$|V_{ub}|$ & $|V_{cb}|$ Determinations

— Review of 2022 and what lies beyond —

23.05.2022

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Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

$$
\begin{array}{c|ccc} & d & s & b \\ u & V_{ud} & V_{us} & V_{ub} \\ c & V_{cd} & V_{cs} & V_{cb} \\ t & V_{td} & V_{ts} & V_{tb} \end{array}
$$

CKM Matrix

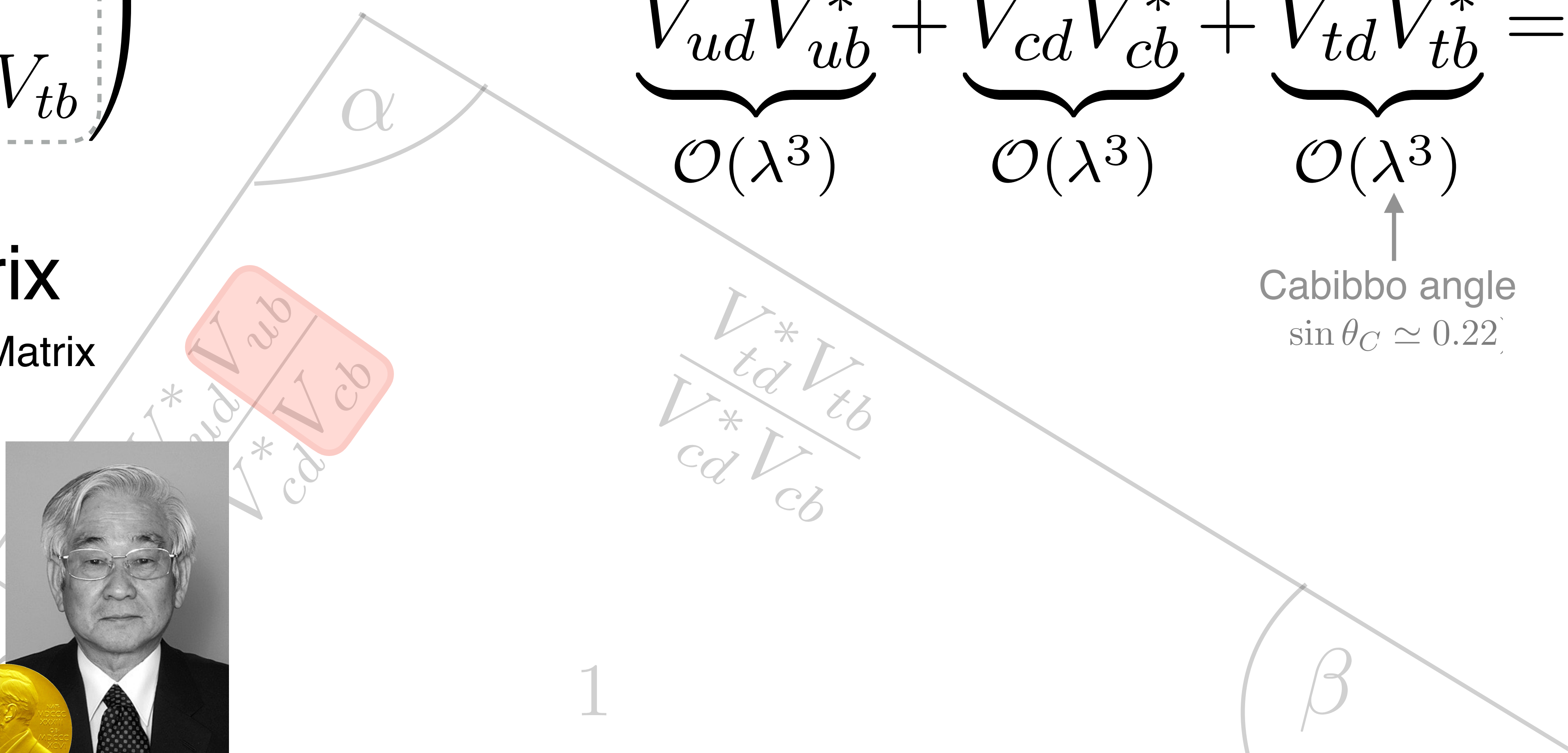
SM: Unitary 3x3 Matrix

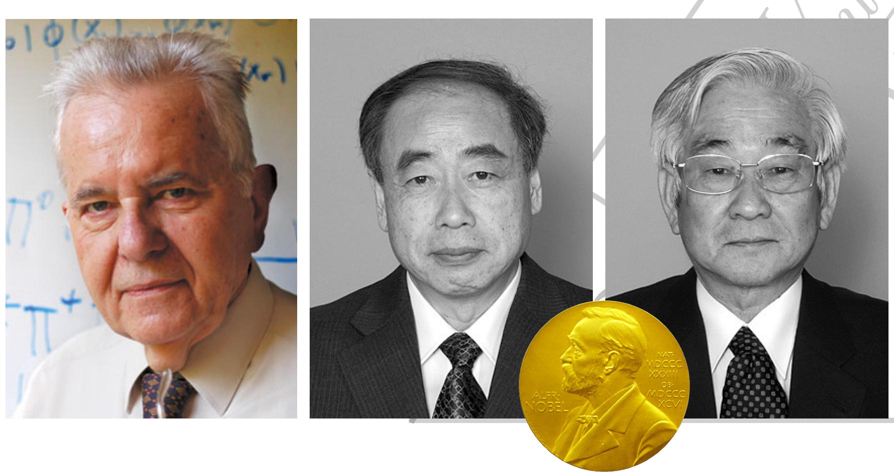
Unitarity $CC^\dagger = 1$

Overconstrain Unitarity condition
→ Potent test of Standard Model

$$
\underbrace{V_{ud}V_{ub}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}V_{cb}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}V_{tb}^*}_{\mathcal{O}(\lambda^3)} = 0
$$

Cabibbo angle $\sin\theta_C \simeq 0.22$

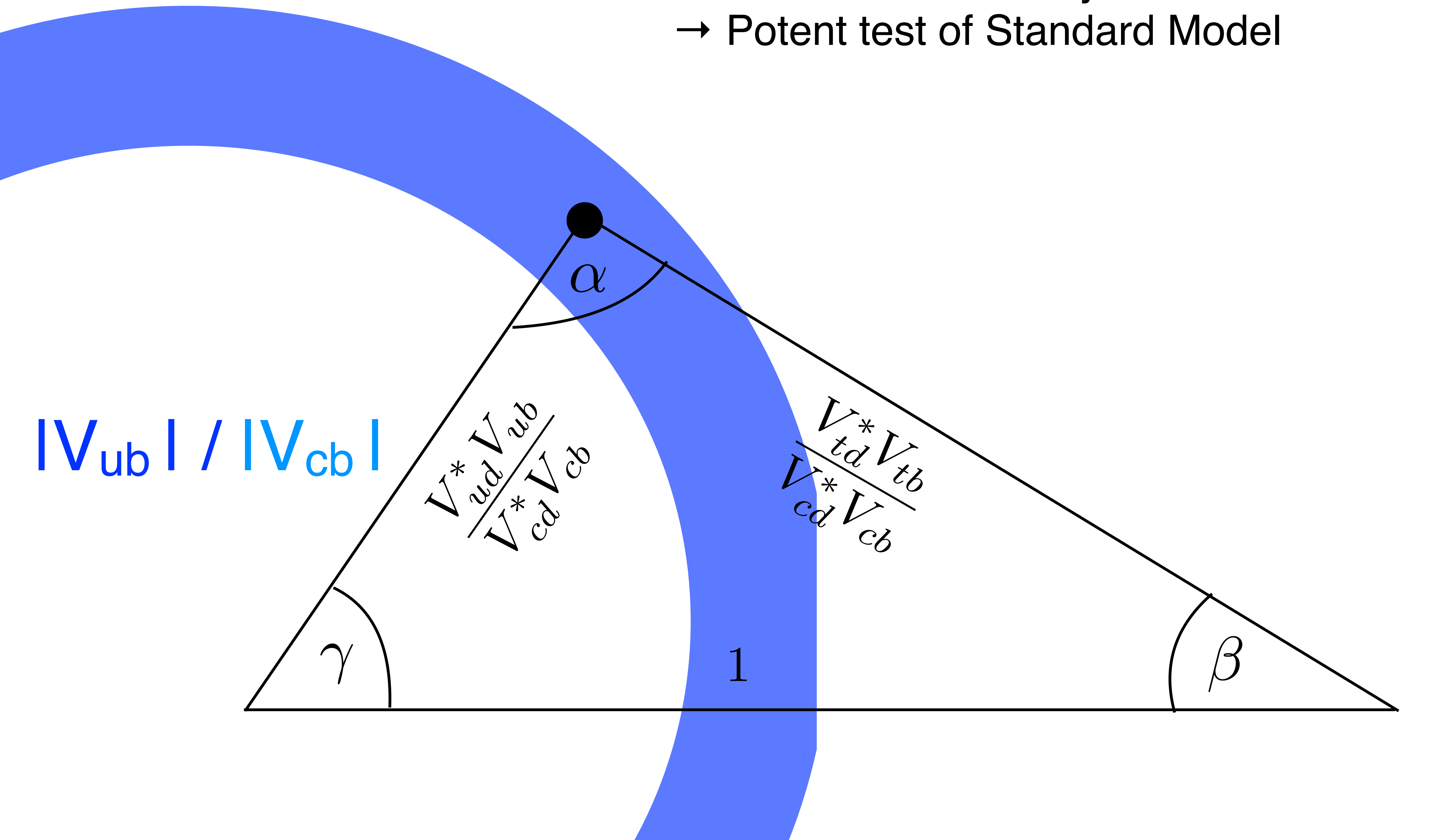




Nobel prize 2008

Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

Overconstrain Unitarity condition
→ Potent test of Standard Model

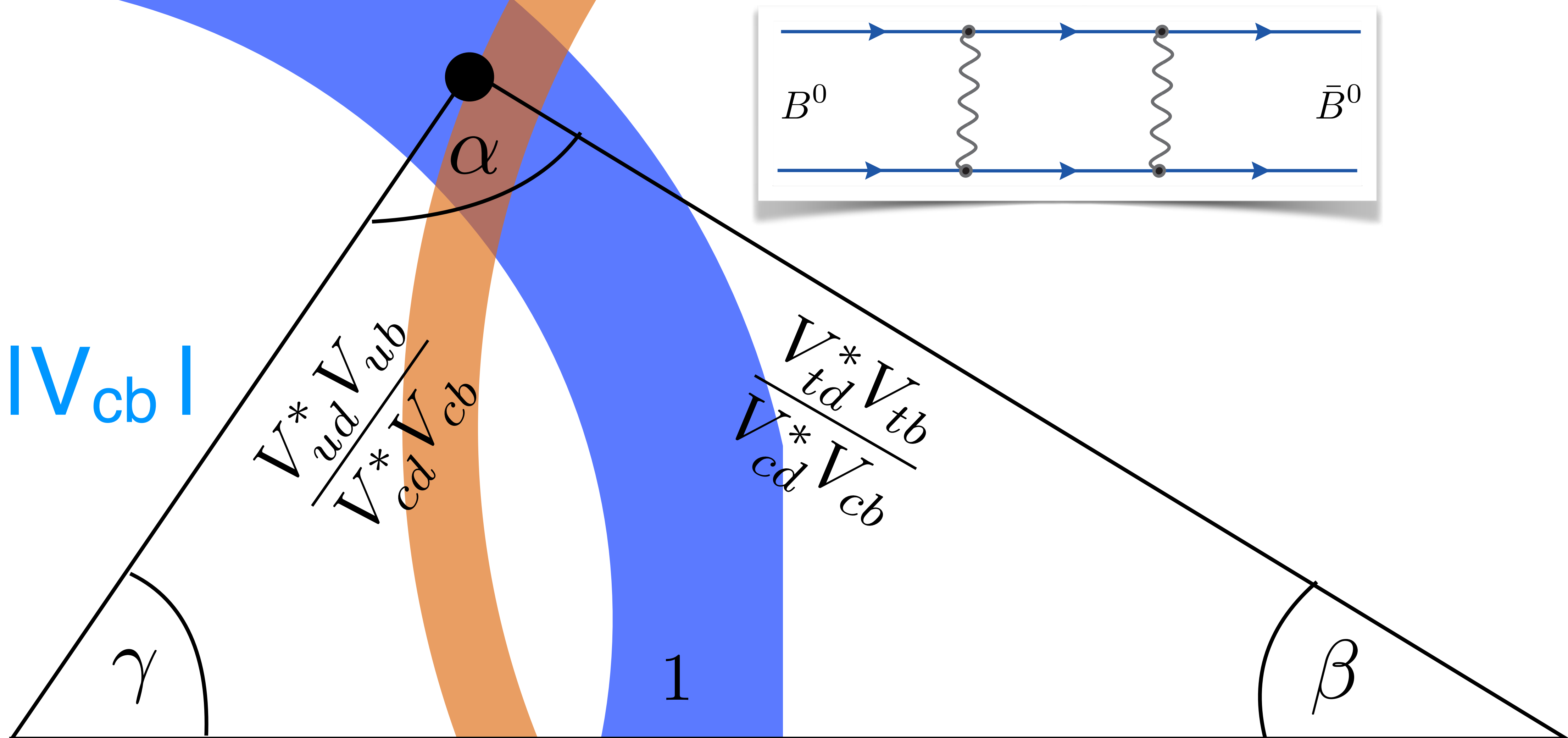


Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

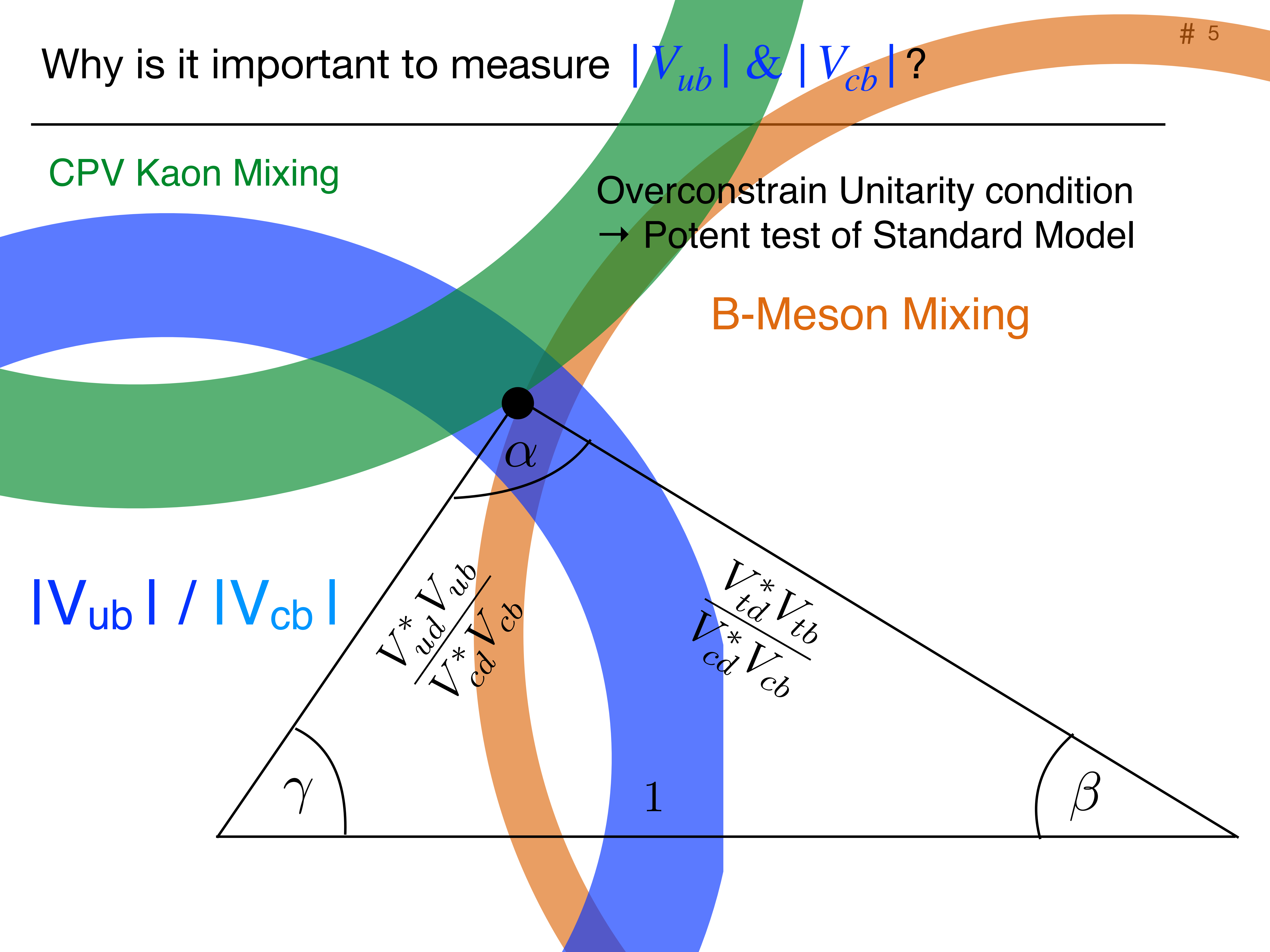
Overconstrain Unitarity condition
→ Potent test of Standard Model

B-Meson Mixing

$|V_{ub}| / |V_{cb}|$

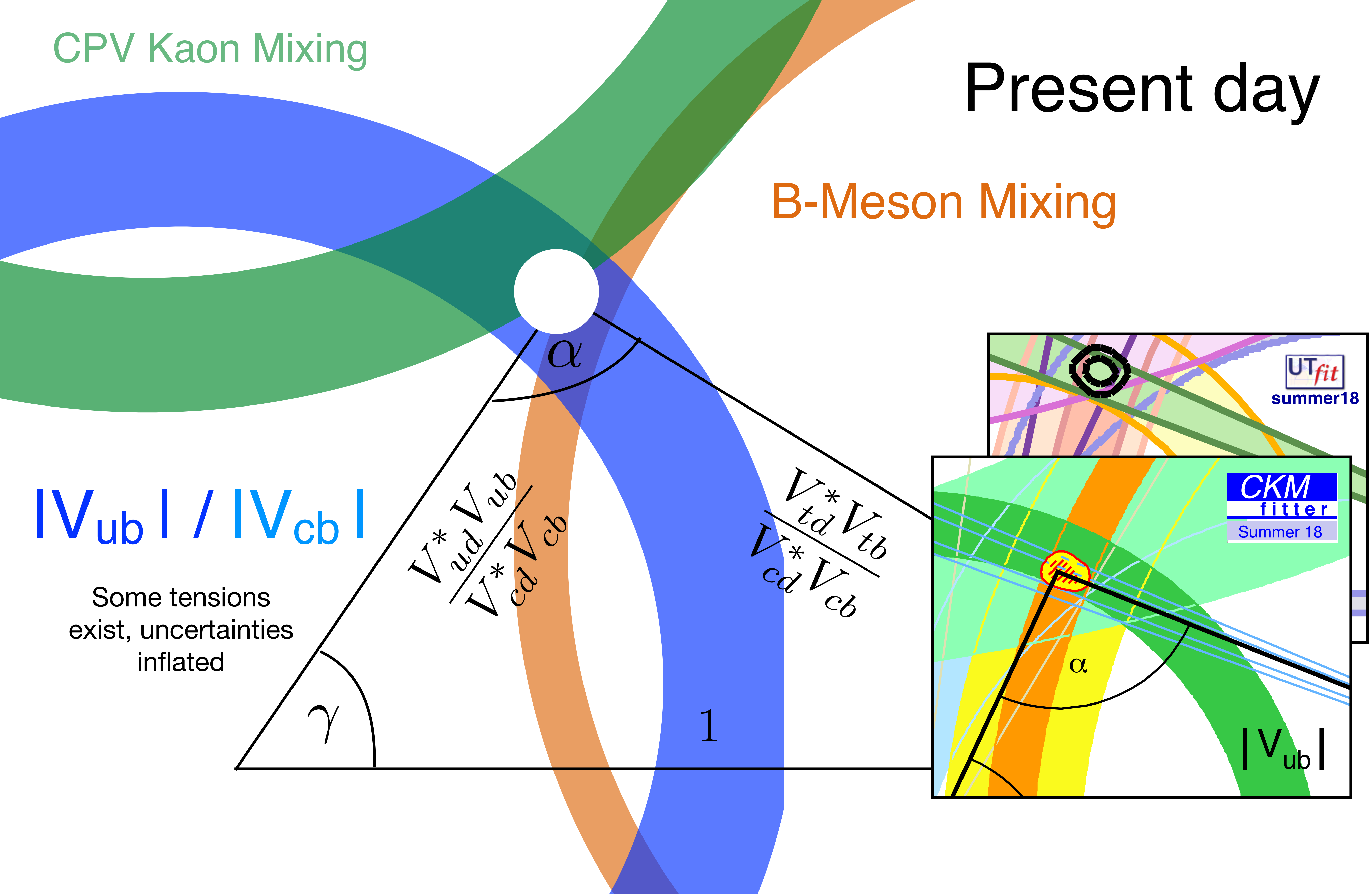


Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?



Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

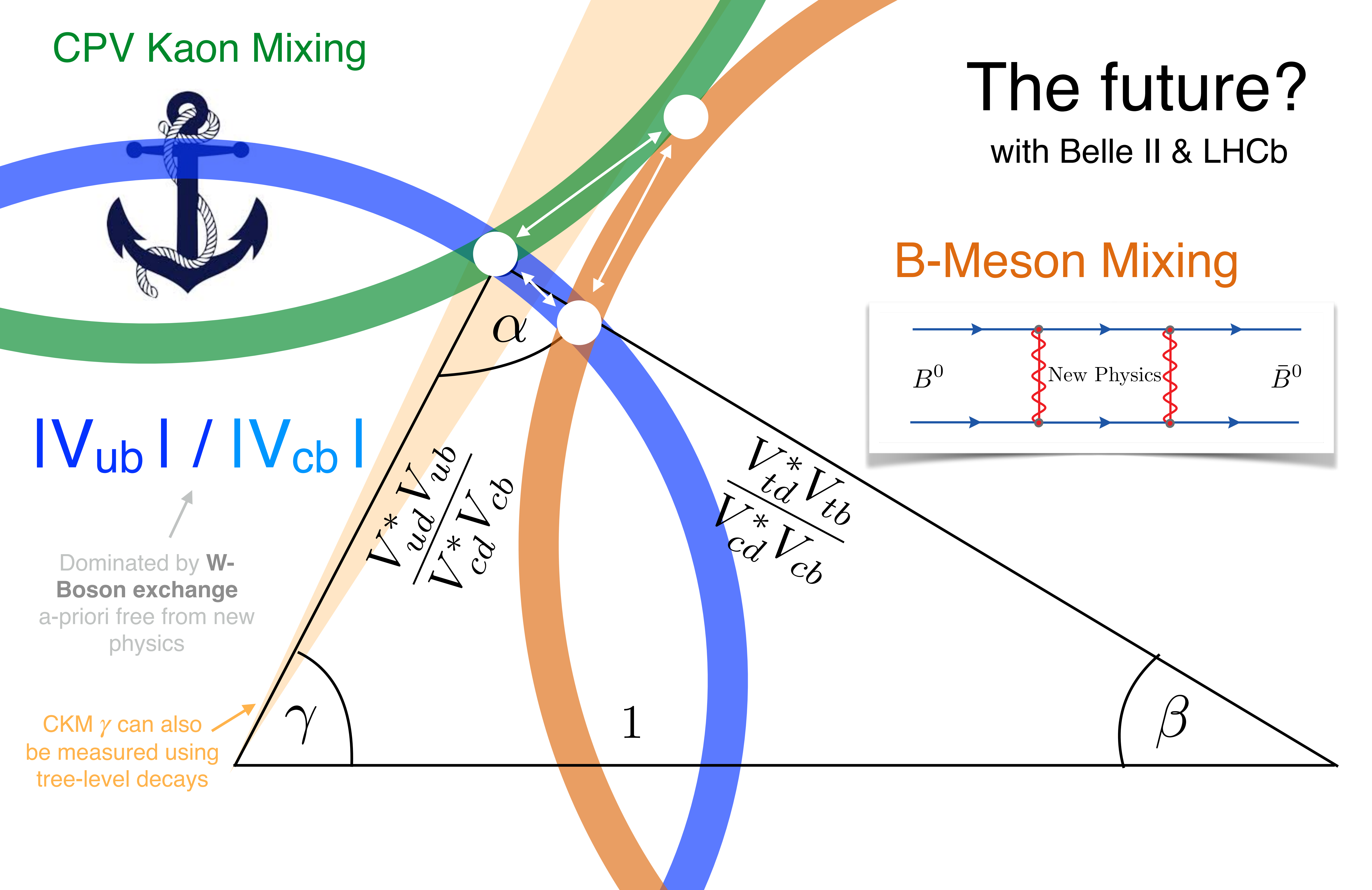
Present day



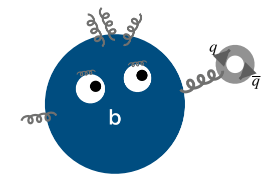
$|V_{ub}|$ / $|V_{cb}|$

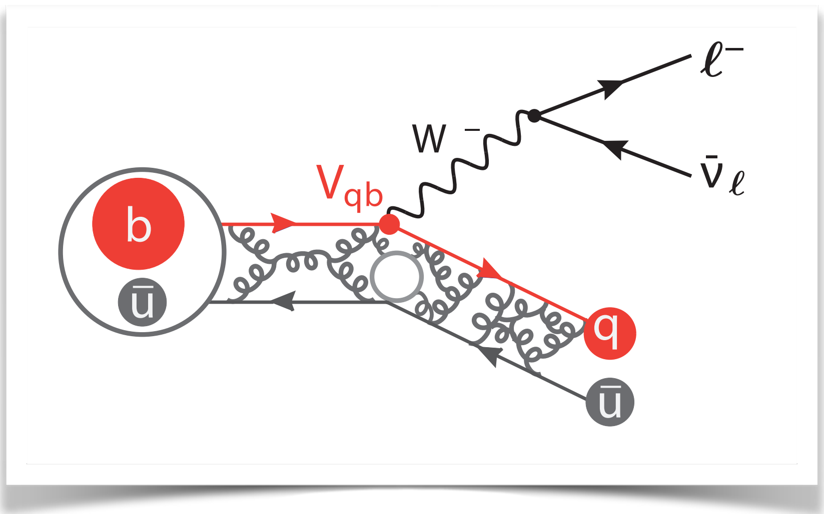
Some tensions exist, uncertainties inflated

Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?



