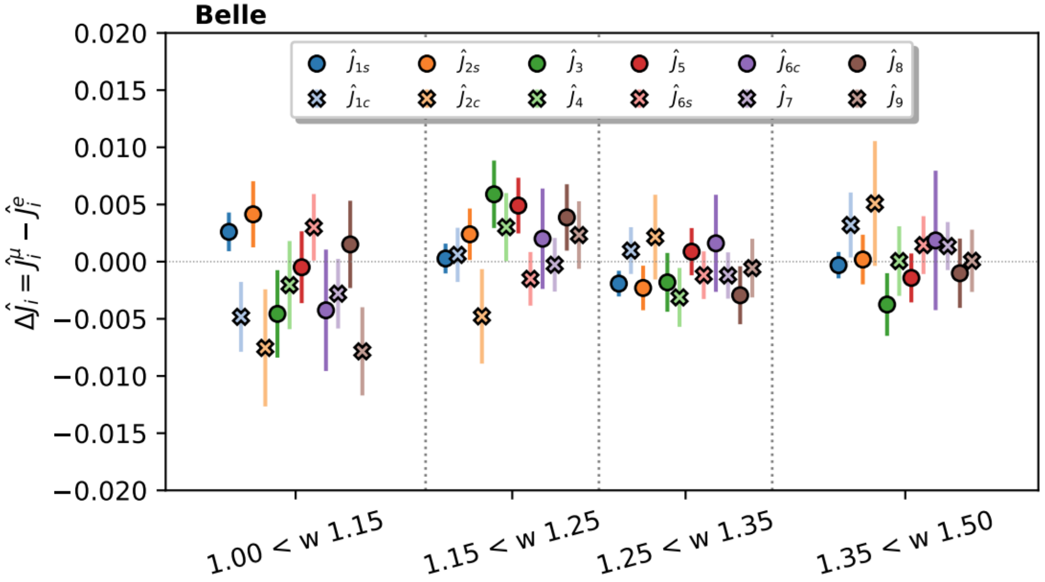
(Accepted by PRL)





Belle, Prim, et al
arXiv:2301.07529
(Published in PRD)

$$\Delta A_{FB} = A_{FB}^{\mu} - A_{FB}^{e} = 0.022 \pm 0.027$$

$$\Delta F_L = F_L^{\mu} - F_L^{e} = 0.034 \pm 0.024$$

Measured over full w range

Summary & Conclusion

- Both measurements rely on the same background subtraction, but extract the angular information in a different way
- Both measurements yield compatible results on V_{cb}, and both show no sign of LFU

- All data is available on HEPData
  - https://www.hepdata.net/record/ins2624324
  - https://www.hepdata.net/record/ins2715684
- Nota bene: The measurements are done on the same collisions data → As of now no correlation between the two measurements have been determined → You cannot use both at the same time!