How can we measure $|V_{ub}|$ & $|V_{cb}|$?





Inclusive $|V_{ub}|$

$$B \to X_u \ell \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

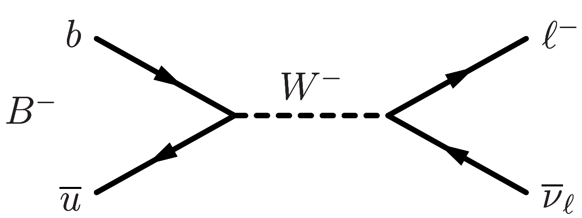
Inclusive $|V_{cb}|$

$$B \to X_c \ell \bar{\nu}_\ell$$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q\,\ell\,\bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Leptonic $|V_{ub}|$



$$\mathcal{B} \propto |V_{ub}|^2 \, f_B^2 \, m_\ell^2$$

B-Meson decay constant

Exclusive $|V_{ub}|$

$$B \to \pi, \rho, \omega \, \ell \bar{\nu}_\ell, \; \Lambda_b \to p \mu \bar{\nu}_\mu$$

$$B_s \to K \mu \, \bar{\nu}_\mu$$

Exclusive $|V_{cb}|$

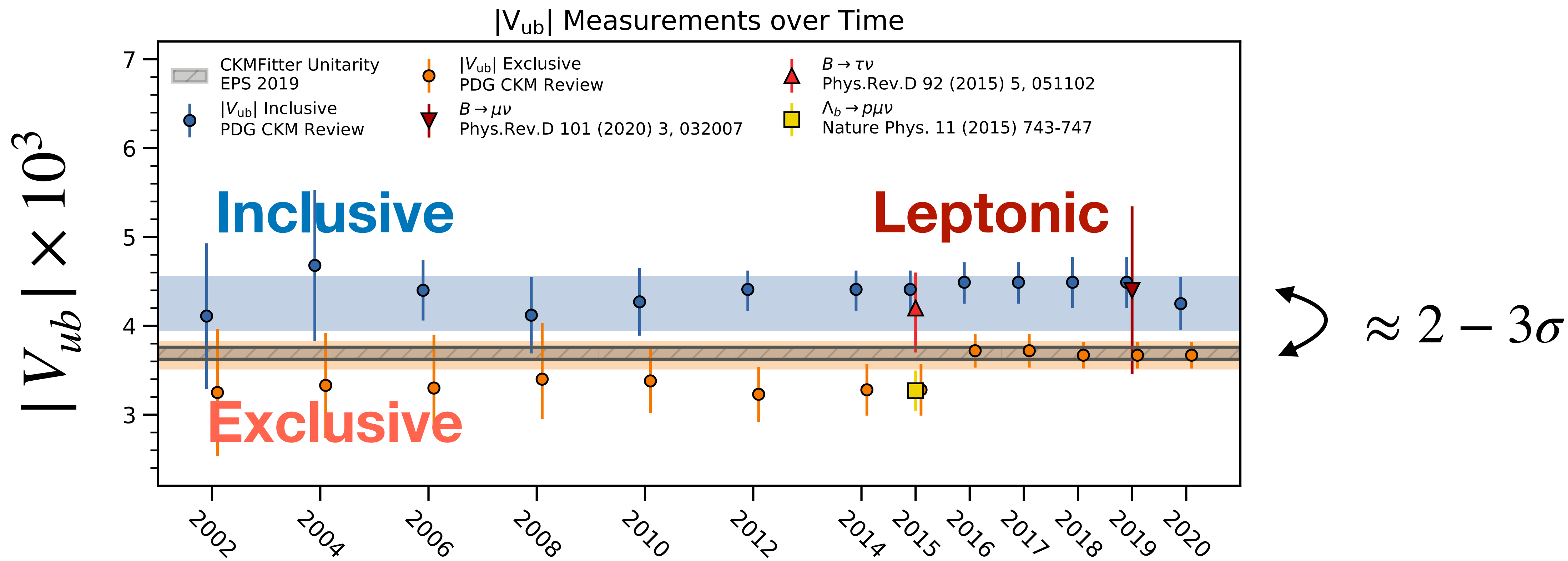
$$B_{(s)} \to D^{(*)}_{(s)} \ell \bar{\nu}_\ell$$

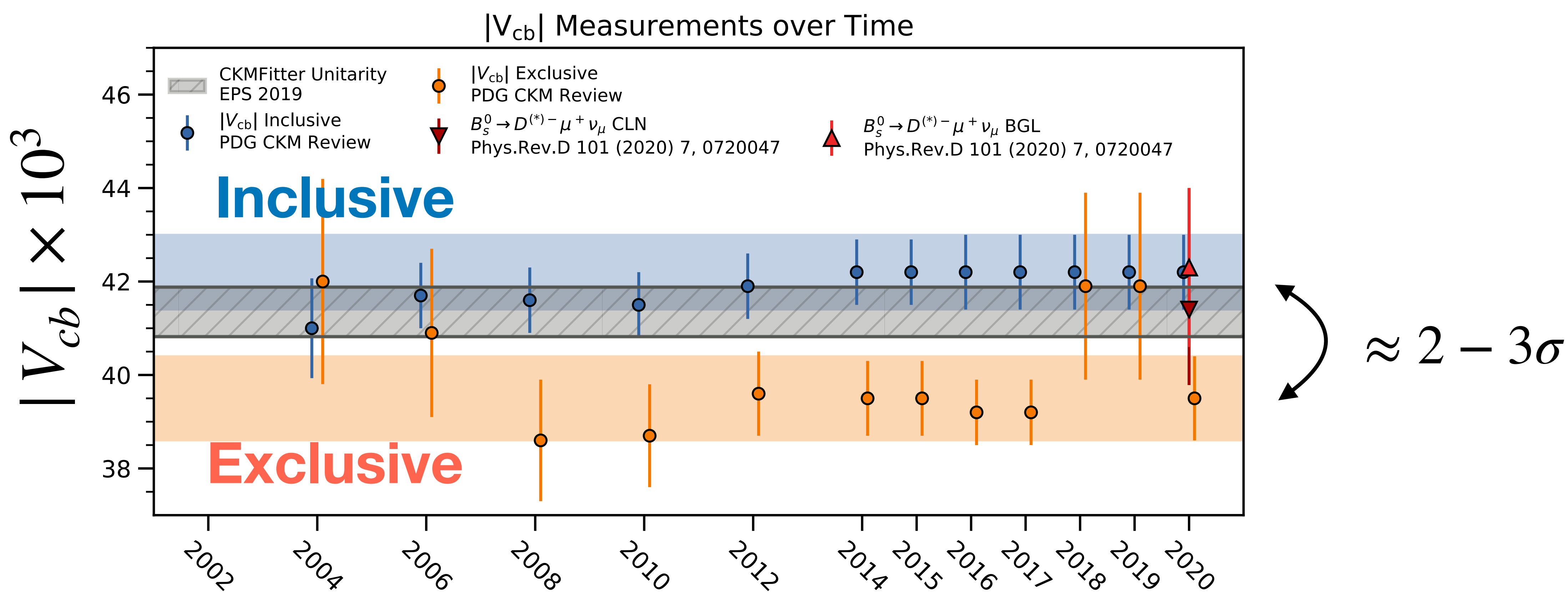
$$\mathcal{B} \propto |V_{qb}|^2 \, f^2$$

Form Factors

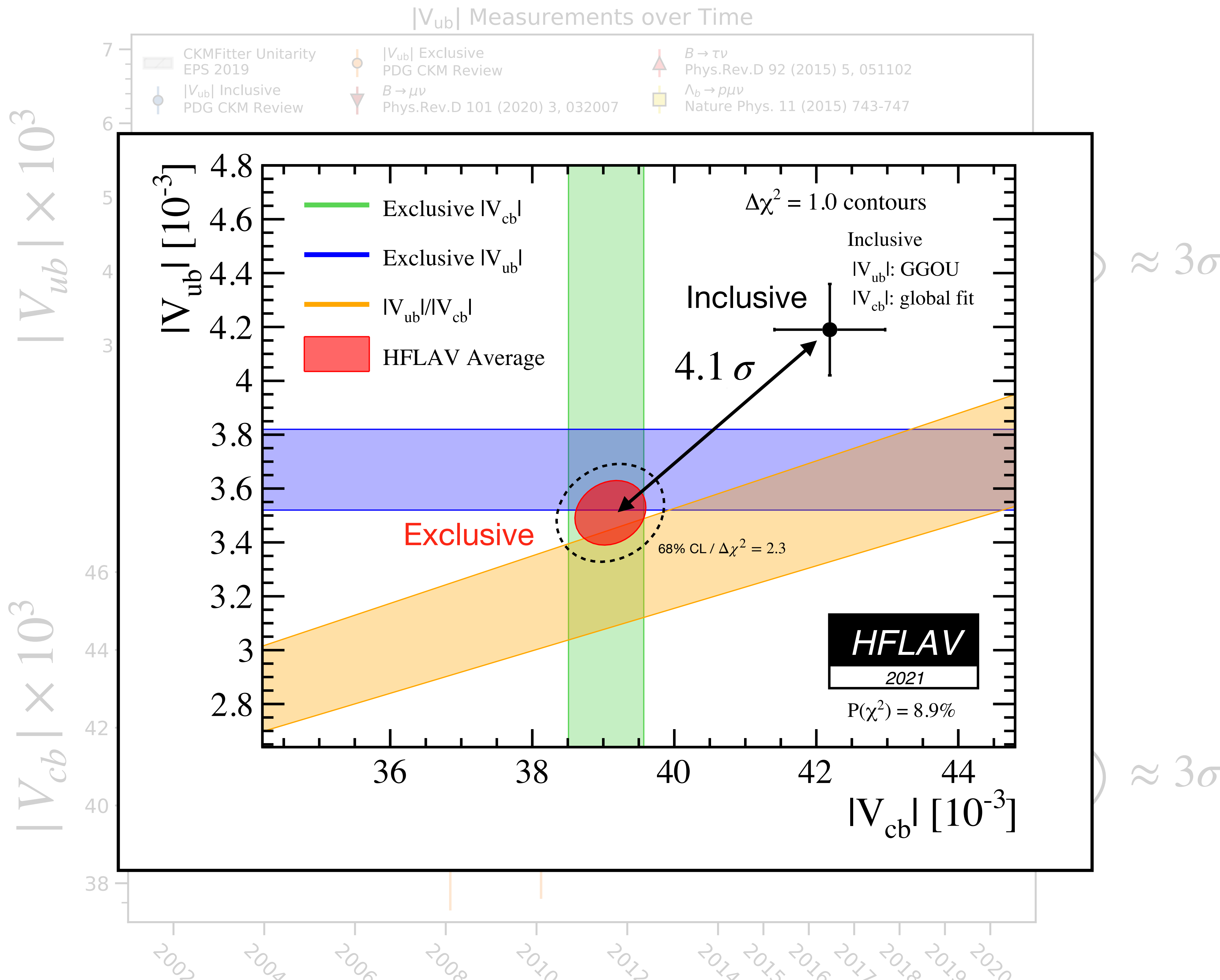
$$\langle B | H_\mu | P \rangle = (p + p')_\mu \, f_+$$

How are we doing?

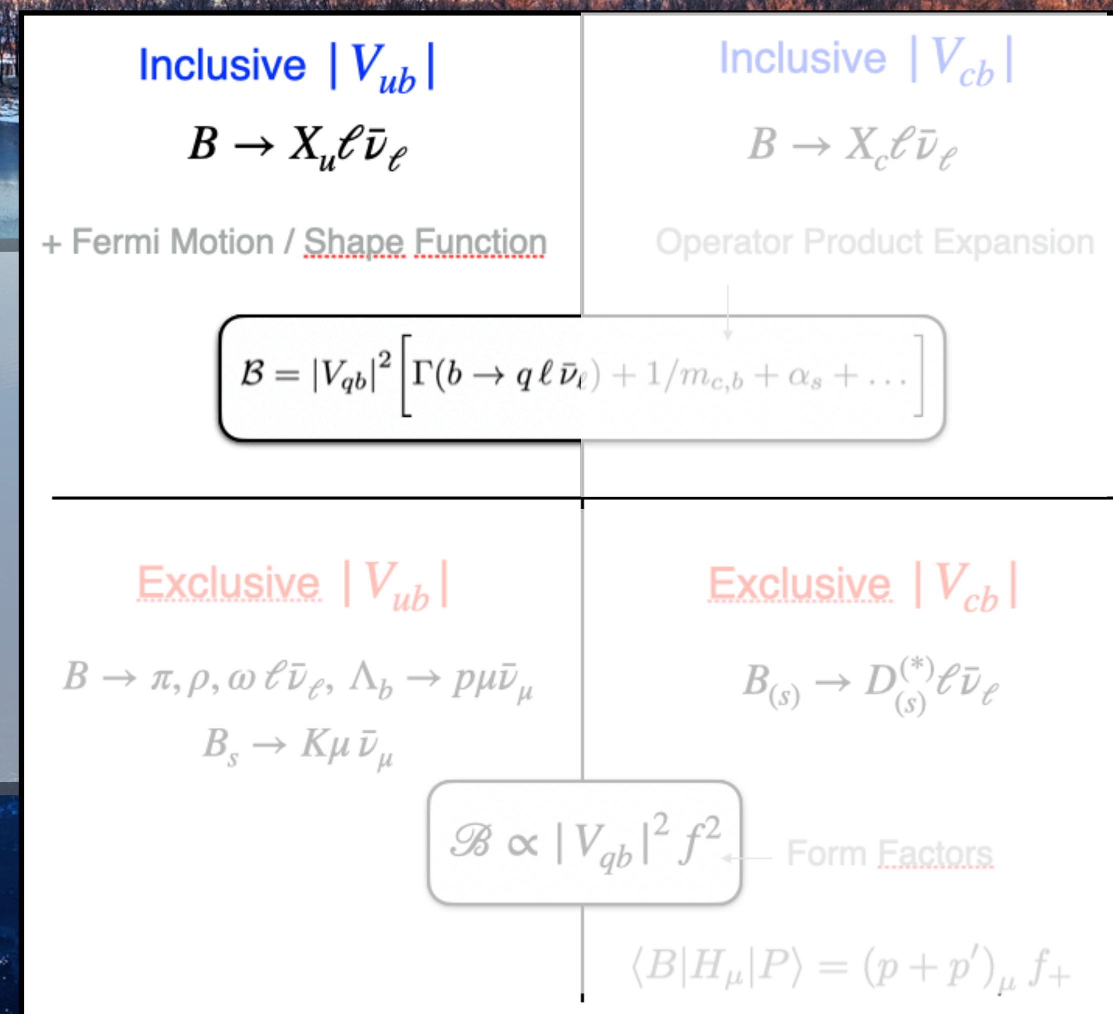




How are we doing?

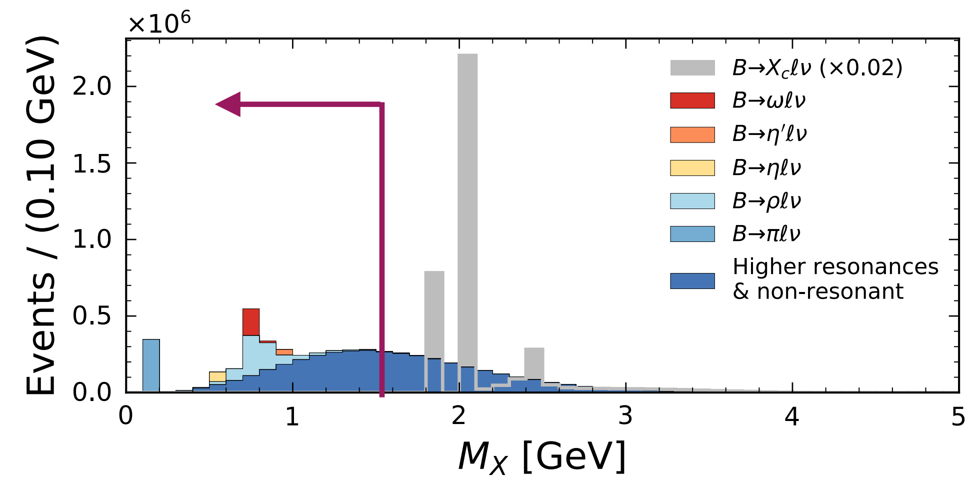


Inclusive $|V_{ub}|$

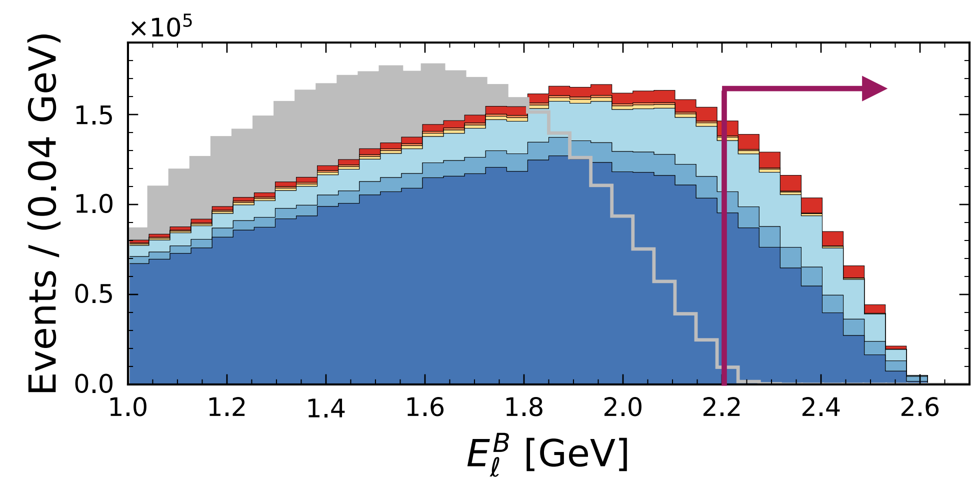


Challenges of measuring inclusive $|V_{ub}|$

Inclusive $B \to X_u \ell \bar{\nu}_\ell$ measurements are extremely challenging due to dominant $B \to X_c \ell \bar{\nu}_\ell$ background



Clean **separation** only **possible** in certain **kinematic regions**, e.g. **lepton endpoint** or **low** M_X



1. Measurement of **partial** branching fractions of inclusive $B \to X_u \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 012008 (2021), arXiv:2102.00020]

2. Measurement of **differential** branching fractions of inclusive $B \to X_u \ell \bar{\nu}_\ell$ decays with hadronic tagging [Phys. Rev. Lett. 127, 261801 (2021), arXiv:2107.13855]

New! 3. New measurement of **ratio** of inclusive $B \to X_u \ell \bar{\nu}_\ell$ / $B \to X_c \ell \bar{\nu}_\ell$ with improved tagging and data-driven background templates [to appear]

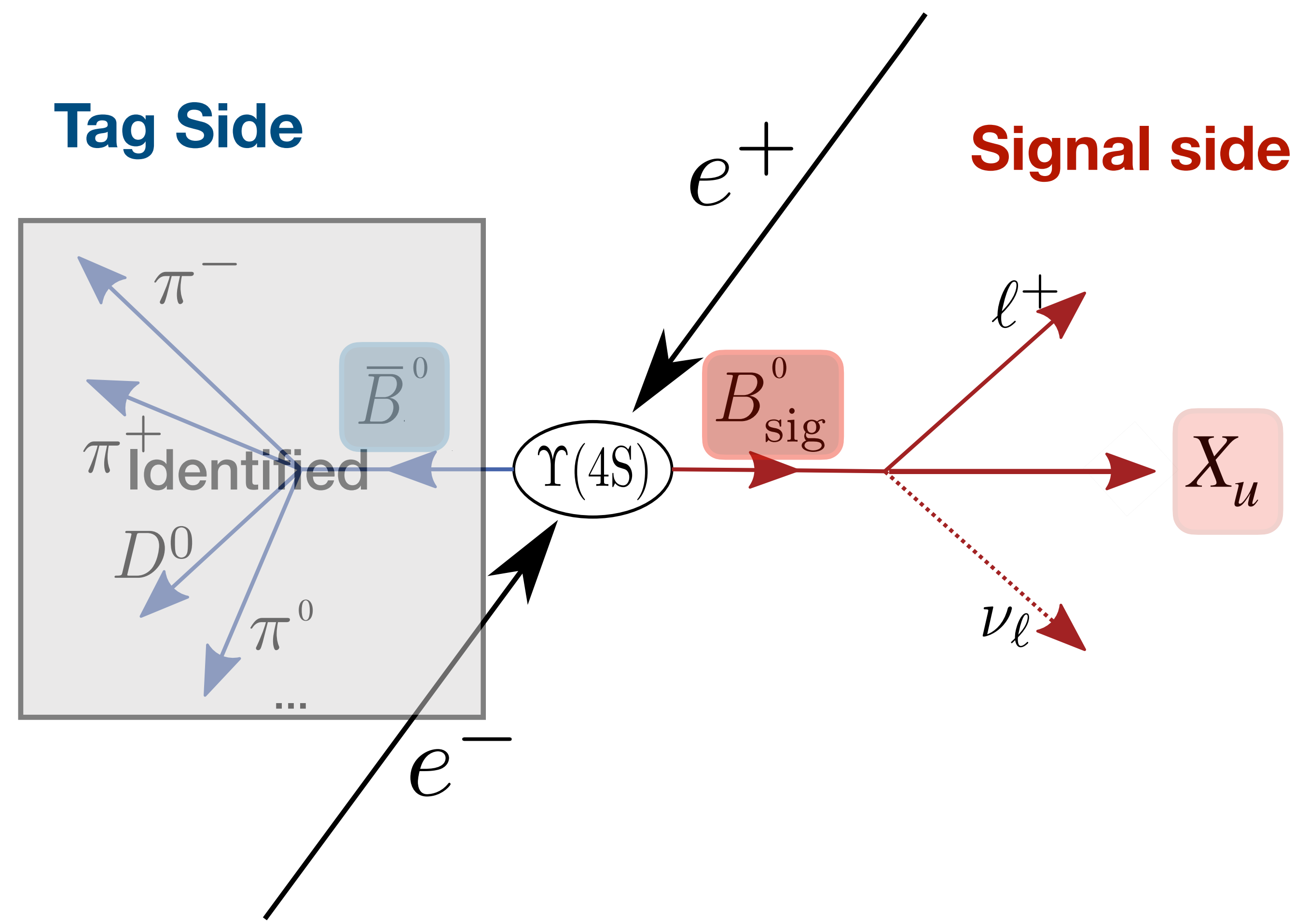
Inclusive

Belle

1. Measurement of **partial** branching fractions of inclusive $B \to X_u \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 012008 (2021), arXiv:2102.00020]

Use **full Belle** data set of **711/fb**

Hadronic tagging with neural networks (ca. 0.2-0.3% efficiency)



Charged Tracks — Neutral Clusters

$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$

$$q^2 = (p_{\rm sig} - p_X)^2 \qquad M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

$$m_{\rm miss}^2 = \left(p_{\rm sig} - p_X - p_\ell \right)^2 \approx m_\nu^2 = 0\,\mathrm{GeV}^2$$

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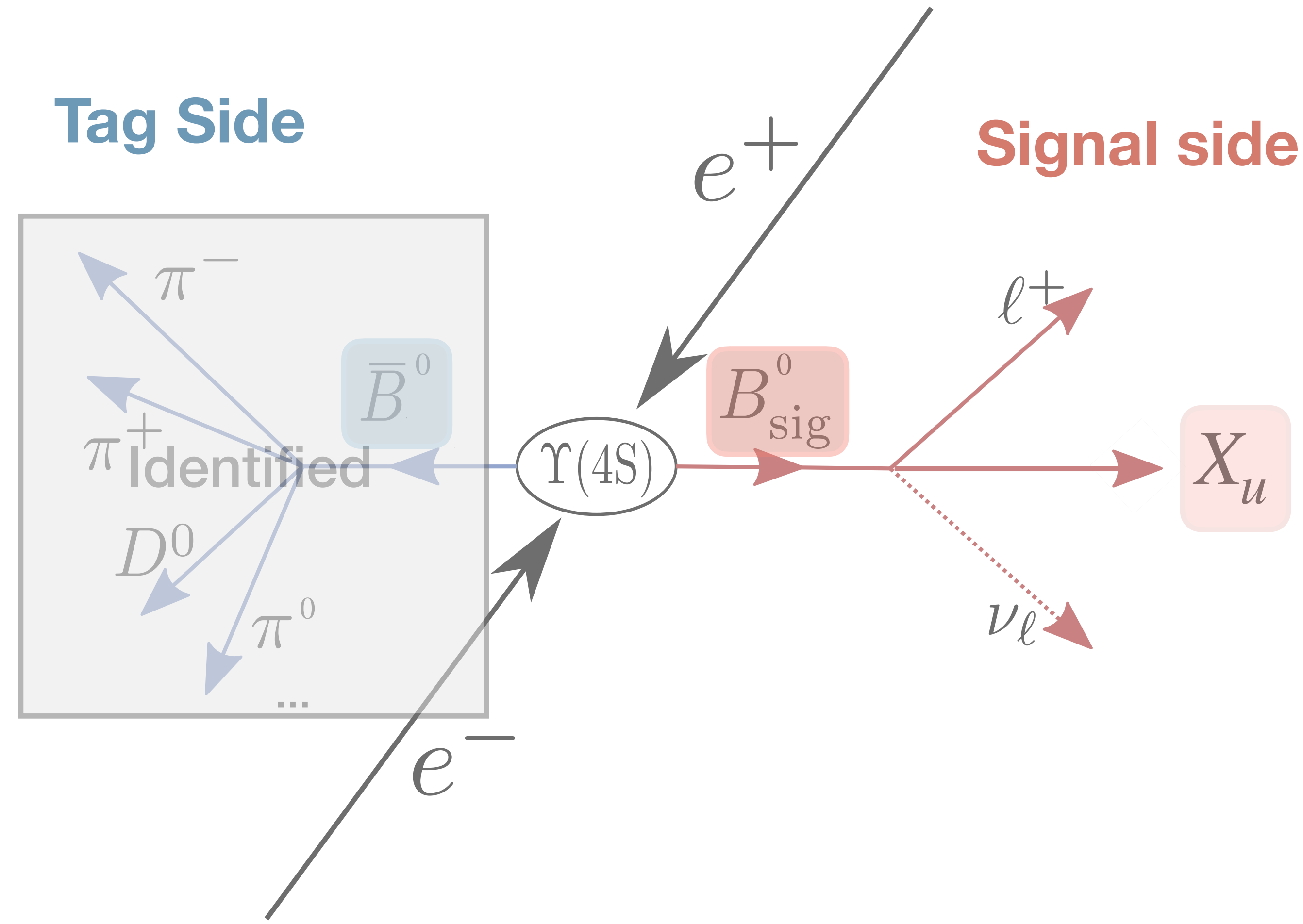
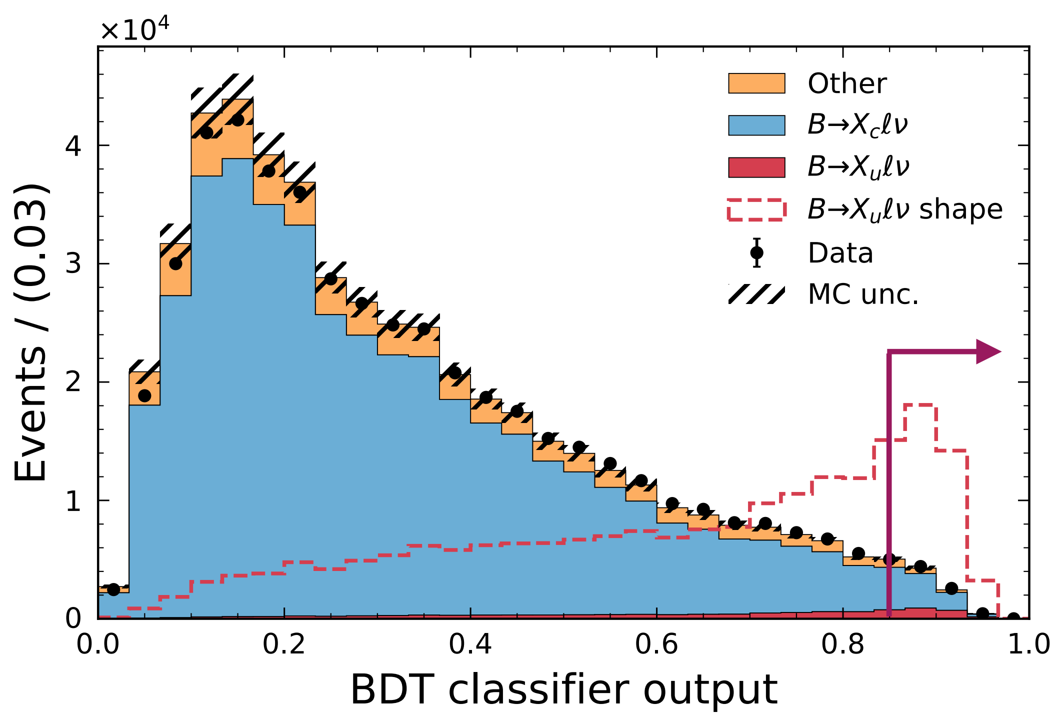
Use **machine learning** (BDTs) to suppress backgrounds with 11 training features, e.g. m^2_{miss}, #K±, #Ks, etc.





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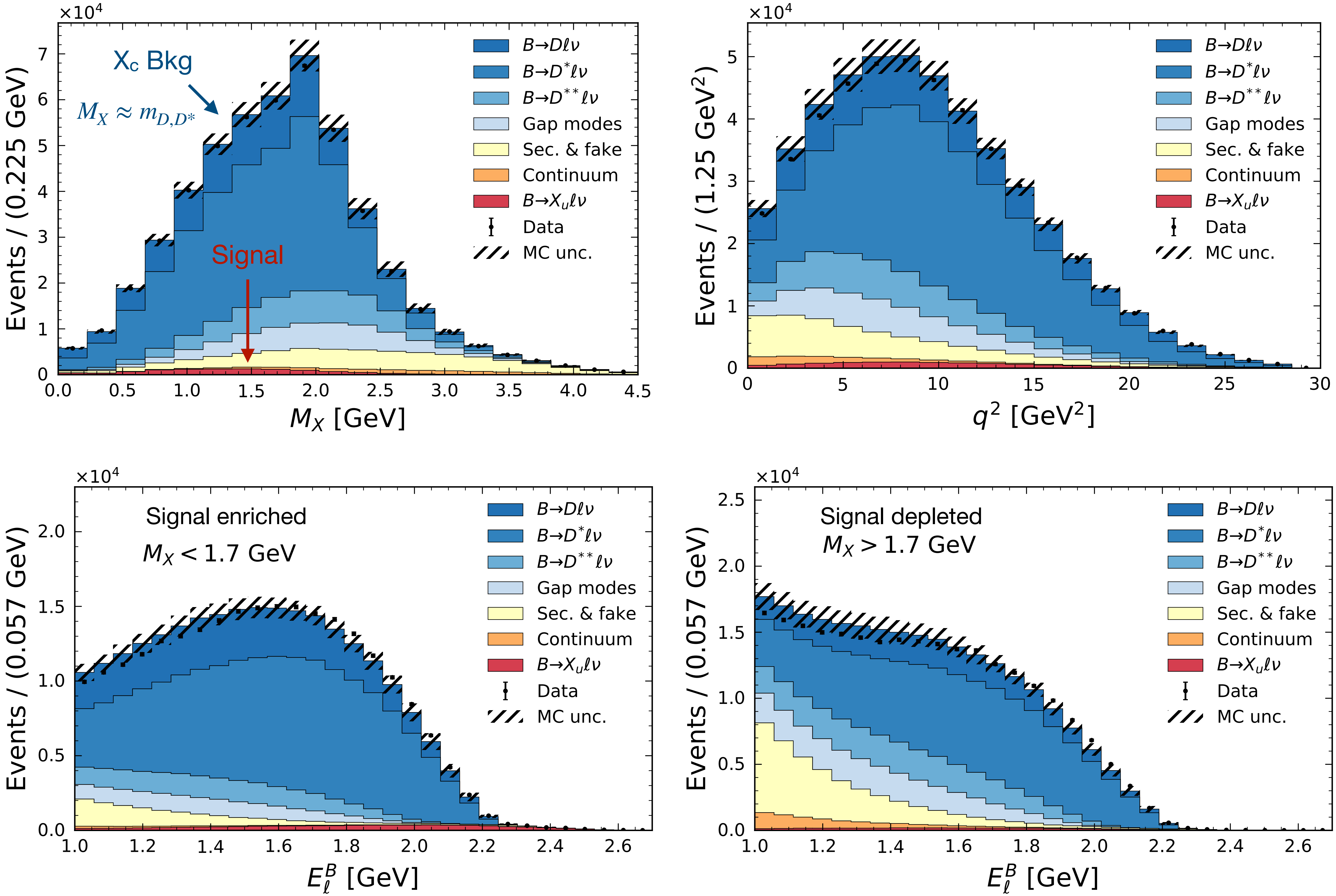
$$m^2_{\mathrm{miss}} = \left(p_{\mathrm{sig}} - p_X - p_\ell\right)^2 \approx m_\nu^2 = 0\,\mathrm{GeV}^2$$

Before BDT selection

Hadronic Mass $M_X = \sqrt{p_X^2}$

Four-momentum transfer squared $q^2 = (p_B - p_X)^2$

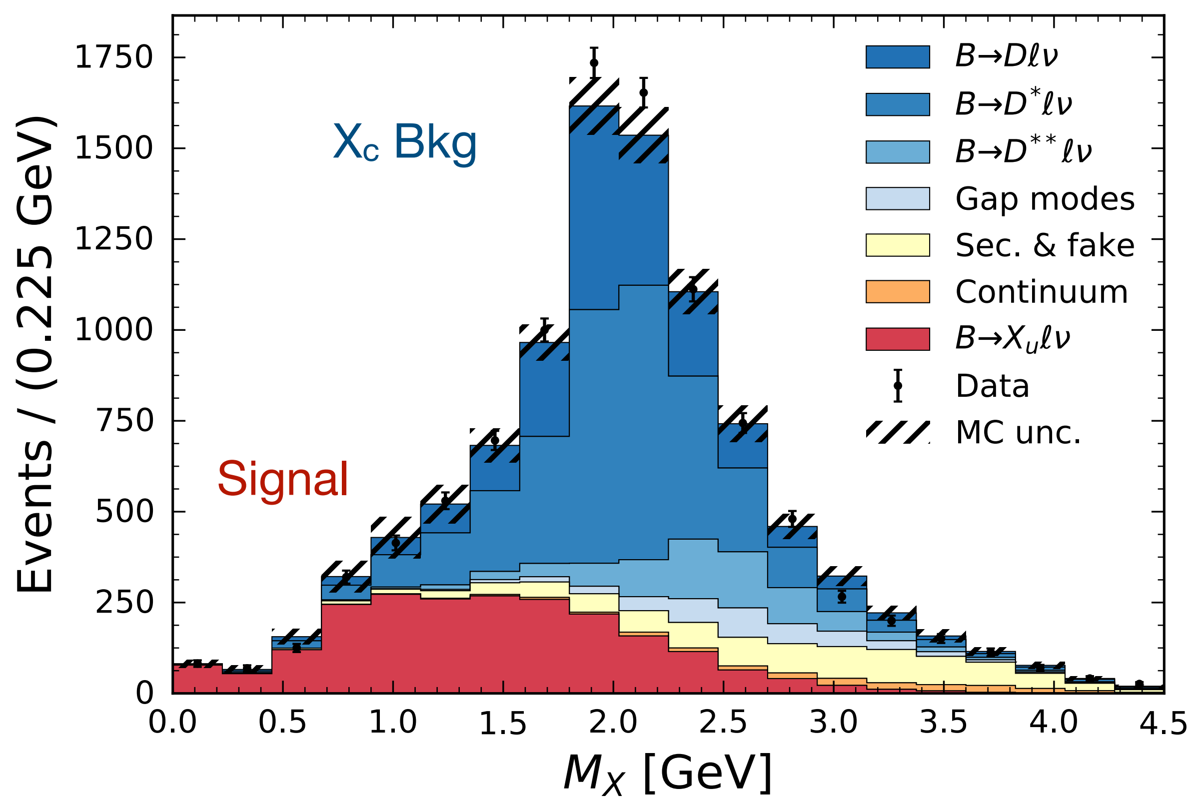


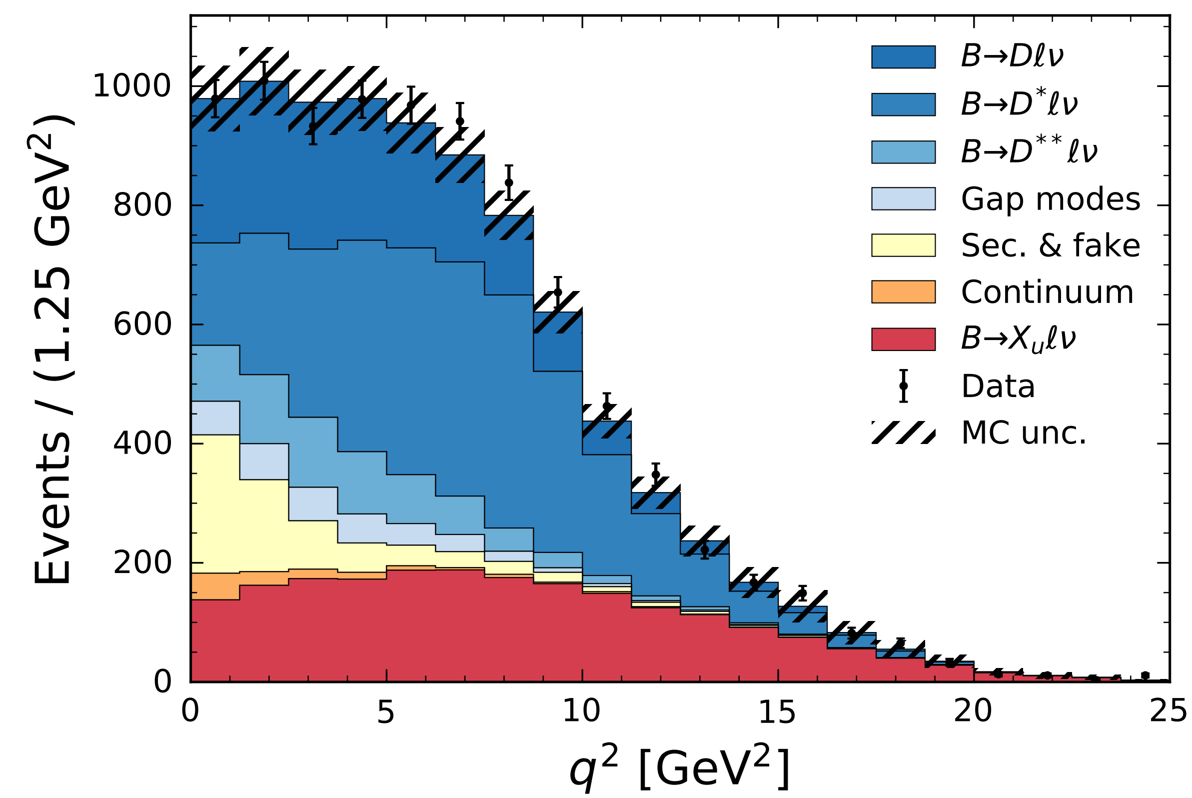
Lepton Energy in signal B rest frame E_ℓ^B

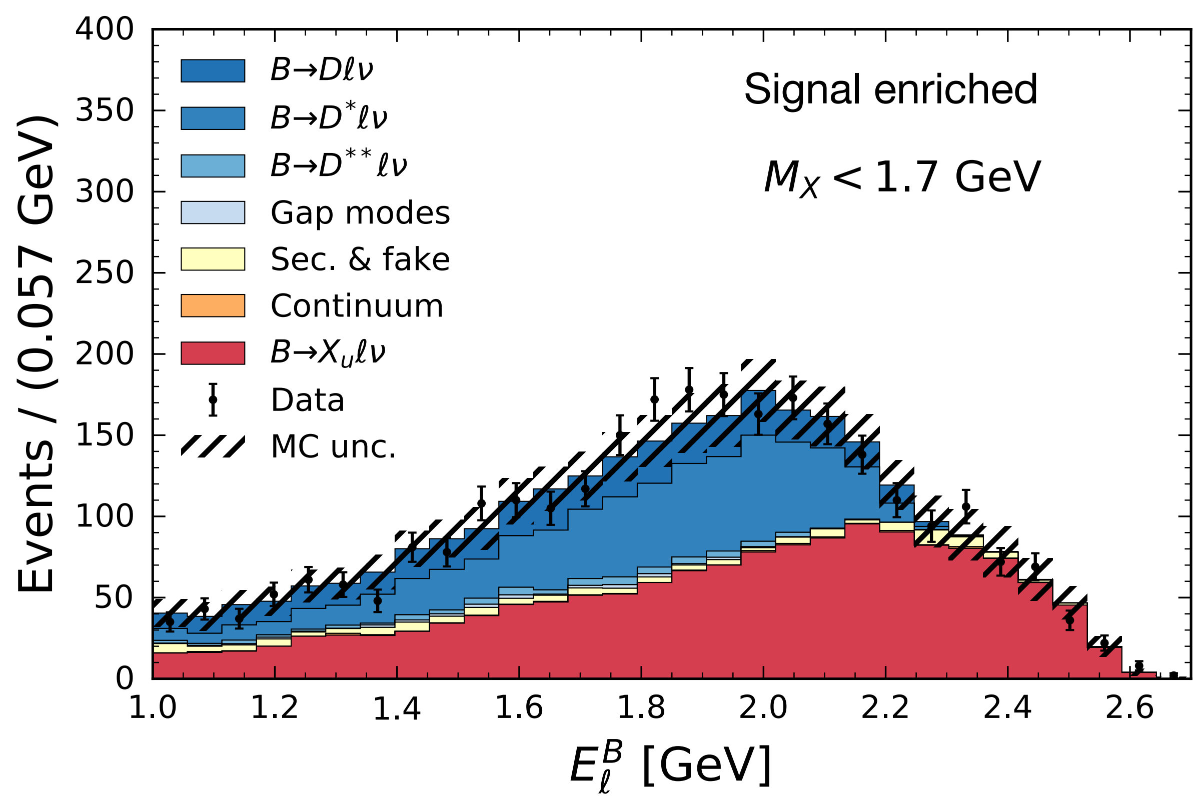
After BDT selection

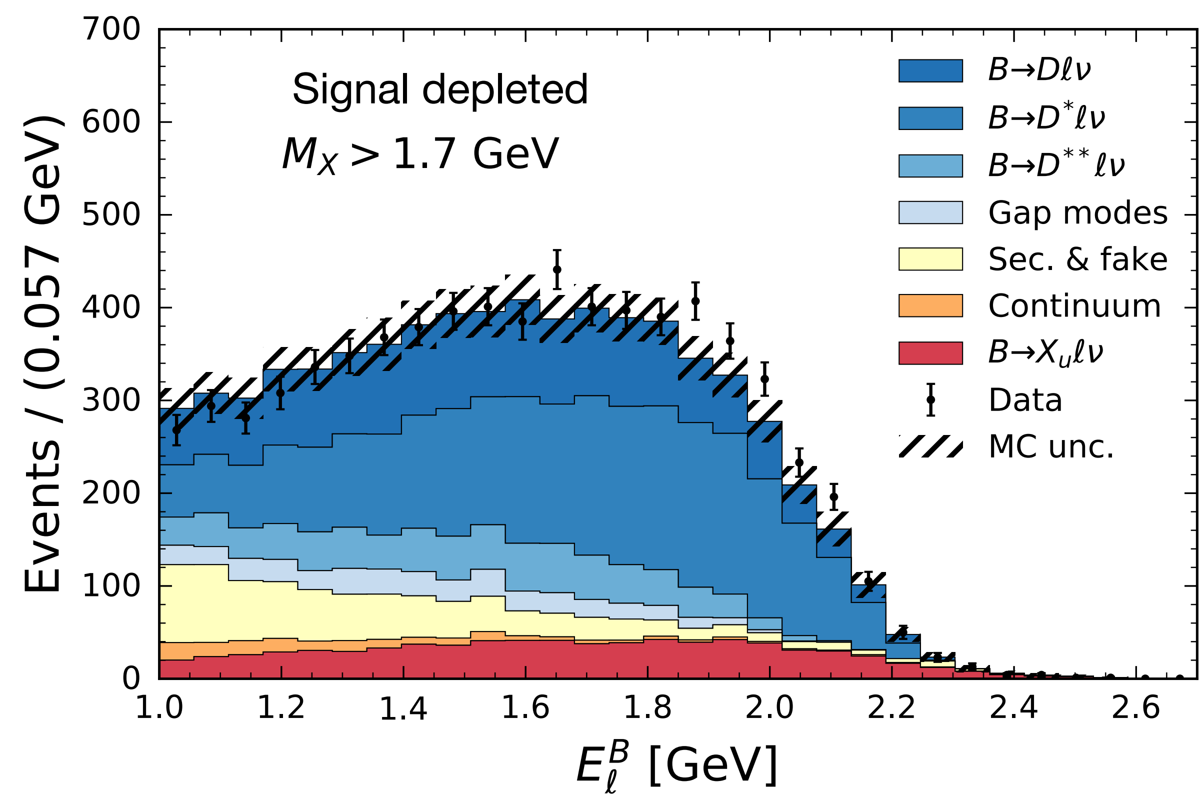
Hadronic Mass $M_X = \sqrt{p_X^2}$

Four-momentum transfer squared $q^2 = (p_B - p_X)^2$









Lepton Energy in signal B restframe E_ℓ^B

Fit kinematic distributions and measure partial BF

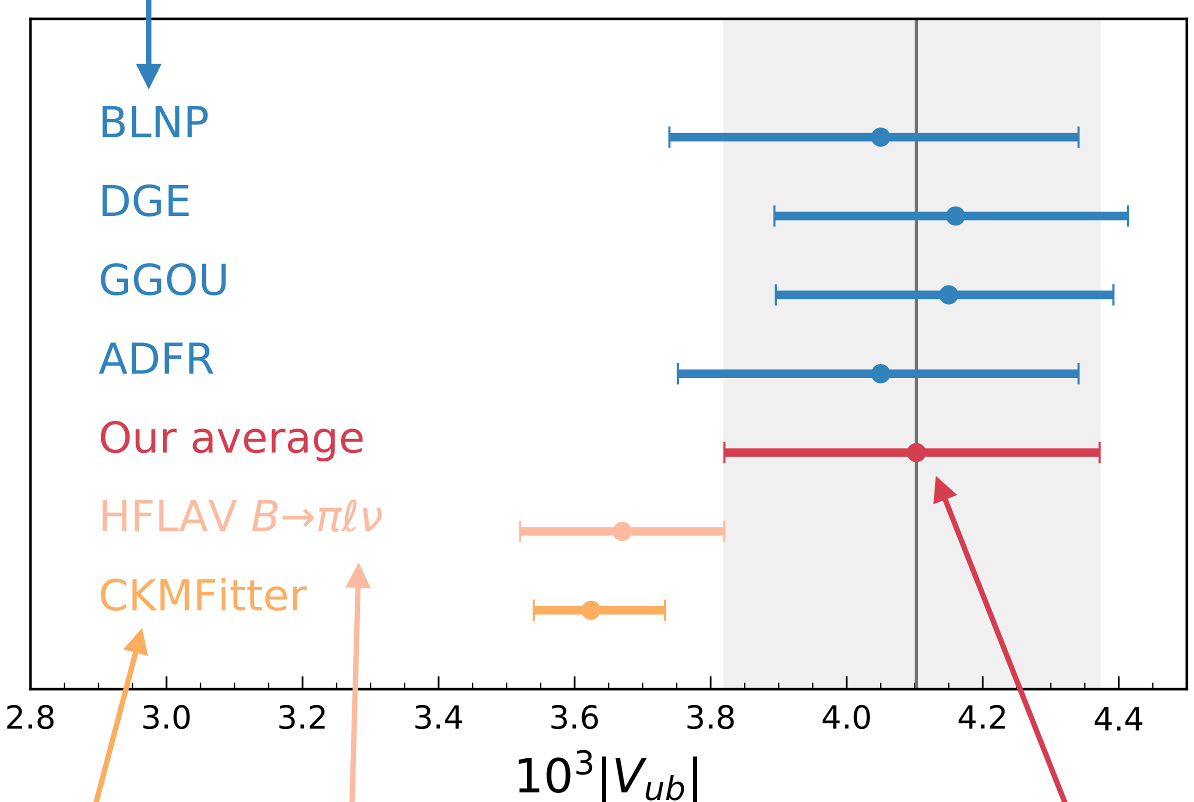
$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u\,\ell^+\,\nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \to X_u\,\ell^+\,\nu_\ell)}}$$

3 phase-space regions

Phase-space region
$M_X < 1.7\,\mathrm{GeV}$
$M_X < 1.7\,\mathrm{GeV},\ q^2 > 8\,\mathrm{GeV}^2$
$E_\ell^B > 1\,\mathrm{GeV}$

4 predictions of the partial rate

Result for most inclusive region with $E_\ell^B > 1\,\mathrm{GeV}$

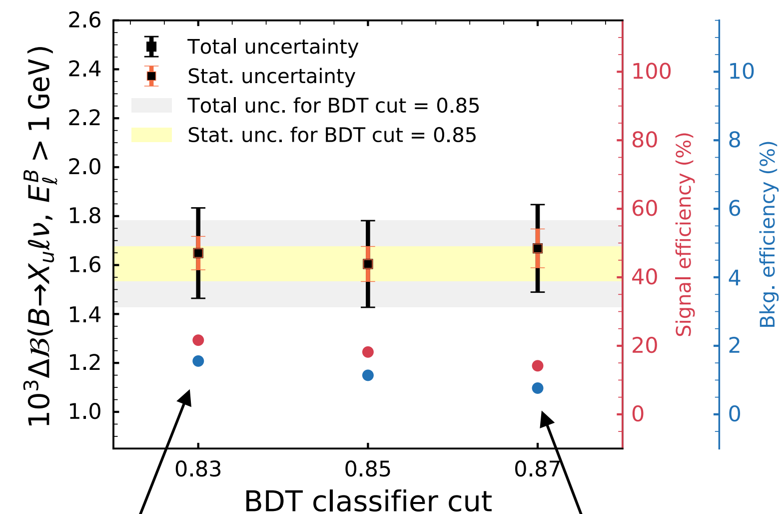


Exclusive Average for $B \to \pi\ell\bar{\nu}_\ell$:
$|V_{ub}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}$

CKM Unitarity:
$|V_{ub}| = (3.62^{+0.11}_{-0.08}) \times 10^{-3}$

Arithmetic average:
$|V_{ub}| = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}$

Stability as a function of BDT cut:



33% more Bkg

37% less Bkg

2.

Measurement of **differential** branching fractions of inclusive $B \to X_u \ell \bar{\nu}_\ell$ decays with hadronic tagging [Phys. Rev. Lett. 127, 261801 (2021), arXiv:2107.13855]

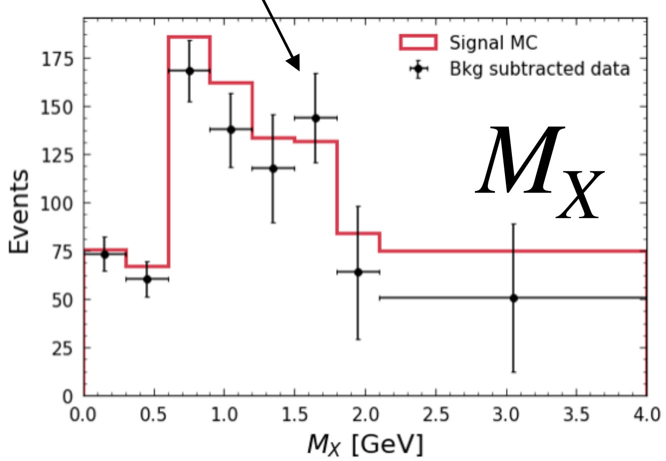
Measurement of **6 kinematic** variables **characterizing** $B \to X_u \ell \bar{\nu}_\ell$ in $E_\ell^B > 1\,\mathrm{GeV}$ region of PS

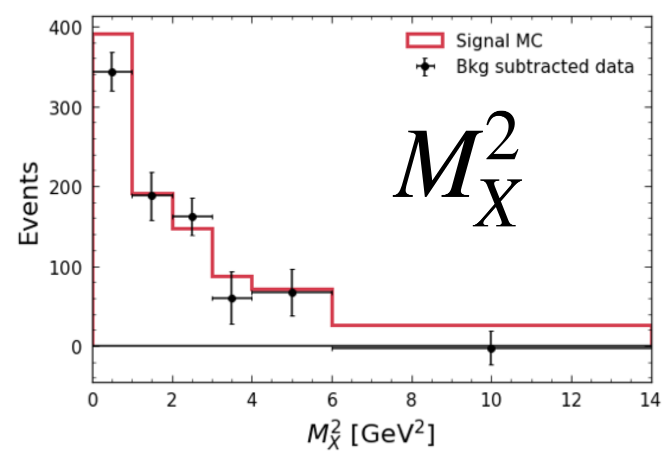
Selection and reconstruction **analogous** to **partial BF** measurement

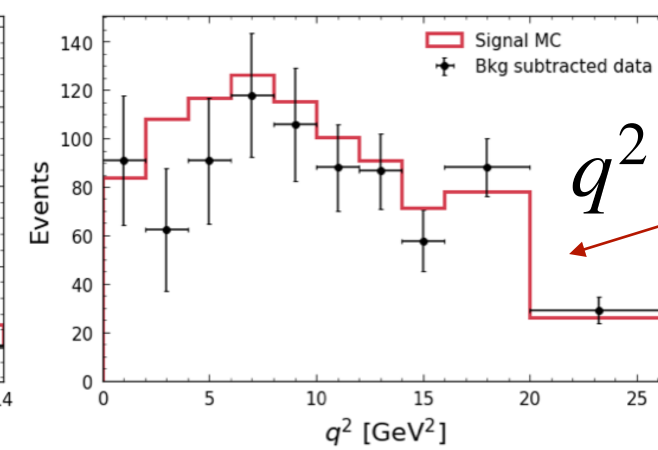
Apply **additional selections** to **improve resolution and background shape uncertainties**

Bkg. subtracted data

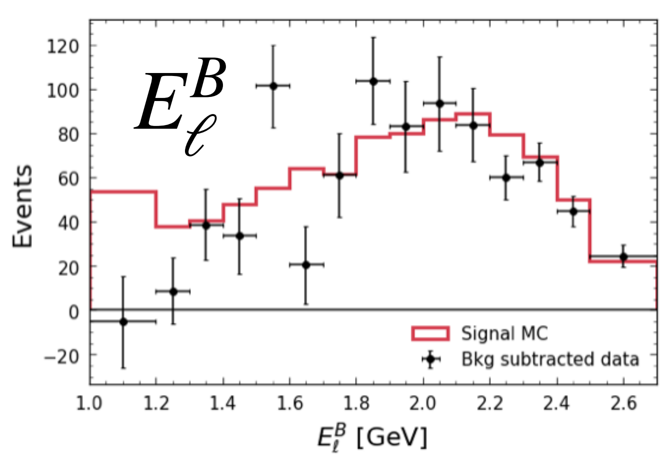
Background subtraction via coarse M_X fit:

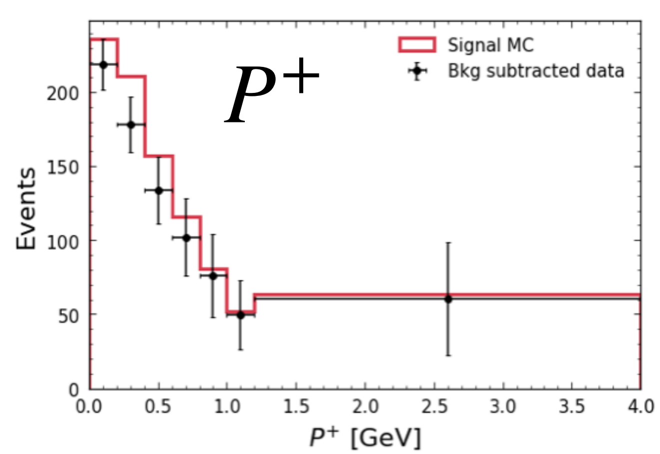


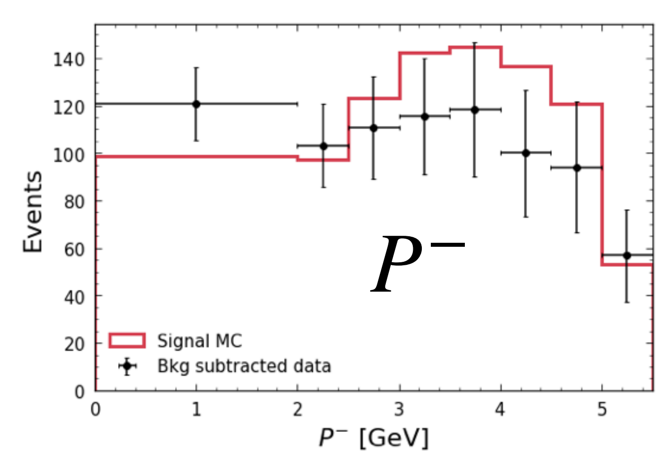




Overlaid **signal MC** (hybrid $B \to X_u \ell \bar{\nu}_\ell$)

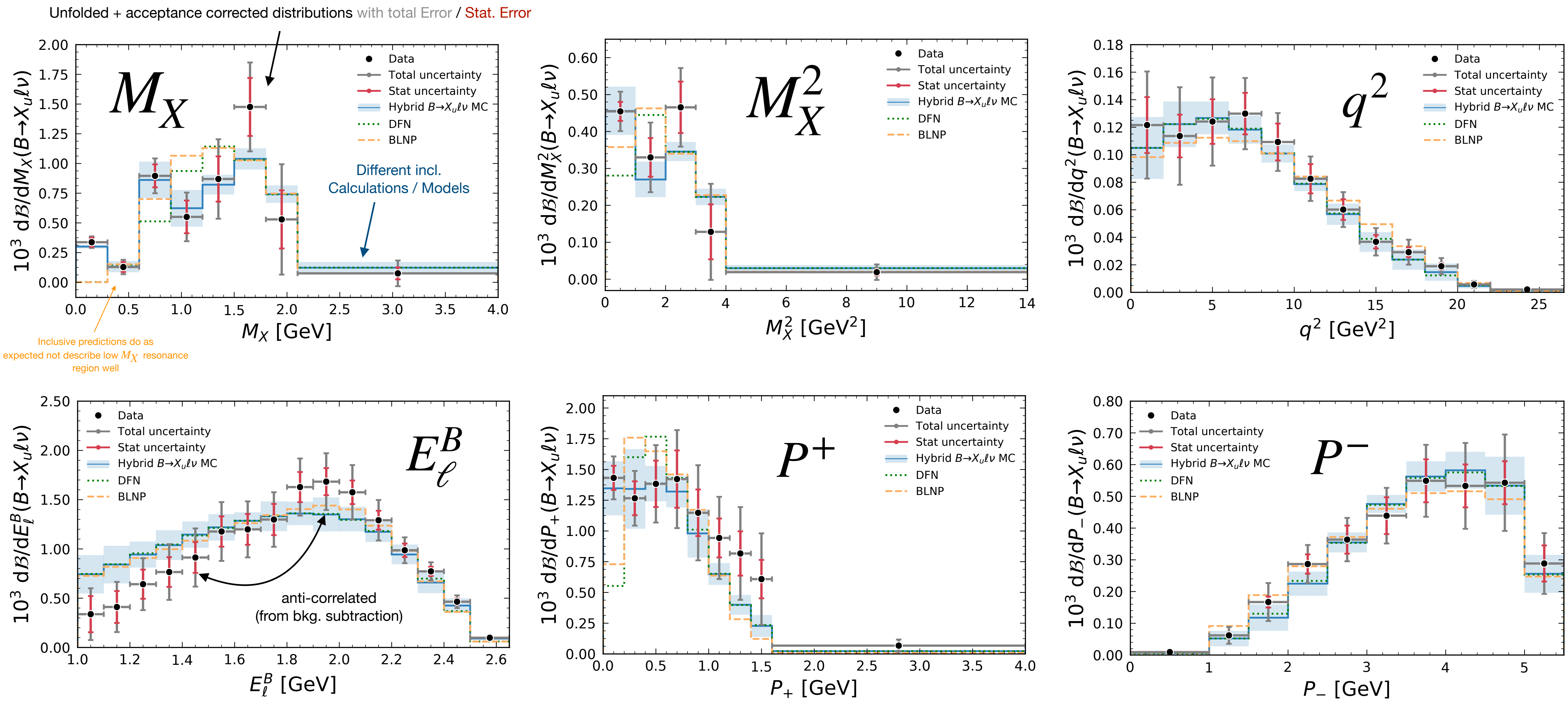






light-cone momenta:
$P_\pm = E_X \mp |P_X|$

Differential Spectra

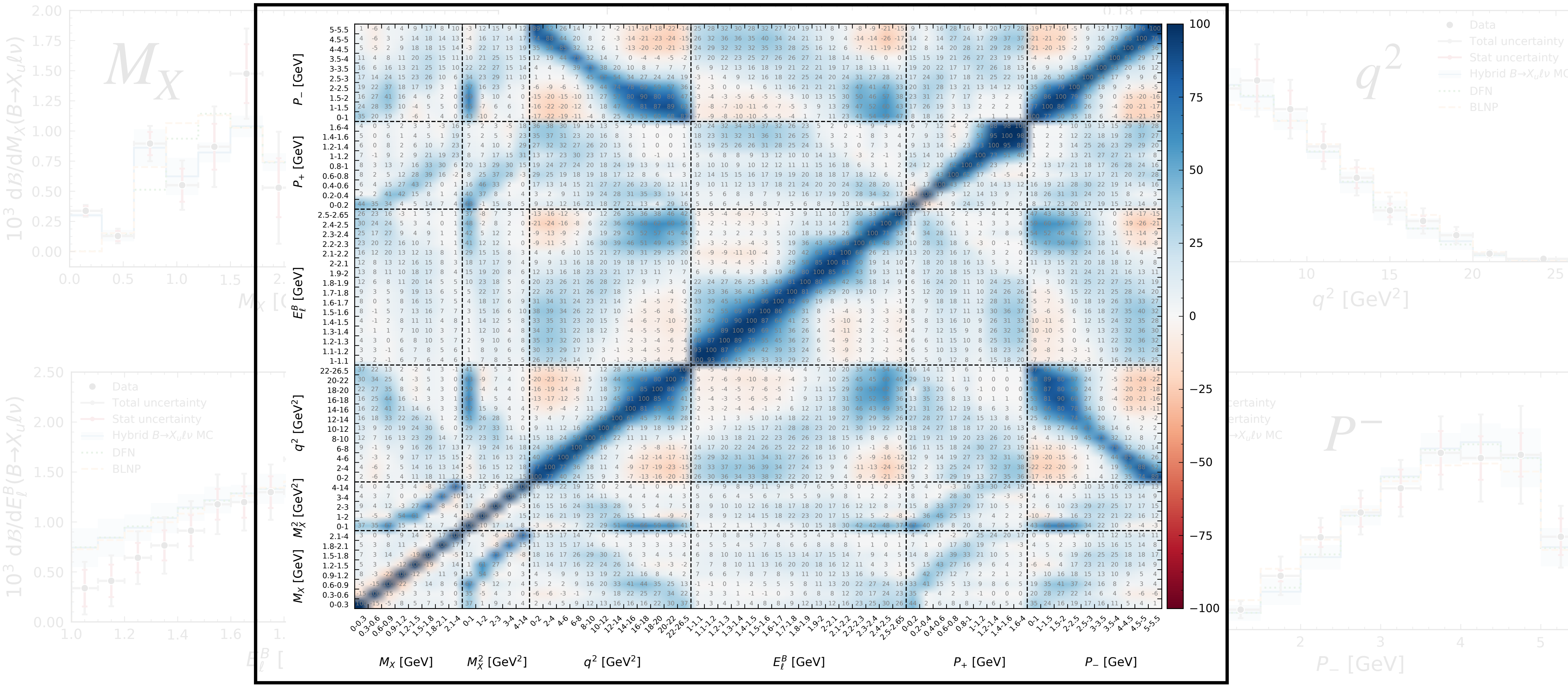


Agreement
(w/o theory uncertainties)

χ^2	E_ℓ^B	M_X	M_X^2	q^2	P_+	P_-
n.d.f.	16	8	5	12	9	10
Hybrid	13.5	2.5	2.6	4.5	1.7	5.2
DFN	16.2	63.2	13.1	18.5	29.3	6.1
BLNP	16.5	61.0	6.3	20.6	23.6	13.7

Differential Spectra

Full experimental correlations



↓

Can be used for future shape-function independent $|V_{ub}|$ determinations

NNVub

P. Gambino, K. Healey, C. Mondino, Phys. Rev. D 94, 014031 (2016), [arXiv:1604.07598]

F. Bernlochner, H. Lacker, Z. Ligeti, I. Stewart, F. Tackmann, K. Tackmann Phys. Rev. Lett. 127, 102001 (2021) [arXiv:2007.04320]

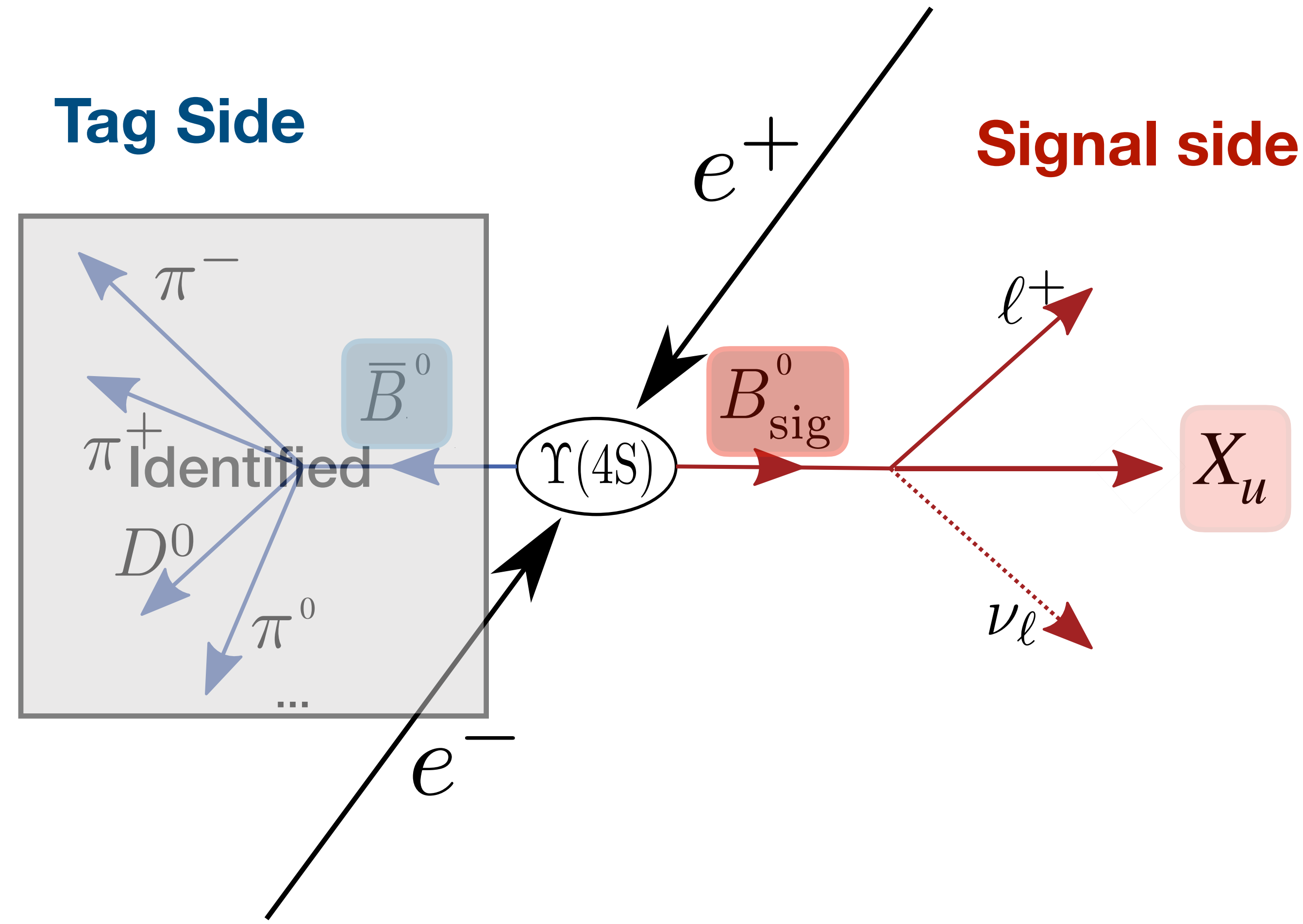
3. New measurement of ratio of inclusive $B \to X_u \ell \bar{\nu}_\ell$ / $B \to X_c \ell \bar{\nu}_\ell$ with improved tagging and data-driven background templates [to appear]

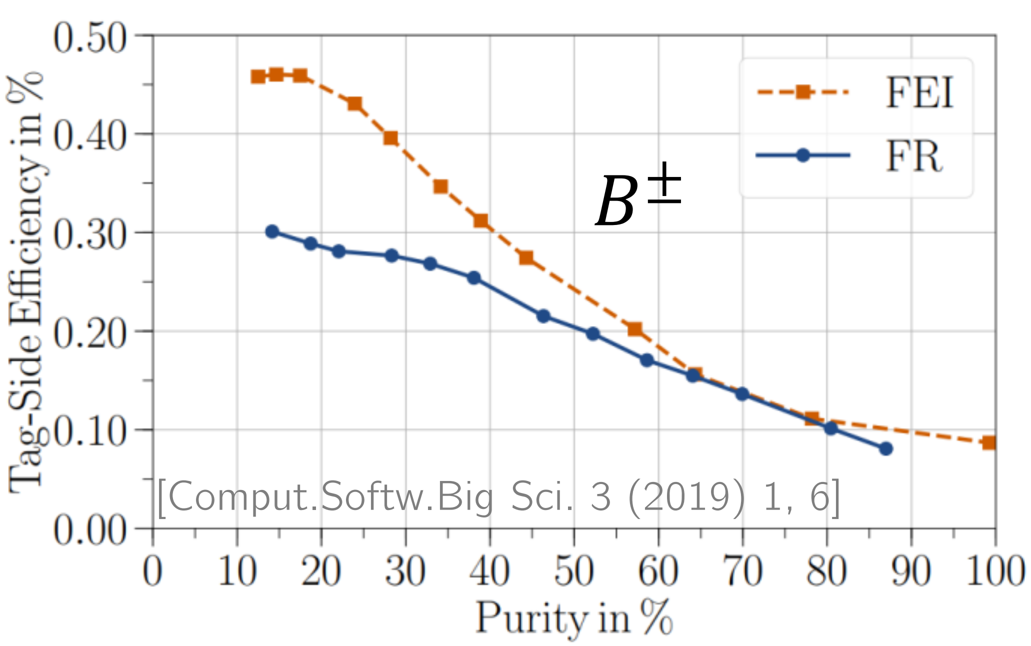
New!

Use **full Belle** data set of **711/fb**

Improved Hadronic Tagging using **Belle II** algorithm (ca. 2 times more efficient)

[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]





Charged Tracks — Neutral Clusters

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$$M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

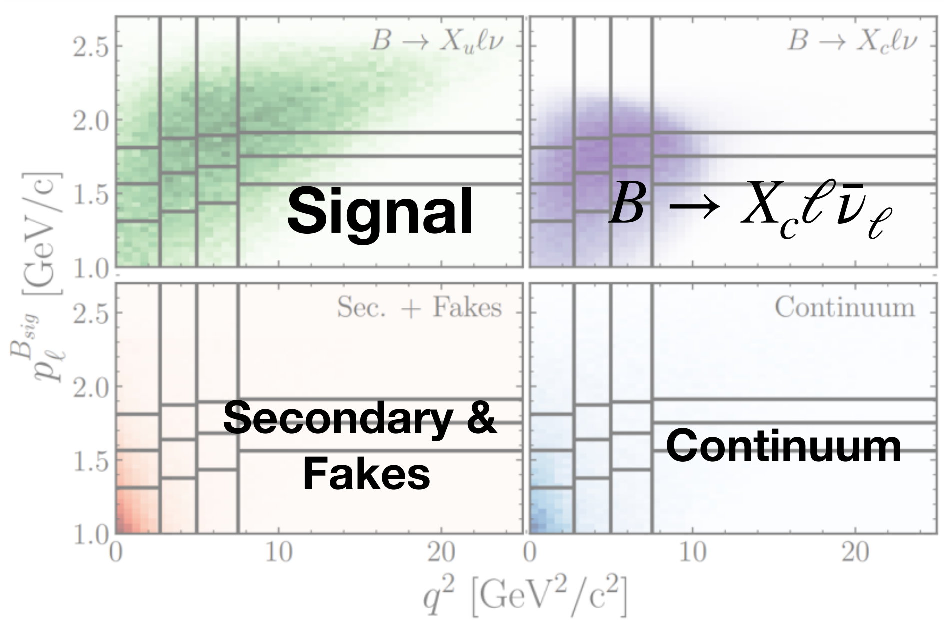
$$m_{\rm miss}^2 = \left(p_{\rm sig} - p_X - p_\ell \right)^2 \approx m_\nu^2 = 0\,{\rm GeV}^2$$

$B \to X_u \ell \bar{\nu}_\ell$ Extraction

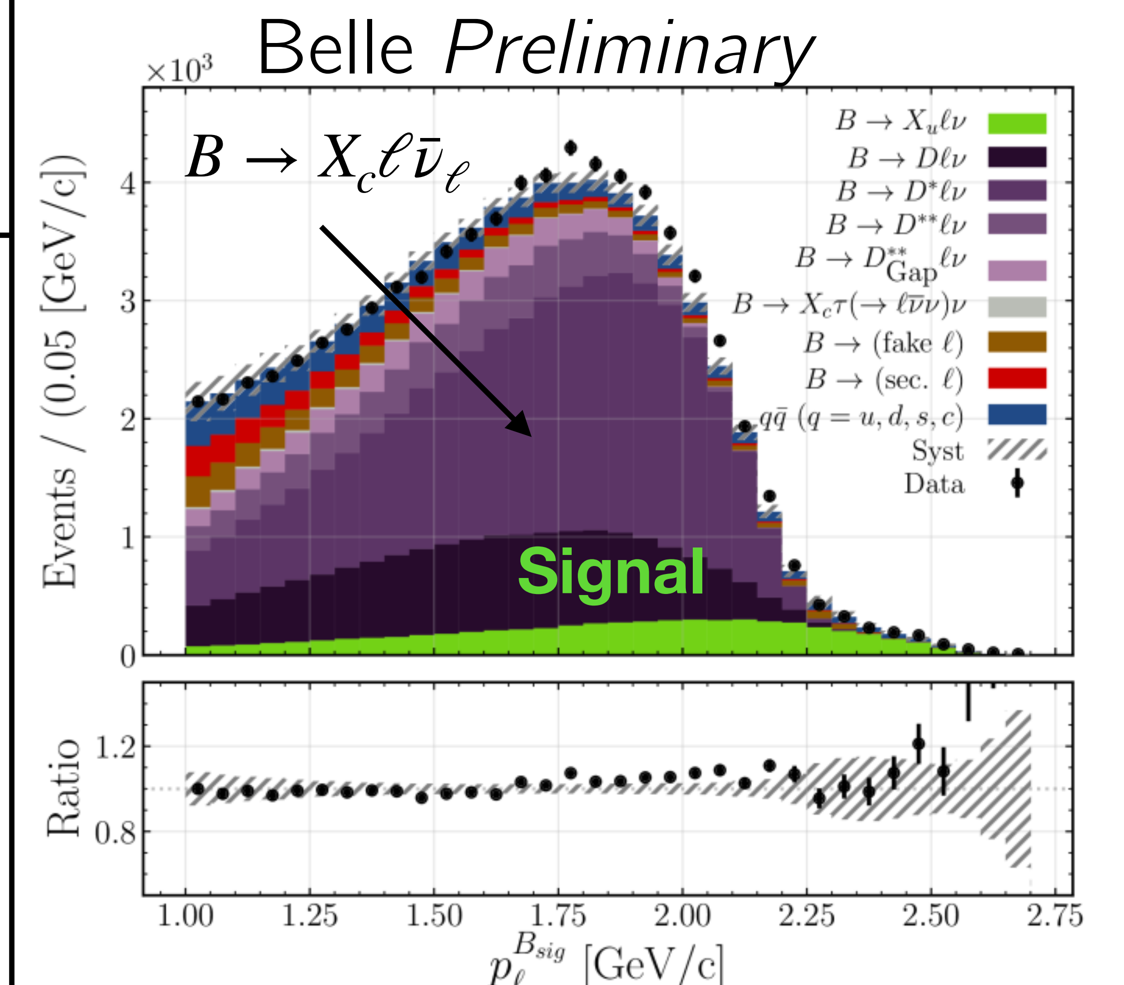
Cut-based selection to suppress $B \to X_c \ell \bar{\nu}_\ell$:

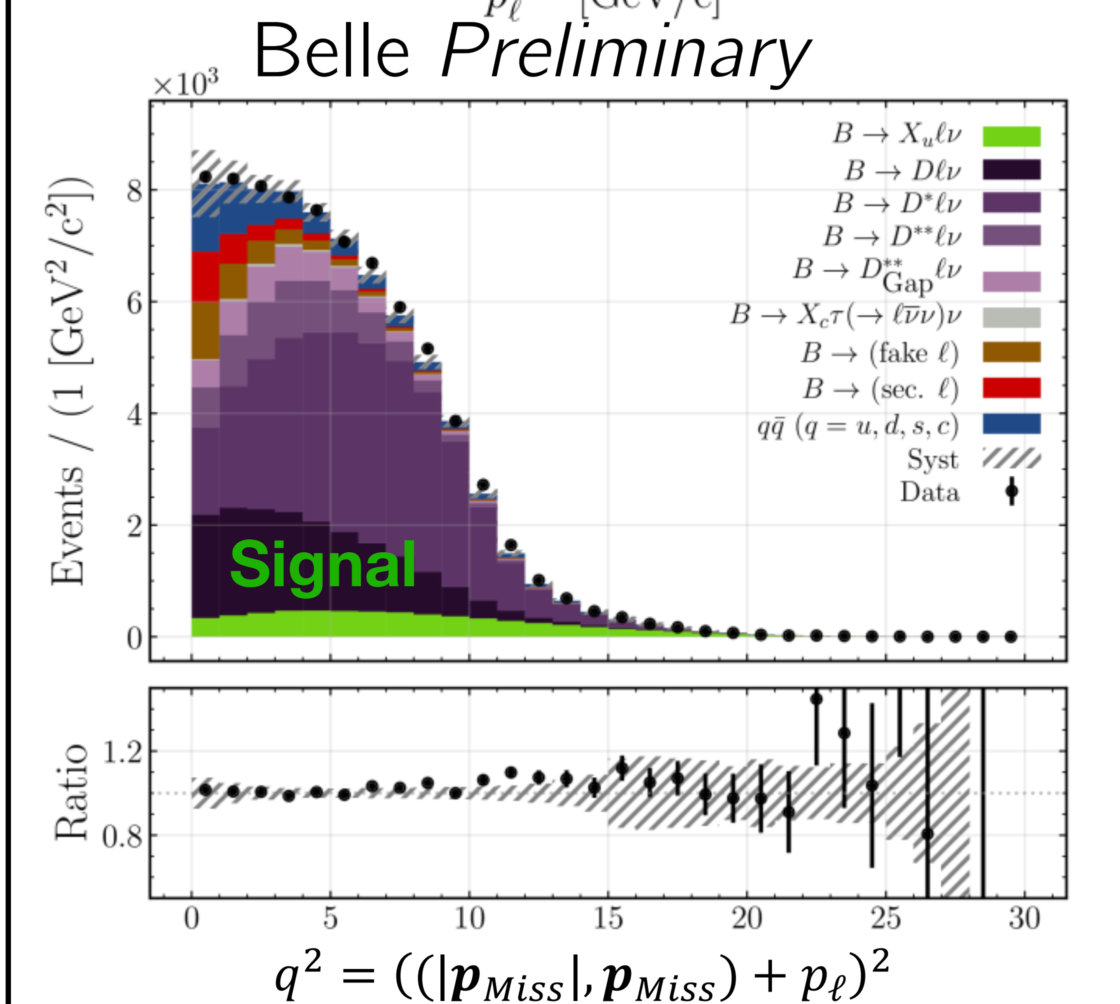
$|m_\nu^2| \approx |m_{Miss}^2| < 0.43\ GeV^2/c^4$
Charged slow pion veto.
Kaon veto: even $N_{K^\pm} + N_{K_S^0}$

Extraction of $B \to X_u \ell \bar{\nu}_\ell$ in 2D fit to $q^2 : p_\ell^B$



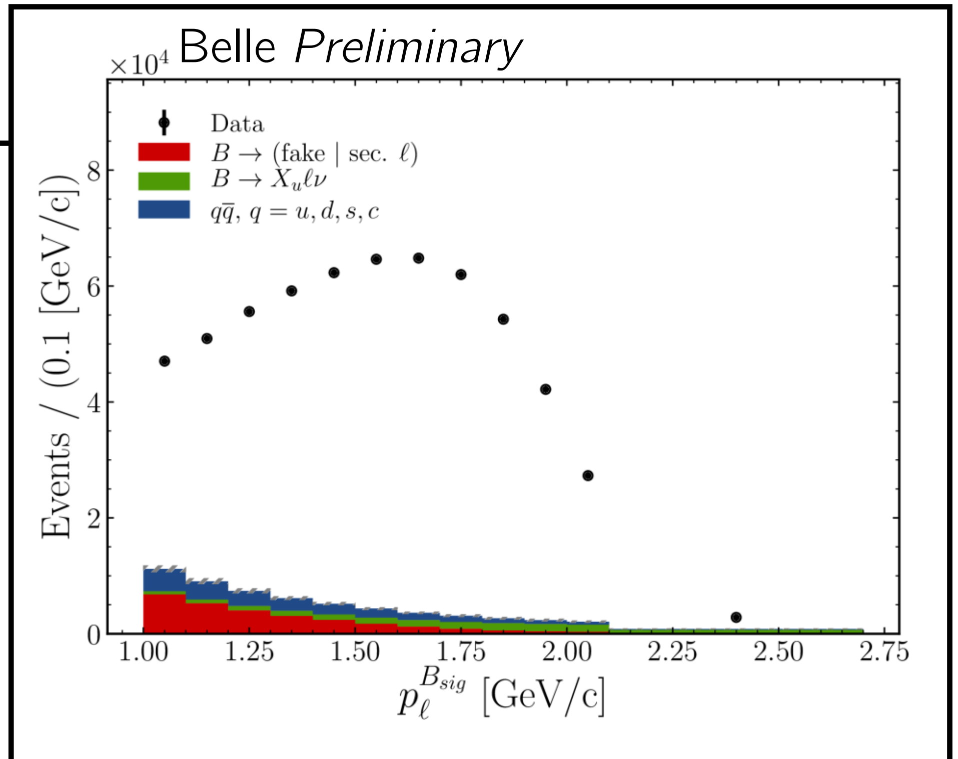
Use $B \to X_c \ell \bar{\nu}_\ell$ shape from **Kaon anti-cut region** with MC based **transfer factors**





$B \to X_u \ell \bar{\nu}_\ell$ / $B \to X_c \ell \bar{\nu}_\ell$ Extraction

Extract $B \to X_c \ell \nu$ yield via simple background subtraction in total $B \to X\ell\nu$ sample.



Determine directly **ratio** of

Belle *Preliminary*

$$\frac{\Delta\mathcal{B}(B \to X_u \ell\nu\colon\ p_\ell^B > 1.0\ GeV/c)}{\Delta\mathcal{B}(B \to X_c \ell\nu\colon\ p_\ell^B > 1.0\ GeV/c)} = 1.95(1 \pm 8.4\%_{stat} \pm 7.2\%_{syst}) \times 10^{-2} \quad \propto \frac{|V_{ub}|^2}{|V_{cb}|^2}$$

Can also convert this for now into a direct determination of $|V_{ub}|$

$$|V_{ub}| = \sqrt{\frac{1}{\tau_B \Delta\Gamma} \frac{\Delta\mathcal{B}(B \to X_u \ell\nu)}{\Delta\mathcal{B}(B \to X_u \ell\nu)} \Delta\mathcal{B}(B \to X_c \ell\nu)}$$

$\tau_B = 1.579 \pm 0.004$ ps

$1.95(1 \pm 0.084 \pm 0.072) \times 10^{-2}$

Belle, 2007 [PRD 75, 032001]: $(8.41 \pm 0.15 \pm 0.17)\%$
Babar, 2010 [PRD 81, 0032003]: $(8.63 \pm 0.17)\%$

Naïve average: $(8.55 \pm 0.13)\%$ - Assume uncorrelated.

$B \to X_u \ell \bar{\nu}_\ell$ / $B \to X_c \ell \bar{\nu}_\ell$ Extraction

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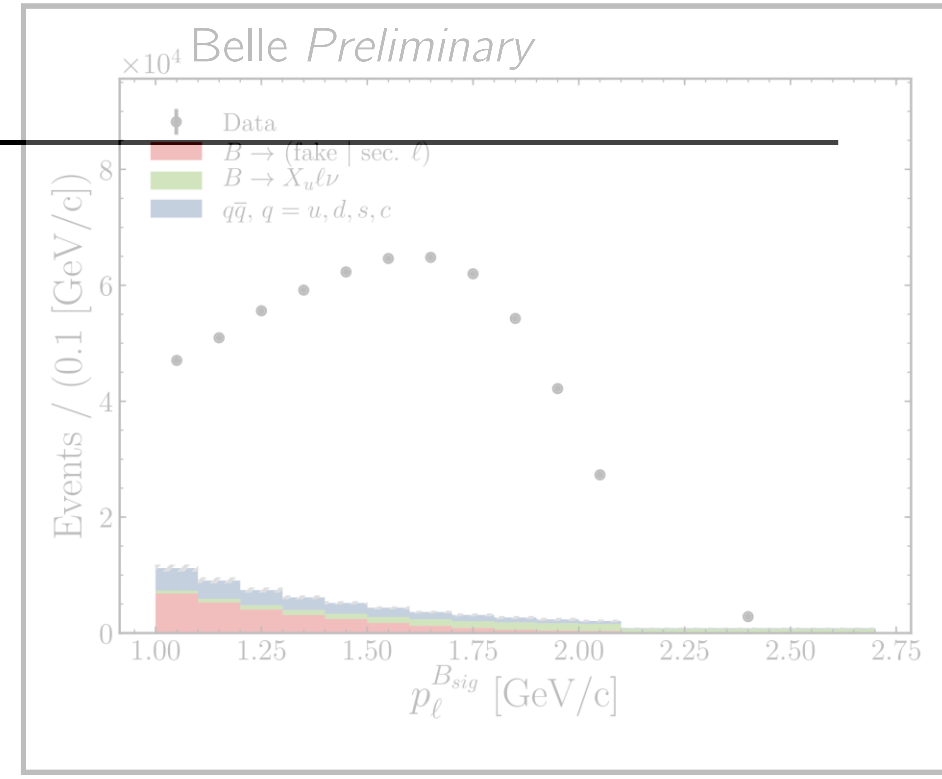

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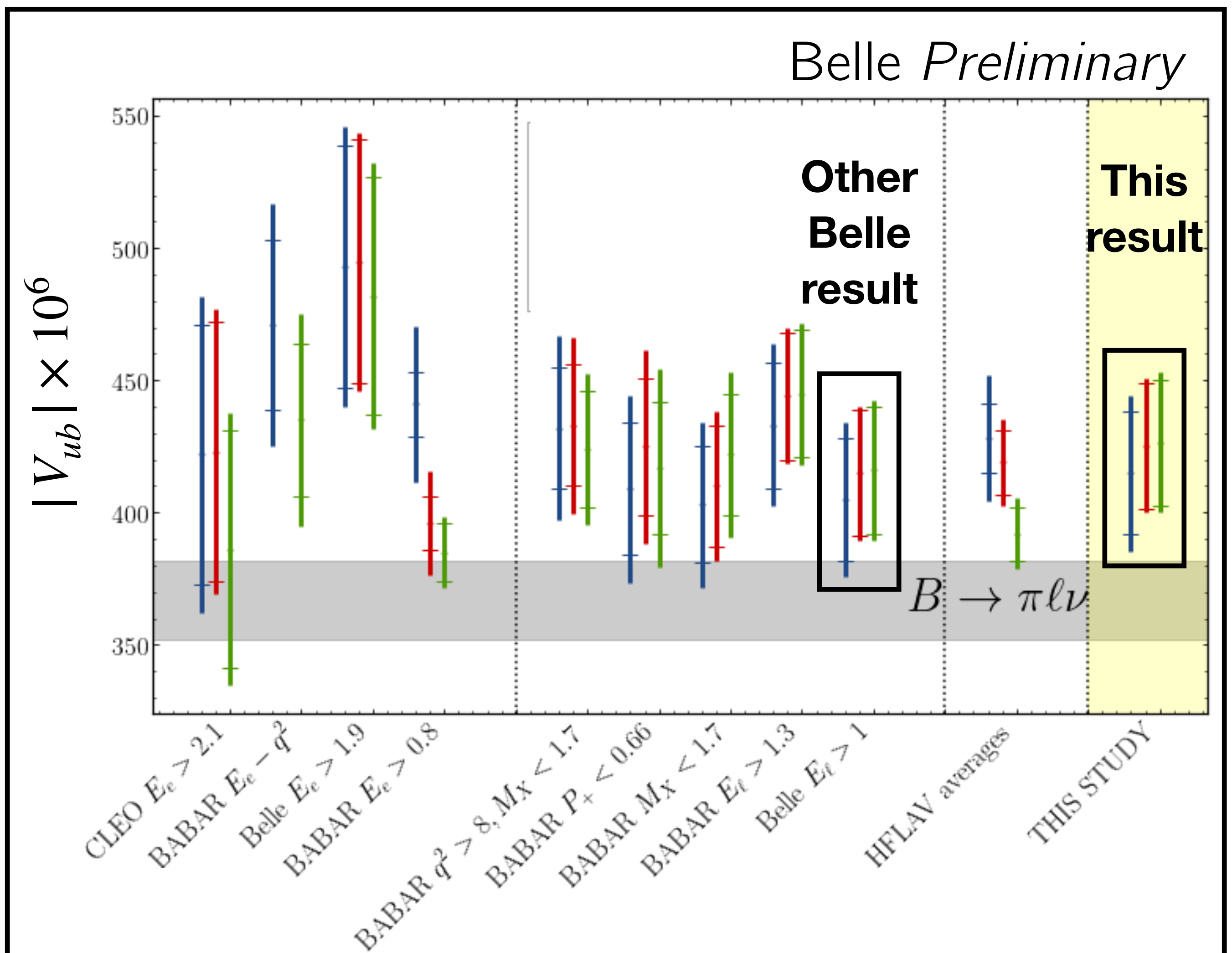


GGOU P. Gambino, P. Giordano, G. Ossola, and N. Uraltsev, JHEP 10, 058 (2007), arXiv:0707.2493 [hep-ph].

BLNP B. O. Lange, M. Neubert, and G. Paz, Phys. Rev. D 72, 073006 (2005), arXiv:hep-ph/0504071.

DGE J. R. Andersen and E. Gardi, JHEP 01, 097 (2006), arXiv:hep-ph/0509360.

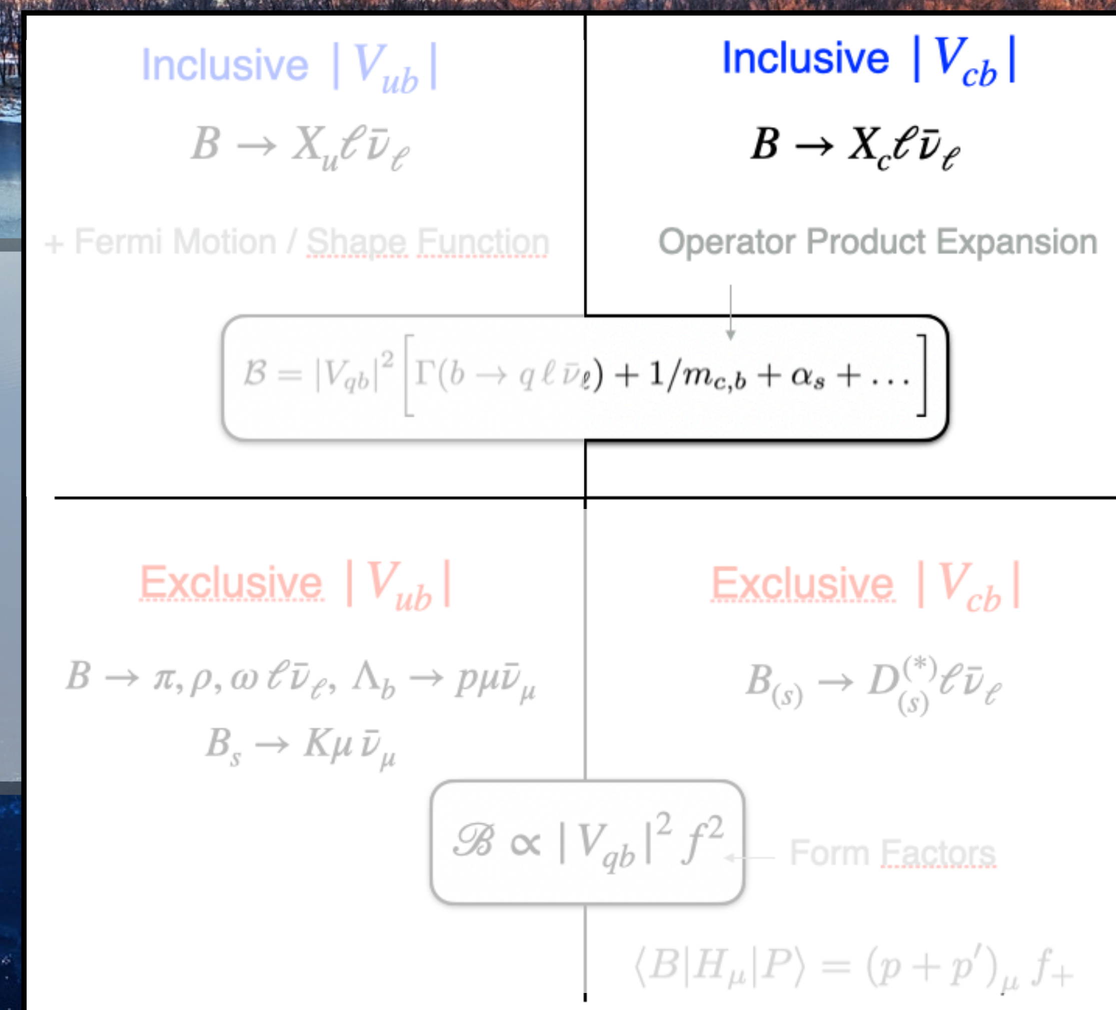
$$|V_{ub}|^{GGOU} = (4.25 \pm 0.18 \pm 0.16 \, {}^{+0.09}_{-0.09}) \times 10^{-3}$$

$$|V_{ub}|^{BLNP} = (4.15 \pm 0.17 \pm 0.15 \, {}^{+0.18}_{-0.20}) \times 10^{-3}$$

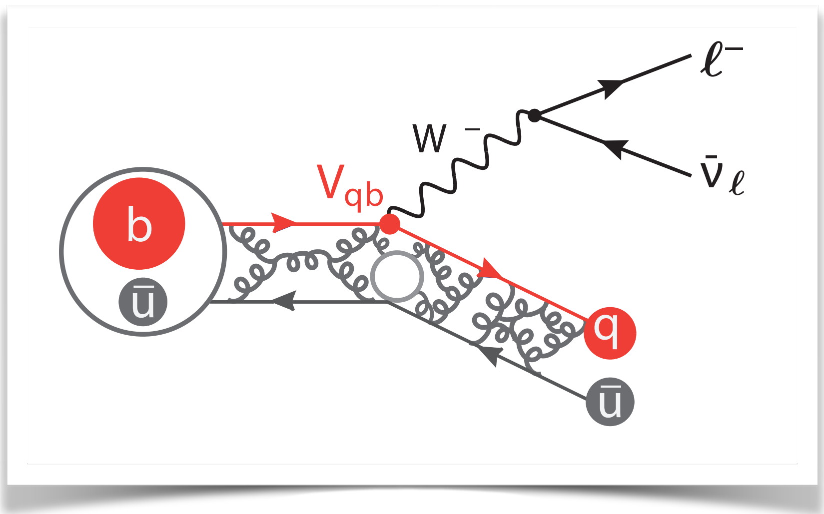
$$|V_{ub}|^{DGE} = (4.26 \pm 0.18 \pm 0.16 \, {}^{+0.11}_{-0.13}) \times 10^{-3}$$

Both Belle results are very compatible with each other

Inclusive $|V_{cb}|$



New Developments in inclusive $|V_{cb}|$



Inclusive $|V_{cb}|$

$$\bar{B} \to X_c\, \ell\, \bar{\nu}_\ell$$

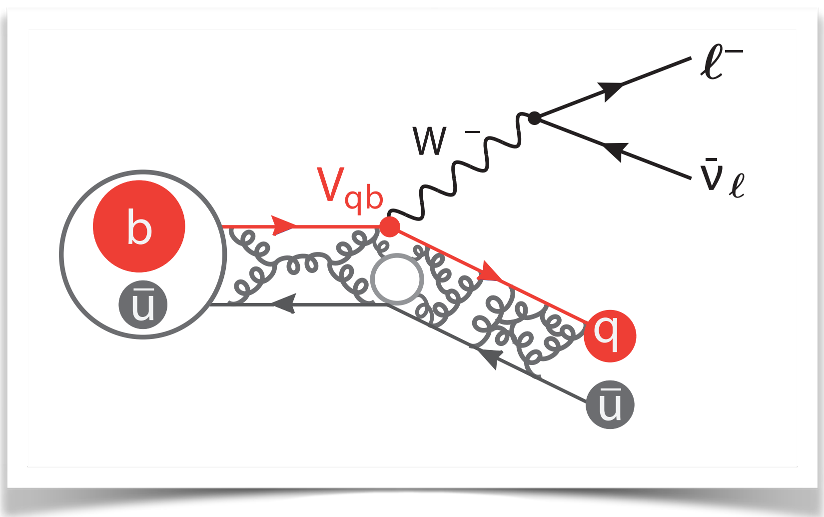
Operator Product Expansion (OPE)

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q\, \ell\, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Established approach: Use hadronic mass moments, lepton energy moments etc. to determine non-perturbative matrix elements (ME) of OPE and extract $|V_{cb}|$

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

New Developments in inclusive $|V_{cb}|$



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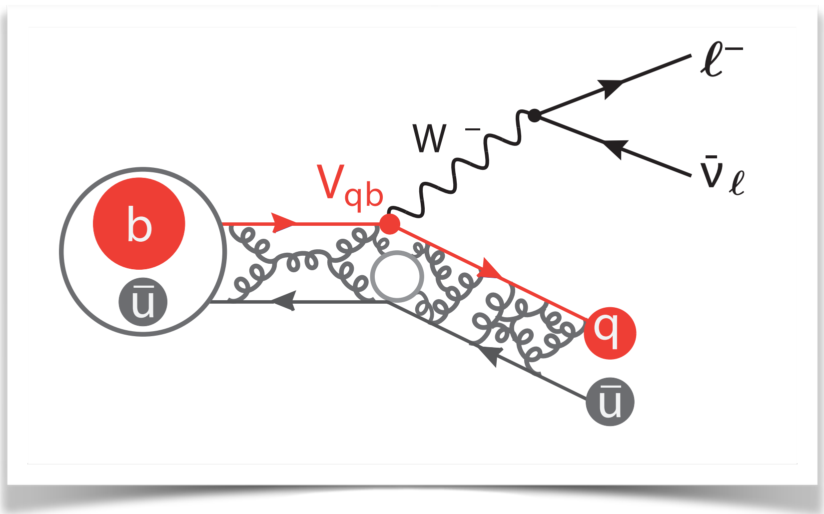
Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

Innovative idea from [JHEP 02 (2019) 177, arXiv:1812.07472] (M. Fael, T. Mannel, K. Vos)

→ Number of ME reduce by exploiting reparametrization invariance, but **not true for every observable** (e.g. not for $\langle M_X \rangle$)

But it **holds** for $\langle q^2 \rangle$ and at $1/m_b^4$ the # of ME reduces from **13 → 8(!)**

New Developments in inclusive $|V_{cb}|$



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Inclusive

1. Measurements of q^2 **moments** of inclusive $B \to X_c \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 112011 (2021), arXiv:2109.01685]

2. Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment [Submitted to PRD, arXiv:2205.06372]

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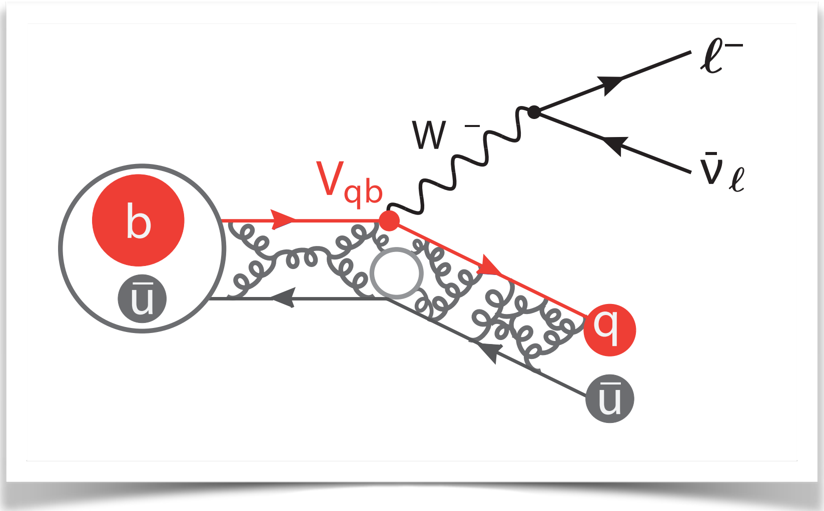

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Traditional approach: Use hadronic mass moments, lepton energy moments etc. to determine non-perturbative matrix elements (ME) of OPE and extract $|V_{cb}|$

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

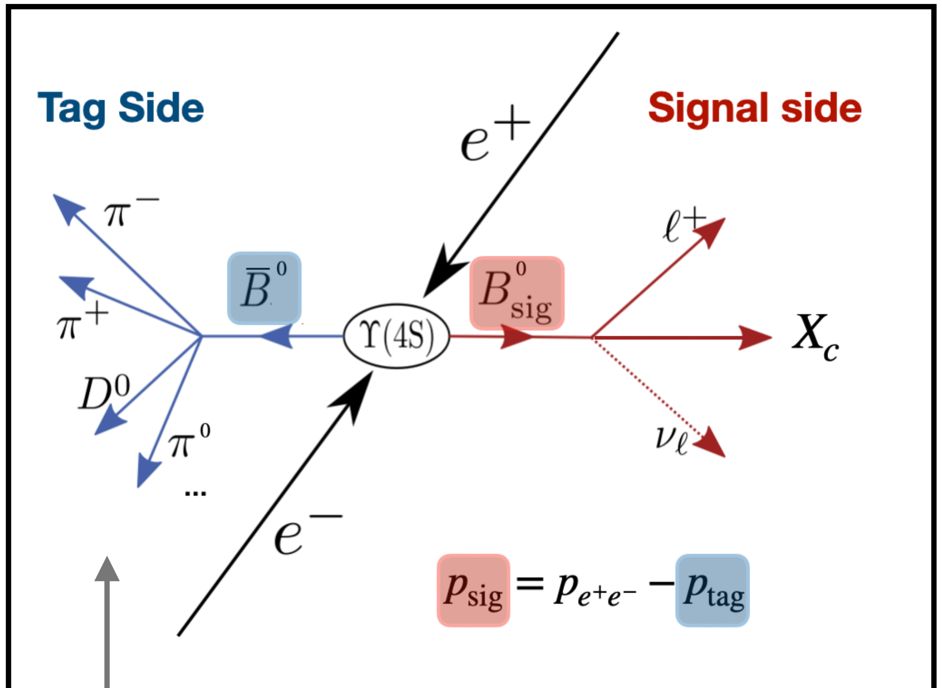
Inclusive

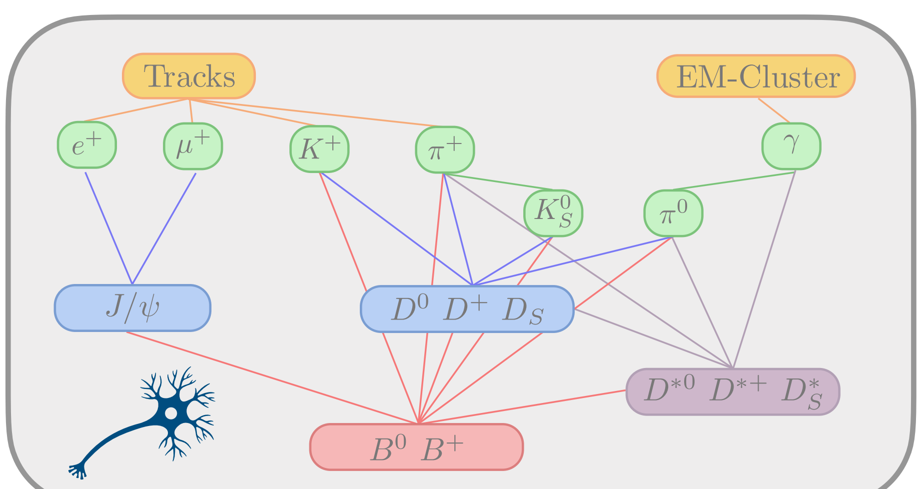
3. Third order correction to the semileptonic $b \to c$ and the muon decays [Phys.Rev.D 104 (2021) 1, 016003, arXiv:2011.13654]
Three loop calculations and inclusive $|V_{cb}|$ [Phys.Lett.B 822 (2021) 136679, arXiv:2107.00604]

4. First determination of V_{cb} from q^2 moments [to appear]

1. Measurements of q^2 **moments** of inclusive $B \to X_c \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 112011 (2021), arXiv:2109.01685]

Key-technique: hadronic tagging

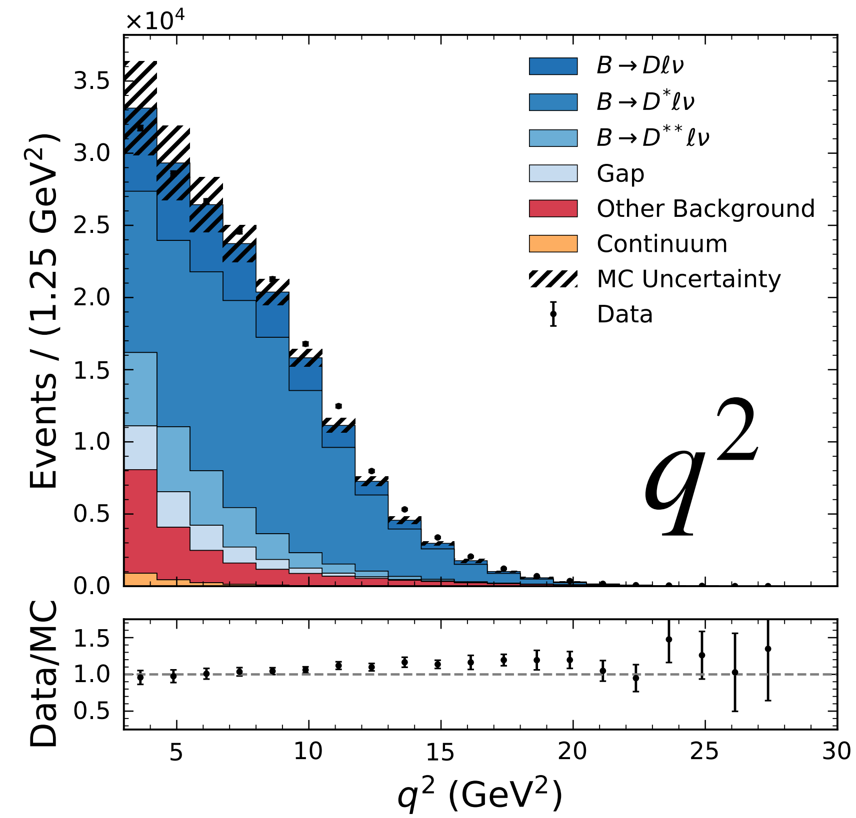


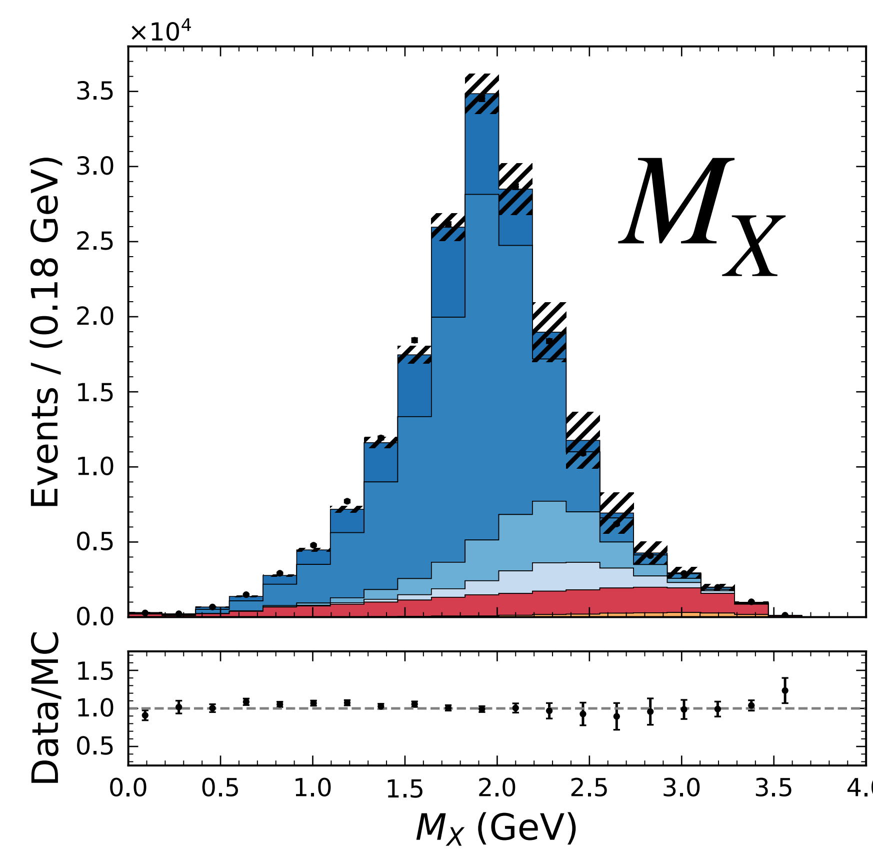


Can identify X$_c$ constituents

$$q^2 = \left(p_{sig} - p_{X_c}\right)^2$$

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$





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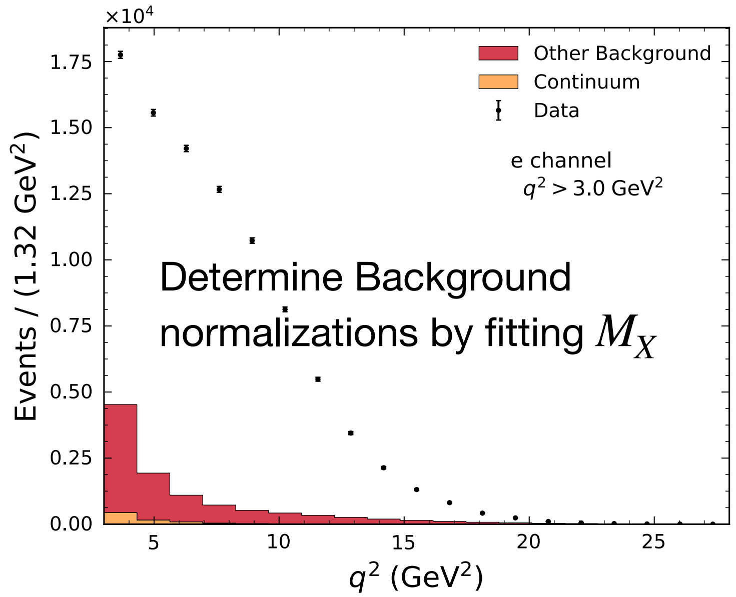
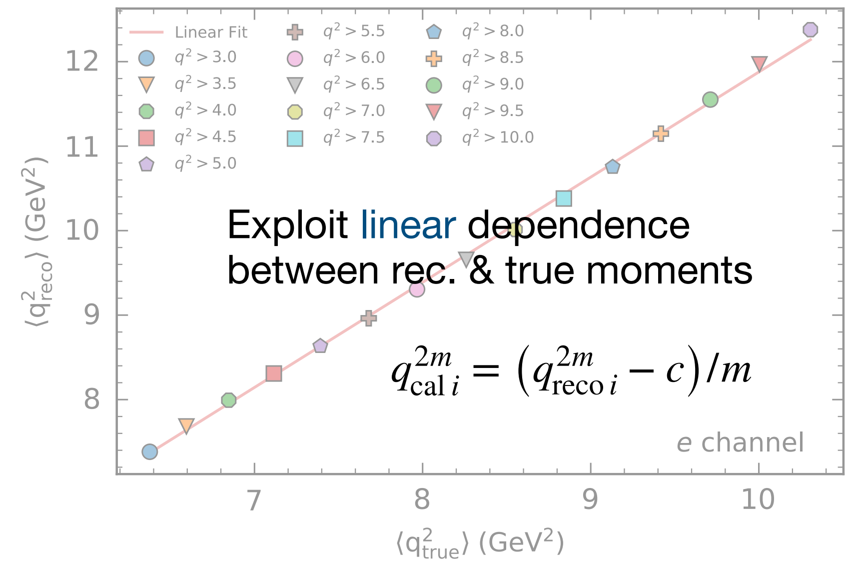




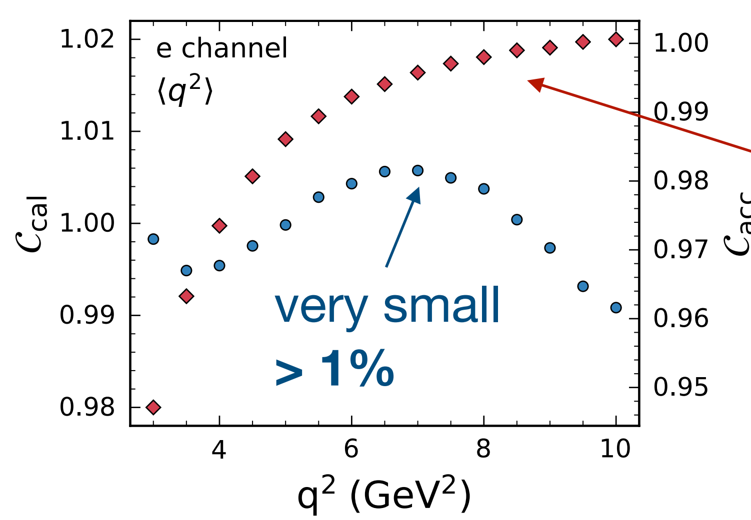
Step #1: Subtract Background

Step #2: Calibrate moment

Event-wise **Master-formula**

$$\langle q^{2m} \rangle = \frac{C_{\mathrm{cal}} \cdot C_{\mathrm{acc}}}{\sum_i^{\mathrm{events}} w(q_i^2)} \times \sum_i^{\mathrm{events}} w(q_i^2) \cdot q^{2m}_{\mathrm{cal}\,i}$$

Step #3: If you fail, try again

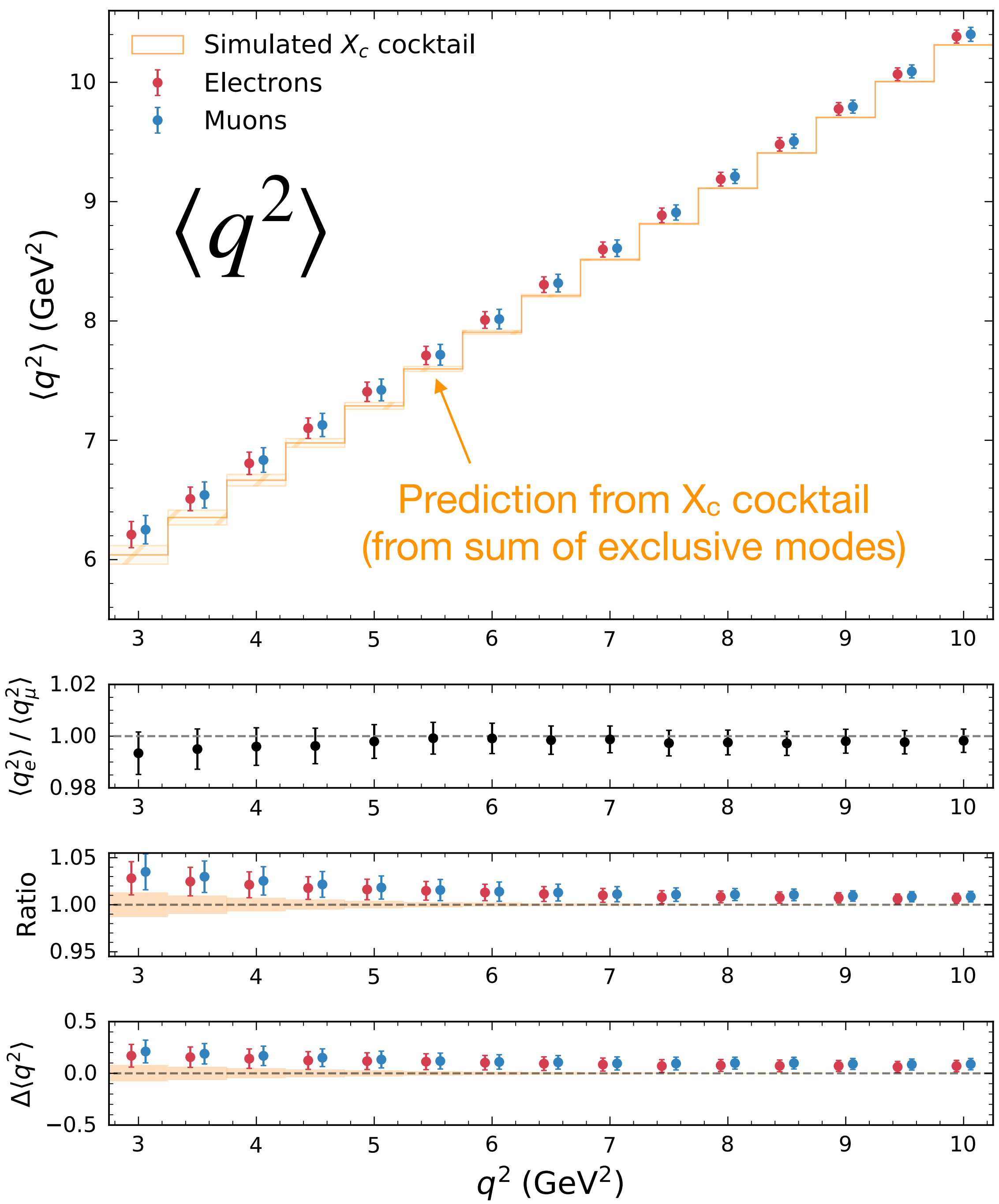


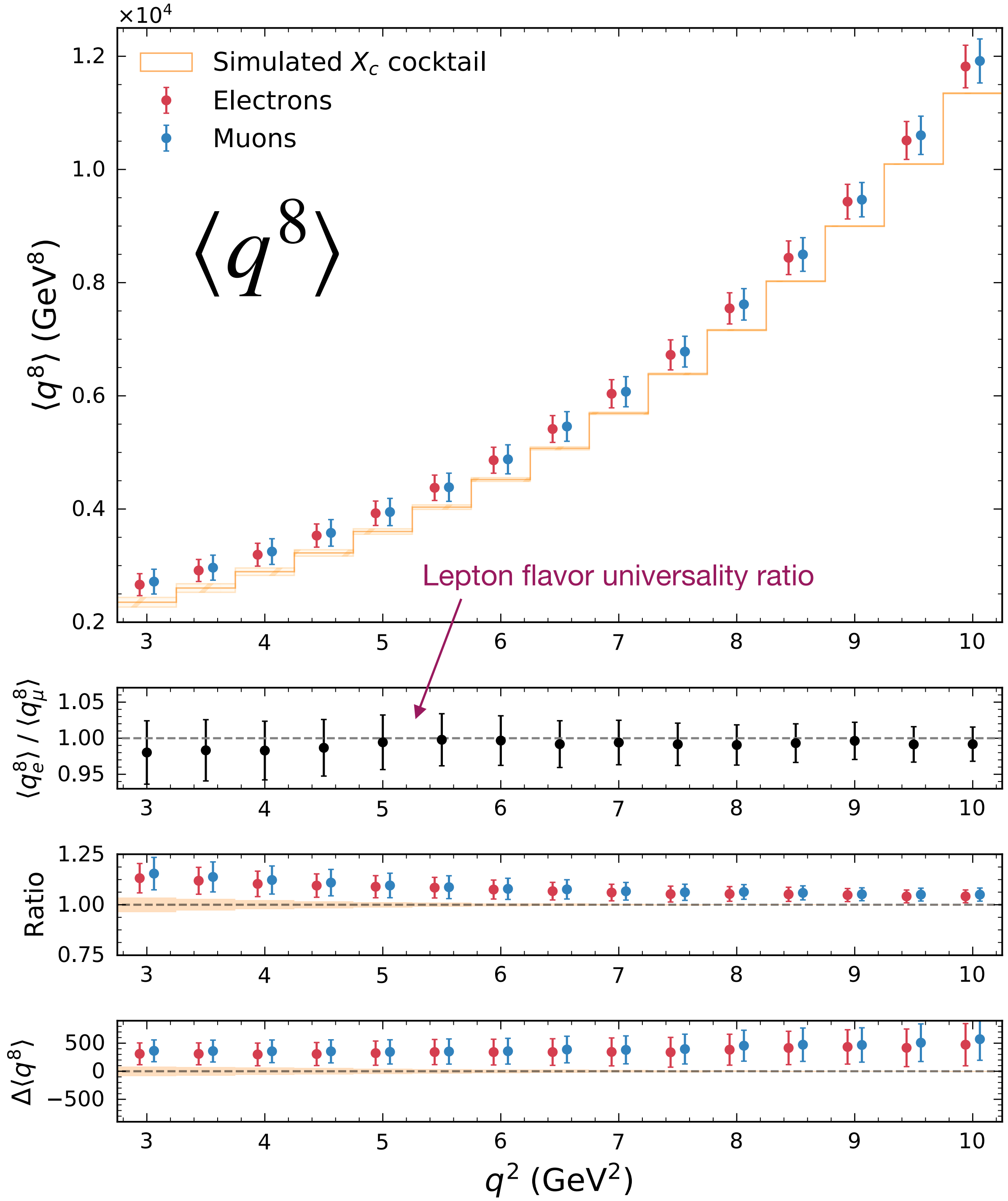
Step #4: Correct for selection effects

Overall event reconstruction itself also **biases** measured moment by **1-2%**

1.

Measurements of q^2 **moments** of inclusive $B \to X_c \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 112011 (2021), arXiv:2109.01685]





2. Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment [Submitted to PRD, arXiv:2205.06372]

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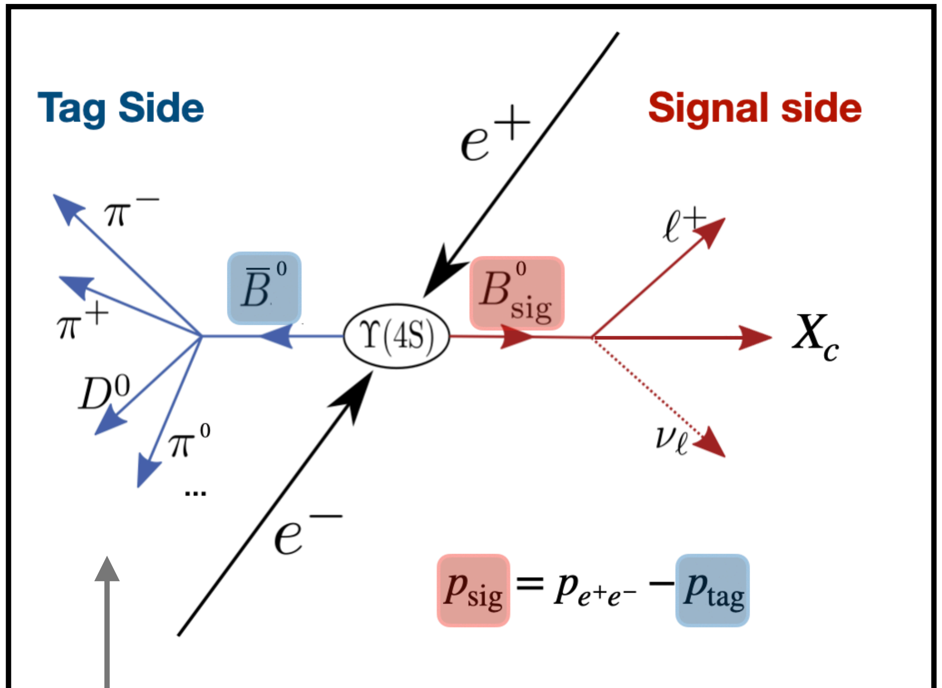

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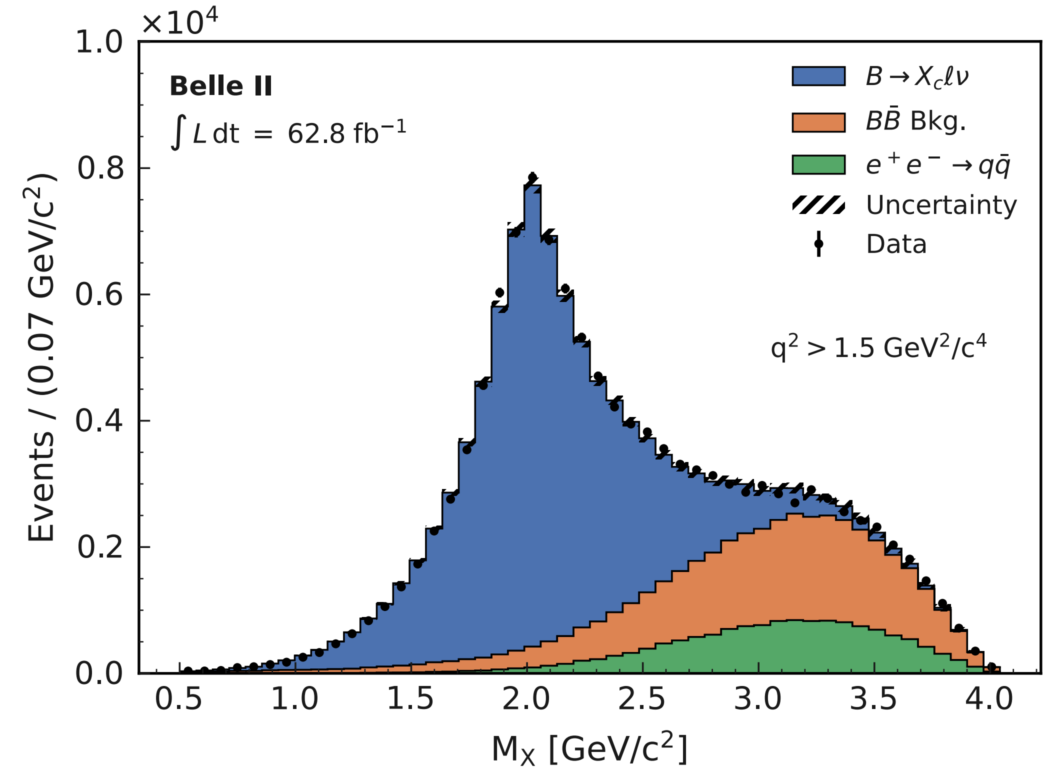

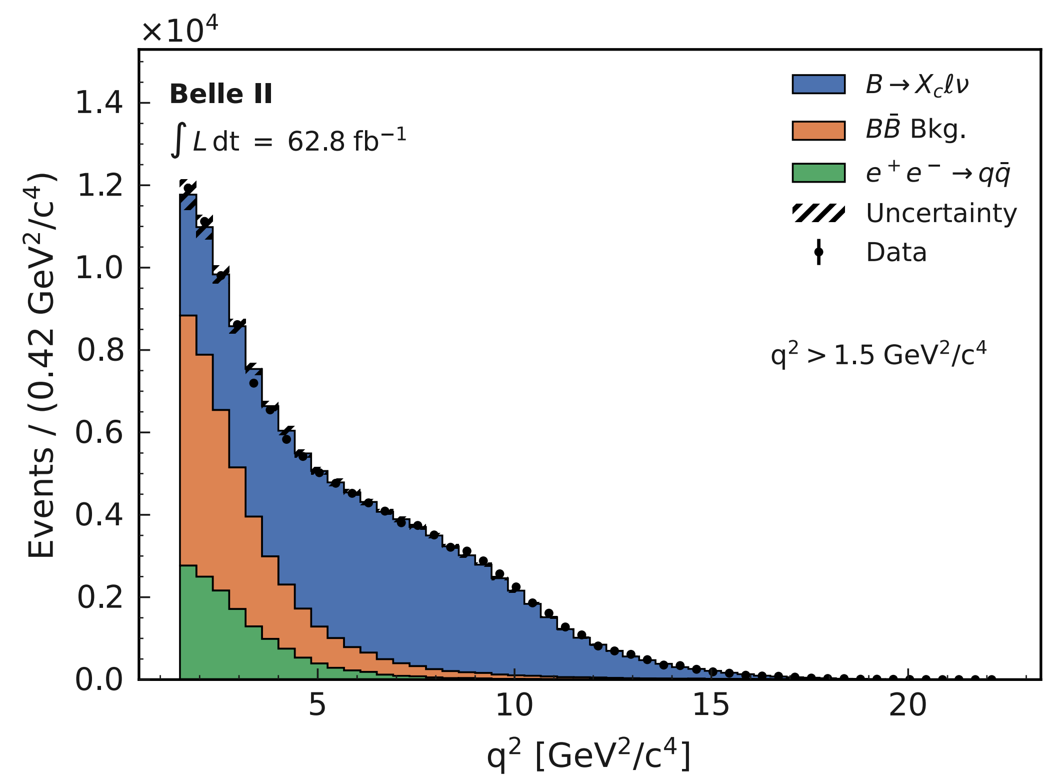

Improved Hadronic Tagging
using **Belle II** algorithm
(ca. 2 times more efficient)

[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]

2. Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment [Submitted to PRD, arXiv:2205.06372]

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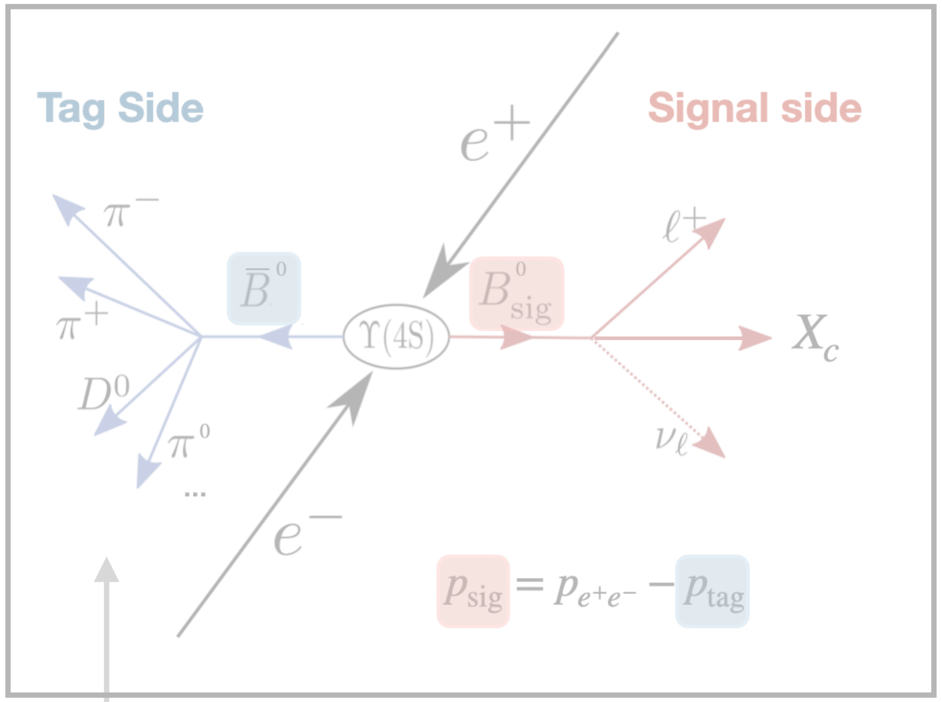

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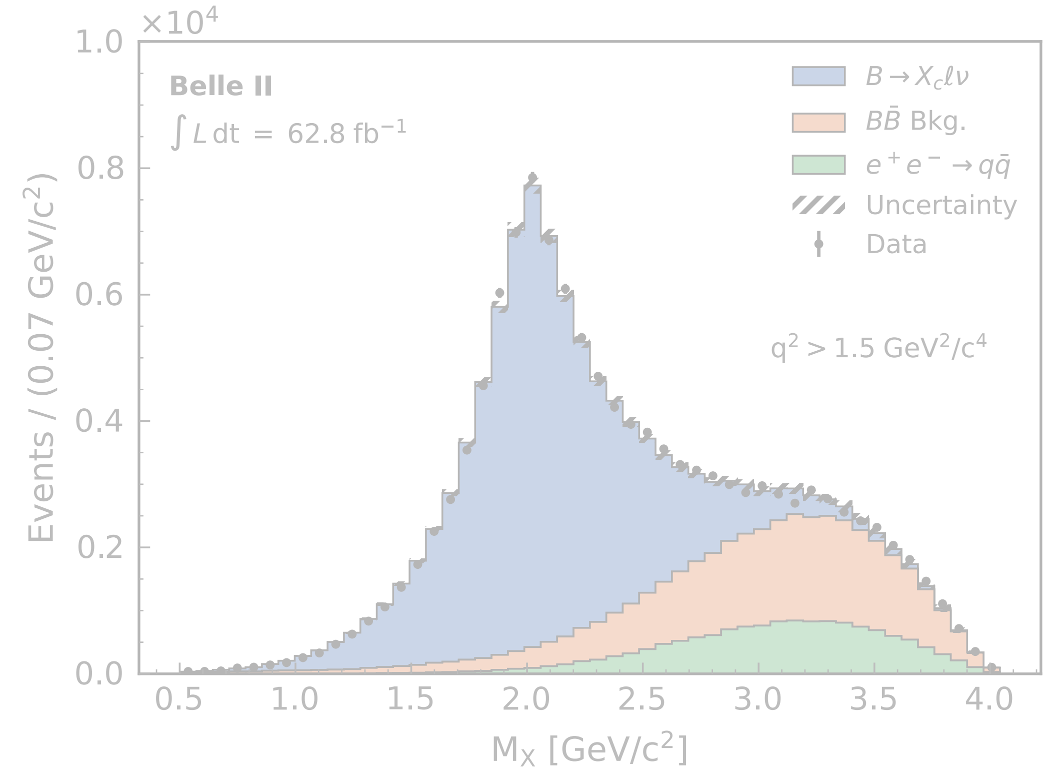
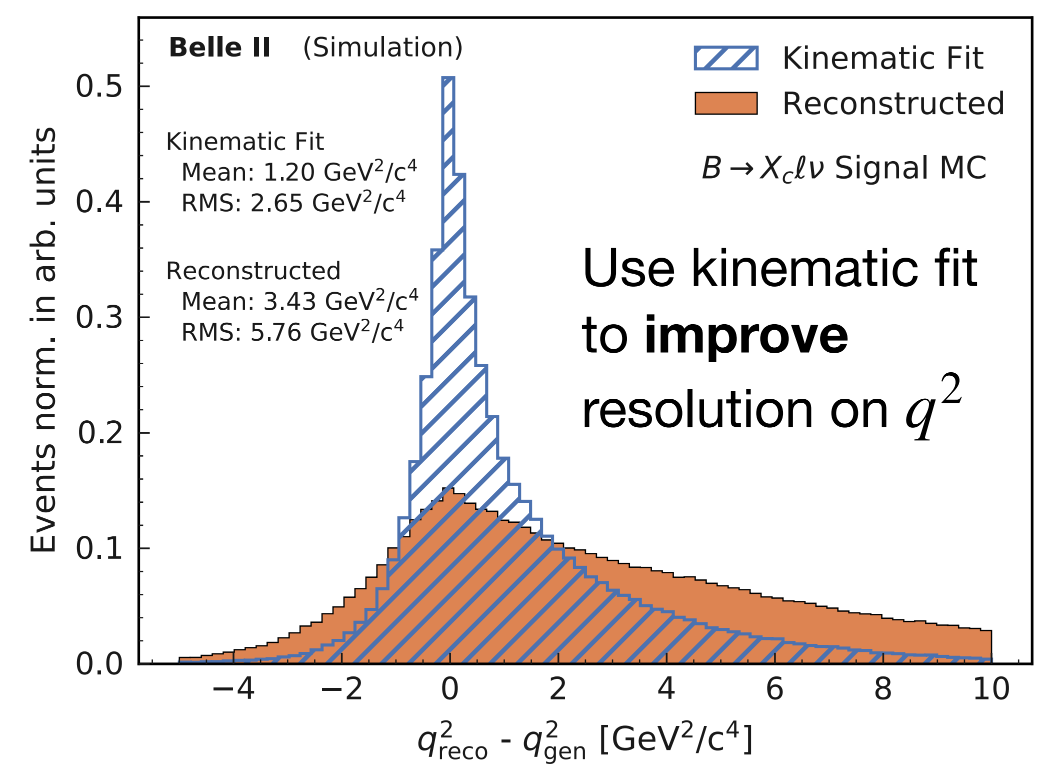
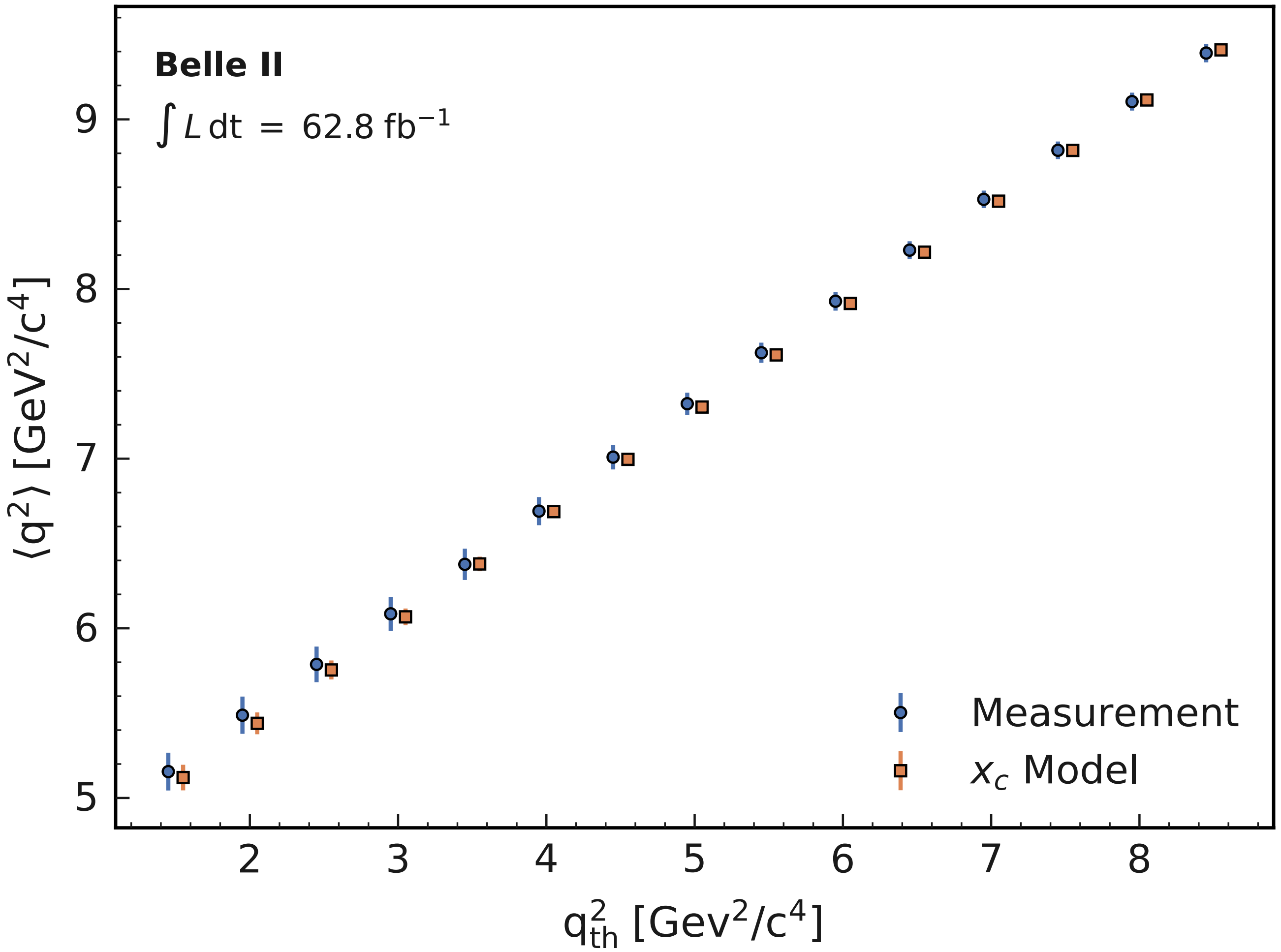
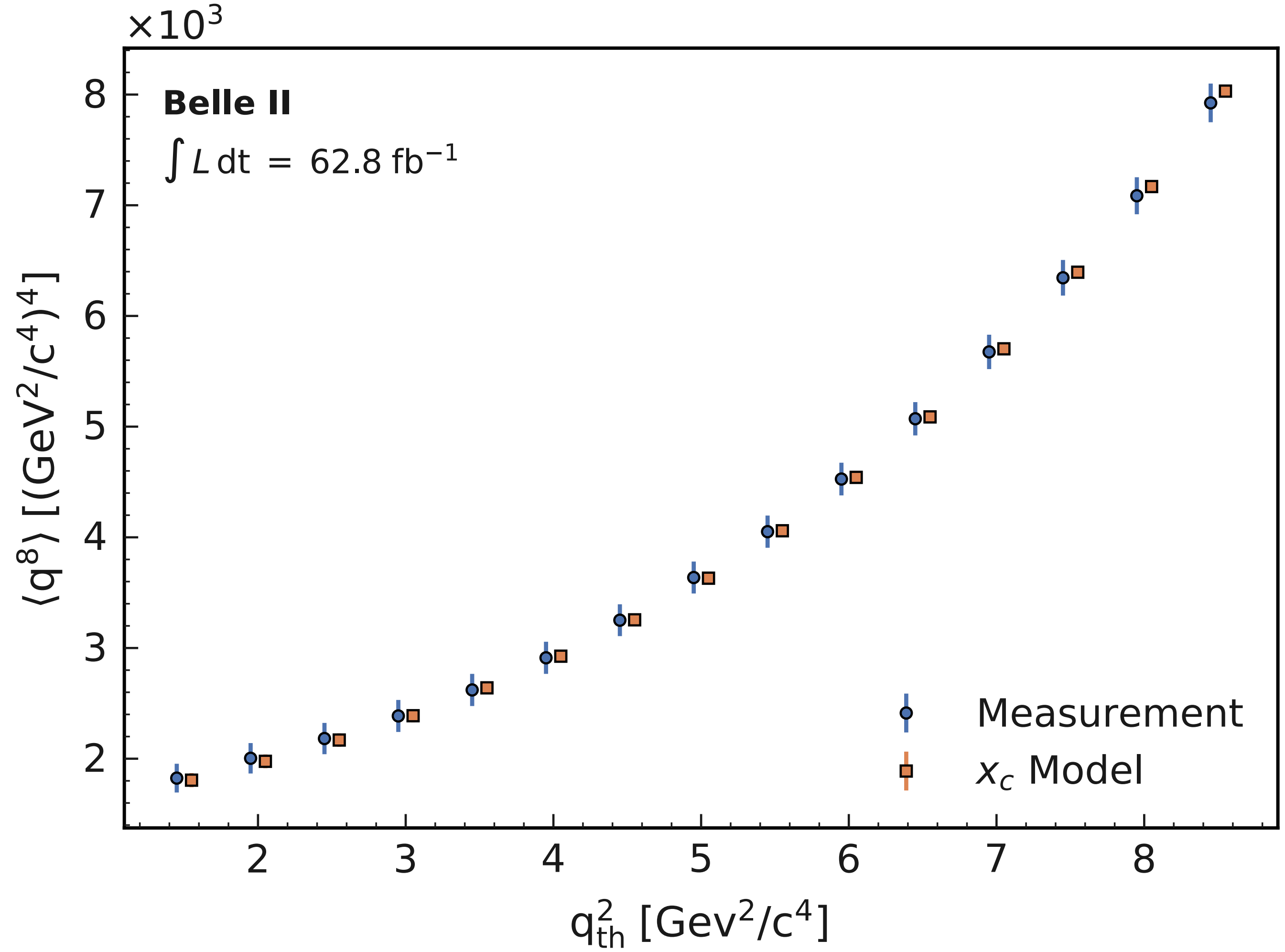


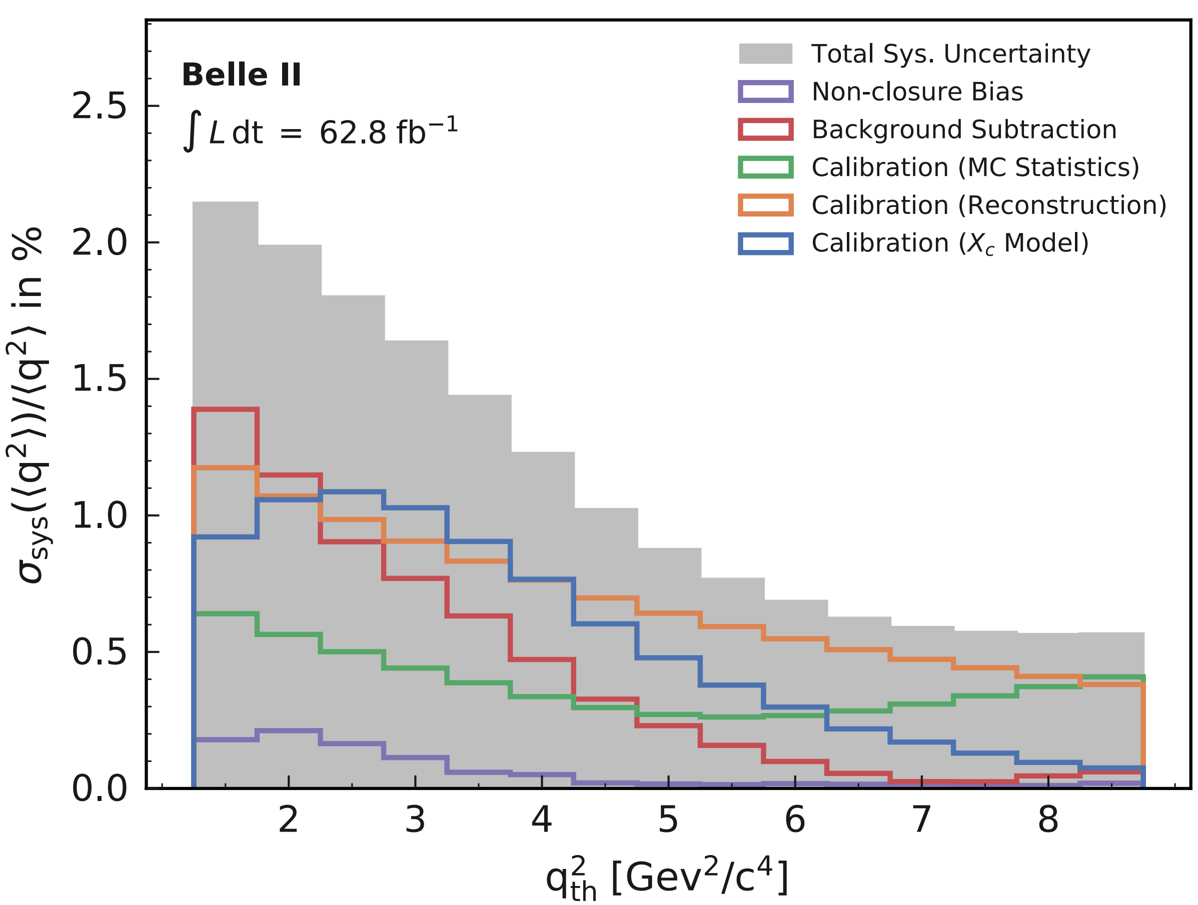
$$q^2 = \left(p_{\rm sig} - p_{X_c}\right)^2$$



Use kinematic fit to **improve** resolution on q^2



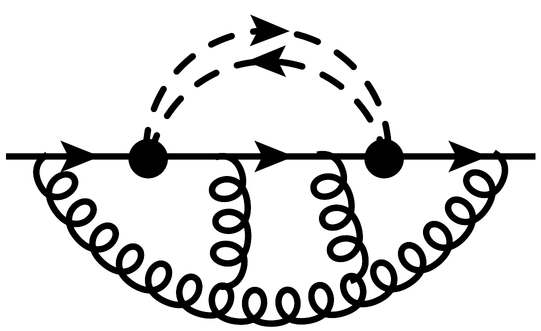




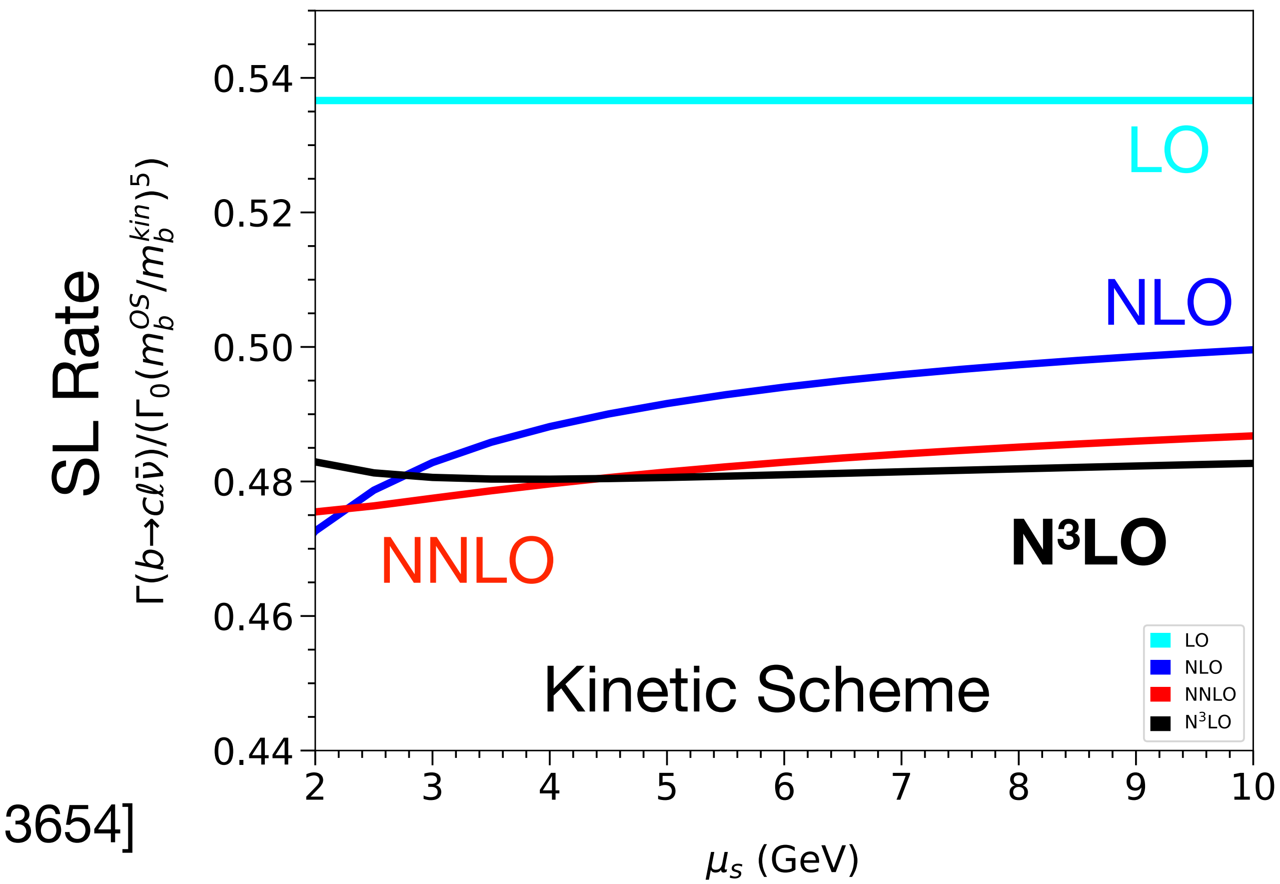
Belle II already reaches **similar precision** to **Belle** and we could lower q^2 threshold to $1.5\,\mathrm{GeV}^2$

Theory progress

Fantastic progress on the theory side:
semileptonic rate @ **N³LO**!



M. Fael, K. Schönwald, M. Steinhauser
[Phys.Rev.D 104 (2021) 1, 016003, arXiv:2011.13654]



Renormalization scale

Updated inclusive fit to $\langle E_\ell \rangle$, $\langle M_X \rangle$ moments:

$$|V_{cb}| = 42.16(30)_{th}(32)_{exp}(25)_\Gamma \ 10^{-3}$$

$$\searrow \ \Delta|V_{cb}|/|V_{cb}| = 1.2\,\%\,!$$

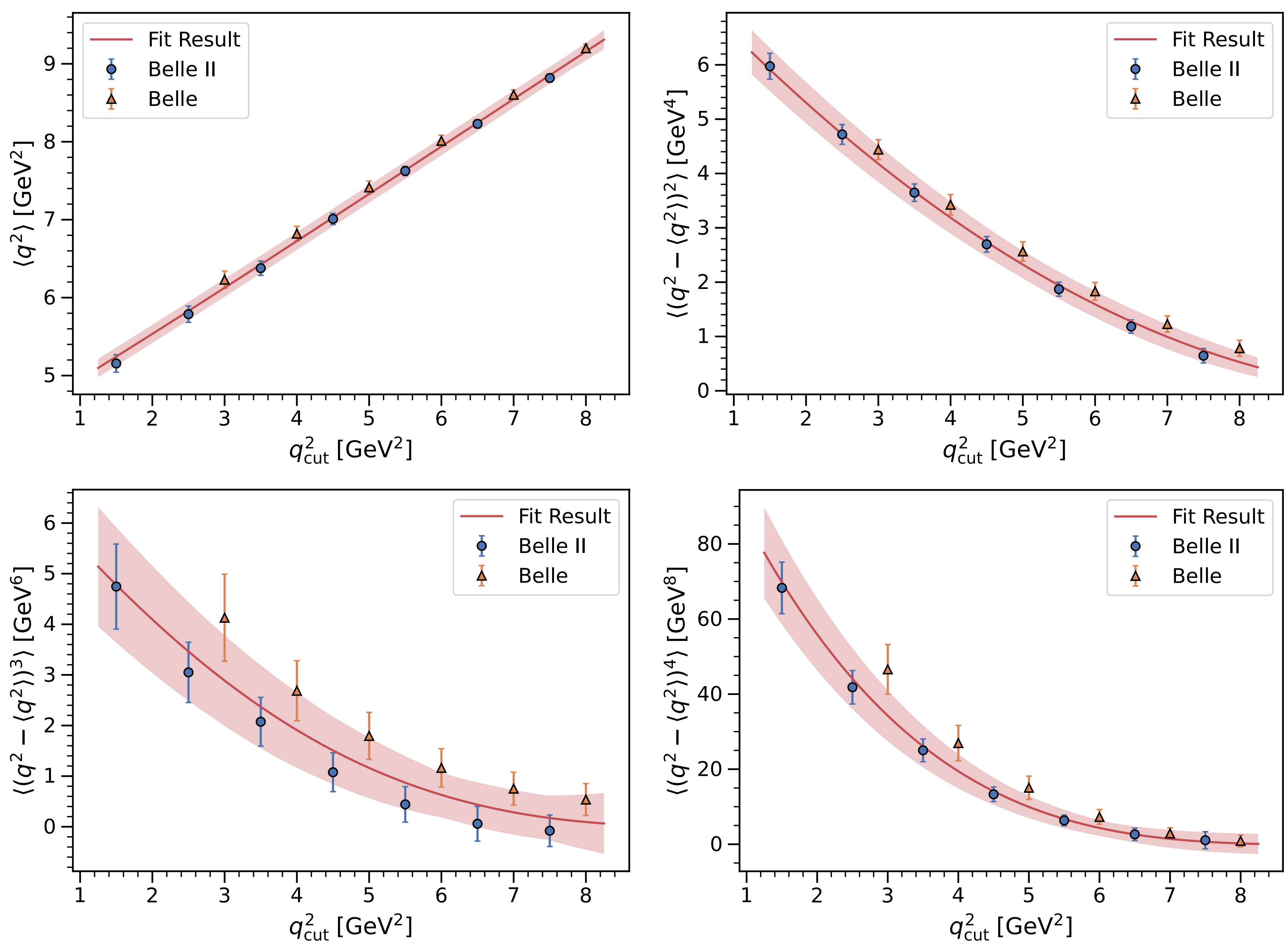
M. Bordone, B. Capdevila, P. Gambino
[Phys.Lett.B 822 (2021) 136679, arXiv:2107.00604]

m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_G^2(m_b)$	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51
1	0.307	-0.141	0.047	0.612	-0.196	-0.064	-0.420
	1	0.018	-0.010	-0.162	0.048	0.028	0.061
		1	0.735	-0.054	0.067	0.172	0.429
			1	-0.157	-0.149	0.091	0.299
				1	0.001	0.013	-0.225
					1	-0.033	-0.005
						1	0.684
							1

$|V_{cb}|$ from q^2 mom.

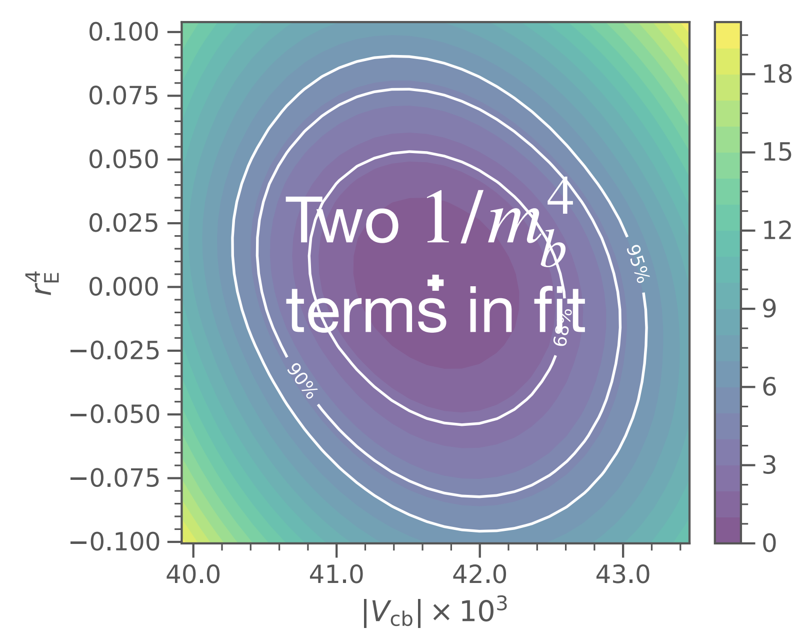
F. Bernlochner, M. Fael, K. Olschwesky, E. Persson, R. Van Tonder, K. Vos, M. Welsch [arXiv:2205.10274]

Also first extraction of $|V_{cb}|$ from q^2 moments:



Included corrections on the mom. predictions

$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			



$$|V_{cb}| = (41.69 \pm 0.59|_{\rm fit} \pm 0.23|_{\rm h.o.}) \cdot 10^{-3} = (41.69 \pm 0.63) \cdot 10^{-3}$$

Exclusive $|V_{cb}|$

Inclusive $|V_{ub}|$

$B \to X_u \ell \bar{\nu}_\ell$

+ Fermi Motion / Shape Function

Inclusive $|V_{cb}|$

$B \to X_c \ell \bar{\nu}_\ell$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q\, \ell\, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Exclusive $|V_{ub}|$

$B \to \pi, \rho, \omega\, \ell \bar{\nu}_\ell,\ \Lambda_b \to p \mu \bar{\nu}_\mu$

$B_s \to K \mu\, \bar{\nu}_\mu$

Exclusive $|V_{cb}|$

$B_{(s)} \to D^{(*)}_{(s)} \ell \bar{\nu}_\ell$

$$\mathscr{B} \propto |V_{qb}|^2 f^2$$

Form Factors

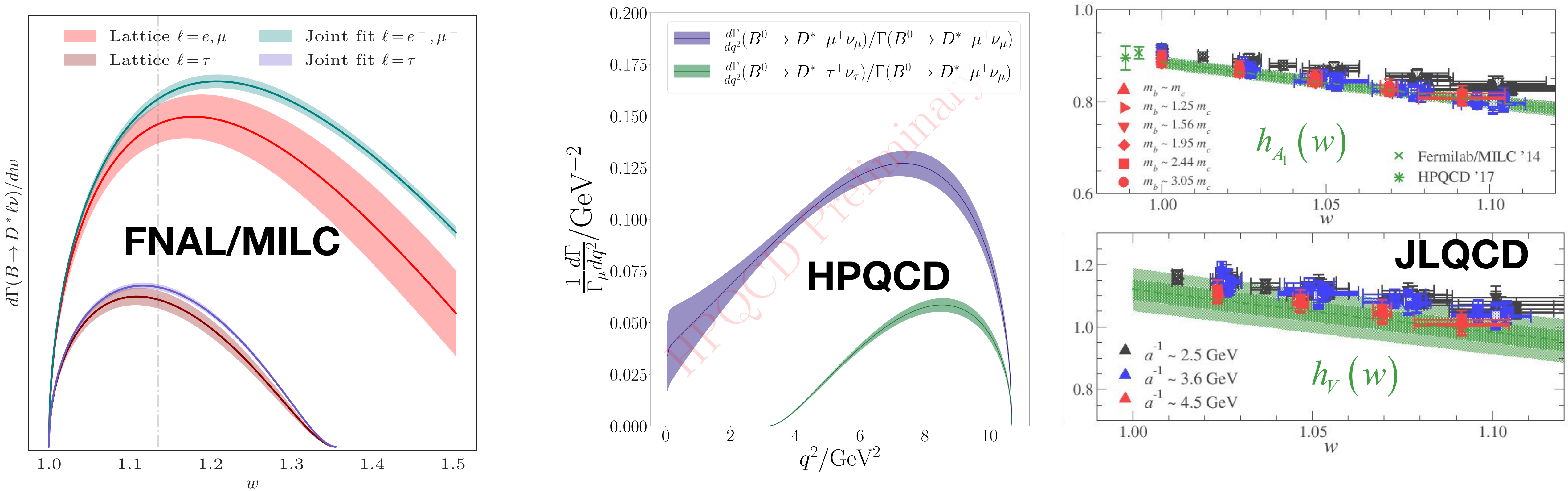
$$\langle B | H_\mu | P \rangle = (p + p')_\mu\, f_+$$

New Developments in exclusive $|V_{cb}|$

Very exciting times:

After more than 10 years in the making, we have first beyond zero recoil LQCD predictions beyond zero recoil for $B \to D^* \ell \bar{\nu}_\ell$:-)

One is finished, two are nearly finished:



A. Bazavov et al. [FNAL/MILC] [Under Review, arXiv:2105.14019]

New Developments in exclusive $|V_{cb}|$

Also experimentally very exciting times:

LHCb keeps producing impressive results probing $B_s \to D_s^{(*)} \ell \bar{\nu}_\ell$ decays,
Belle II also presented first determinations of $|V_{cb}|$ using $B \to D^* \ell \bar{\nu}_\ell$

Small taste of what there is to come from both experiments !

Exclusive

1. Measurement of $|V_{cb}|$ with $B_s \to D_s^{(*)} \mu \bar{\nu}_\mu$ decays
[Phys. Rev. D **101**, 072004, arXiv:2001.03225]

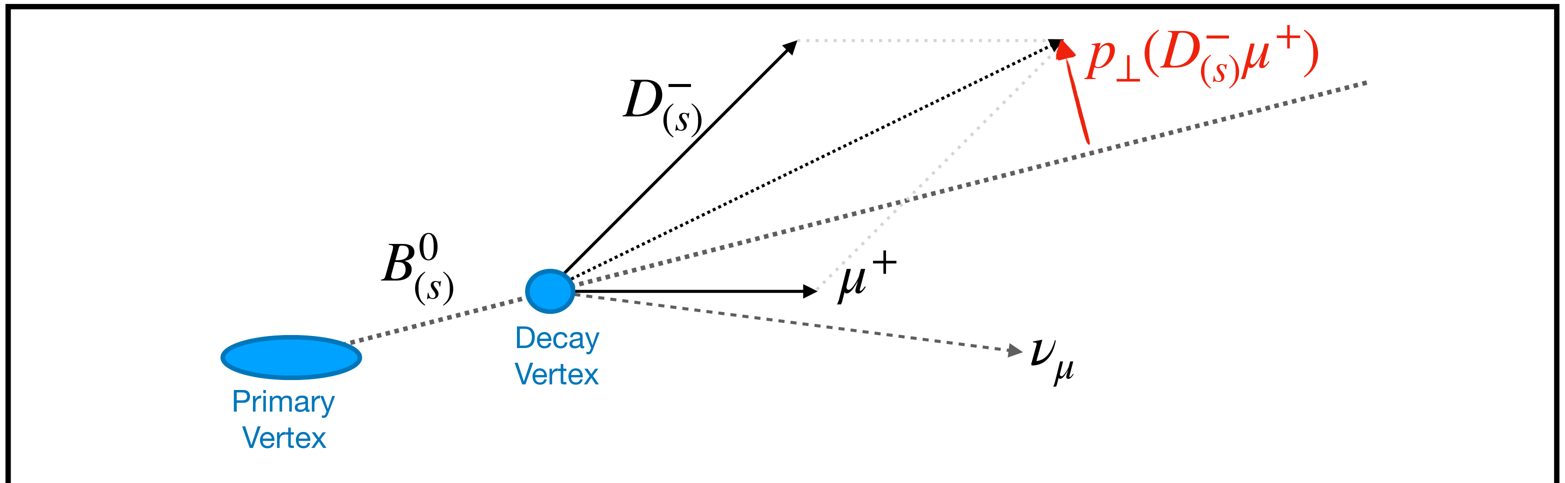
2. First glimpse at $|V_{cb}|$ in $B^0 \to D^{(*)-} \ell^+ \nu_\ell$ with Belle II data
[Preliminary]

1.

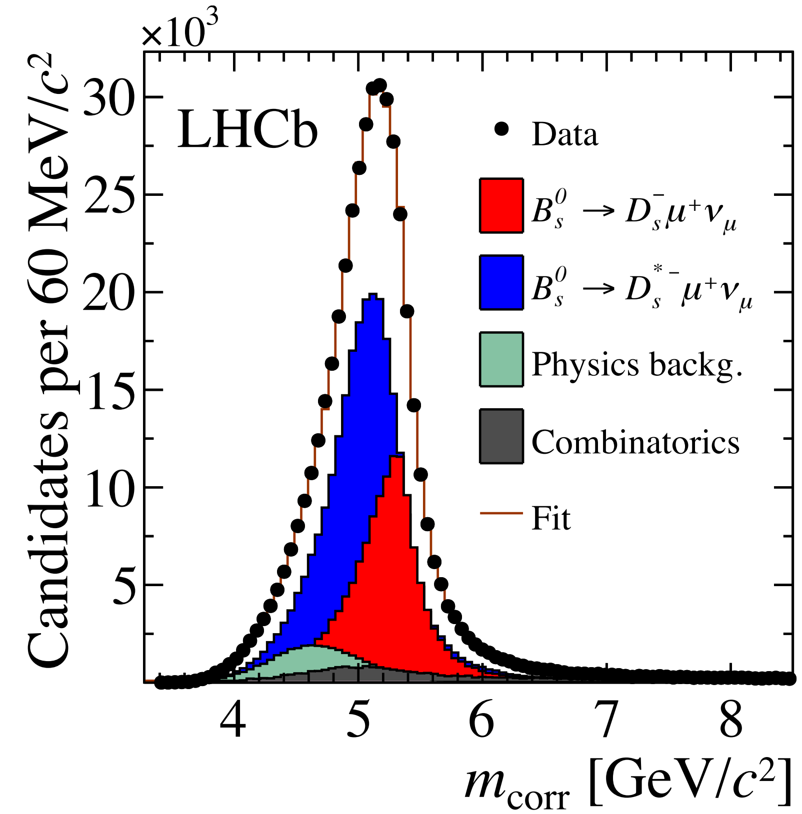
Measurement of $|V_{cb}|$ with $B_s \to D_s^{(*)} \mu \bar{\nu}_\mu$ decays

[Phys. Rev. D **101**, 072004, arXiv:2001.03225]

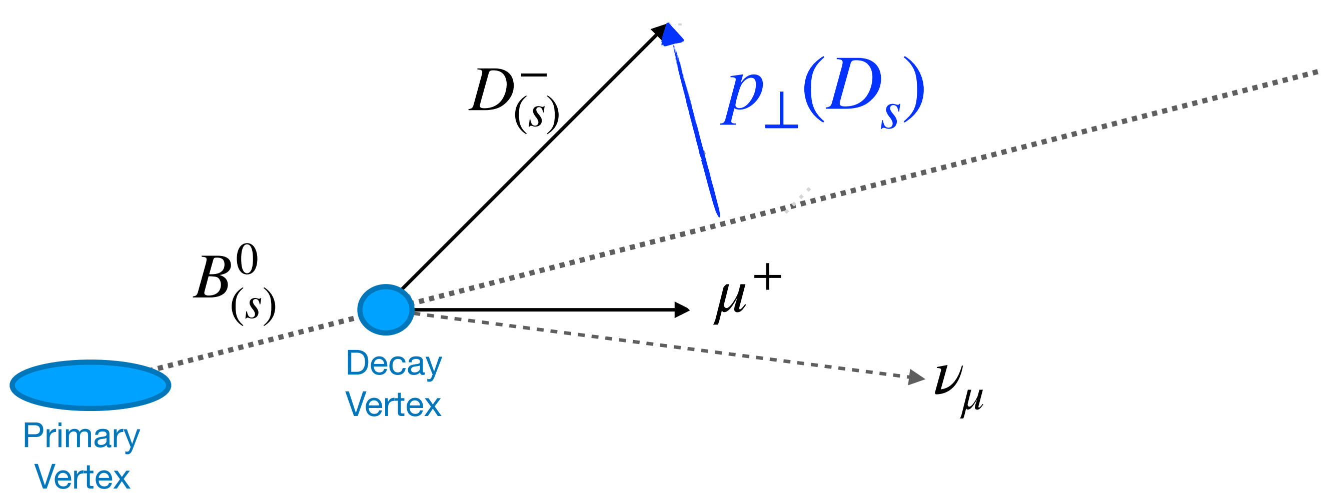
Leverage large **separation** of **decay vertex** from **primary vertex** to reconstruct B_s **flight direction**; reconstruct *corrected* mass m_{corr}:

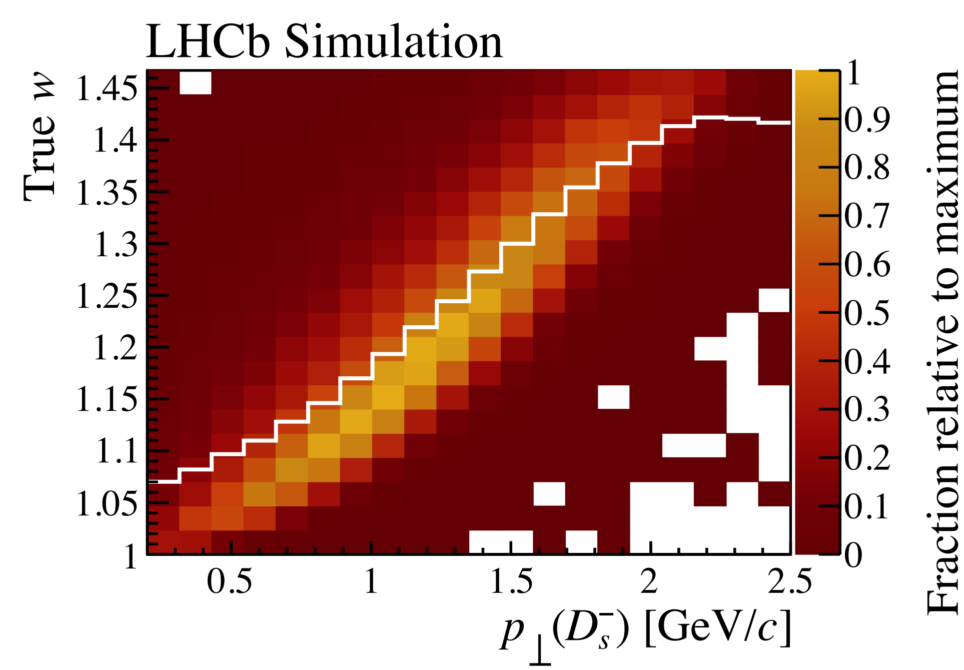


$$m_{corr} \equiv \sqrt{m^2(D_s^- \mu^+) + p_\perp^2(D_s^- \mu^+)} + p_\perp(D_s^- \mu^+)$$



Exploit $p_\perp(D_s)$ correlation with *w* to fit form factors

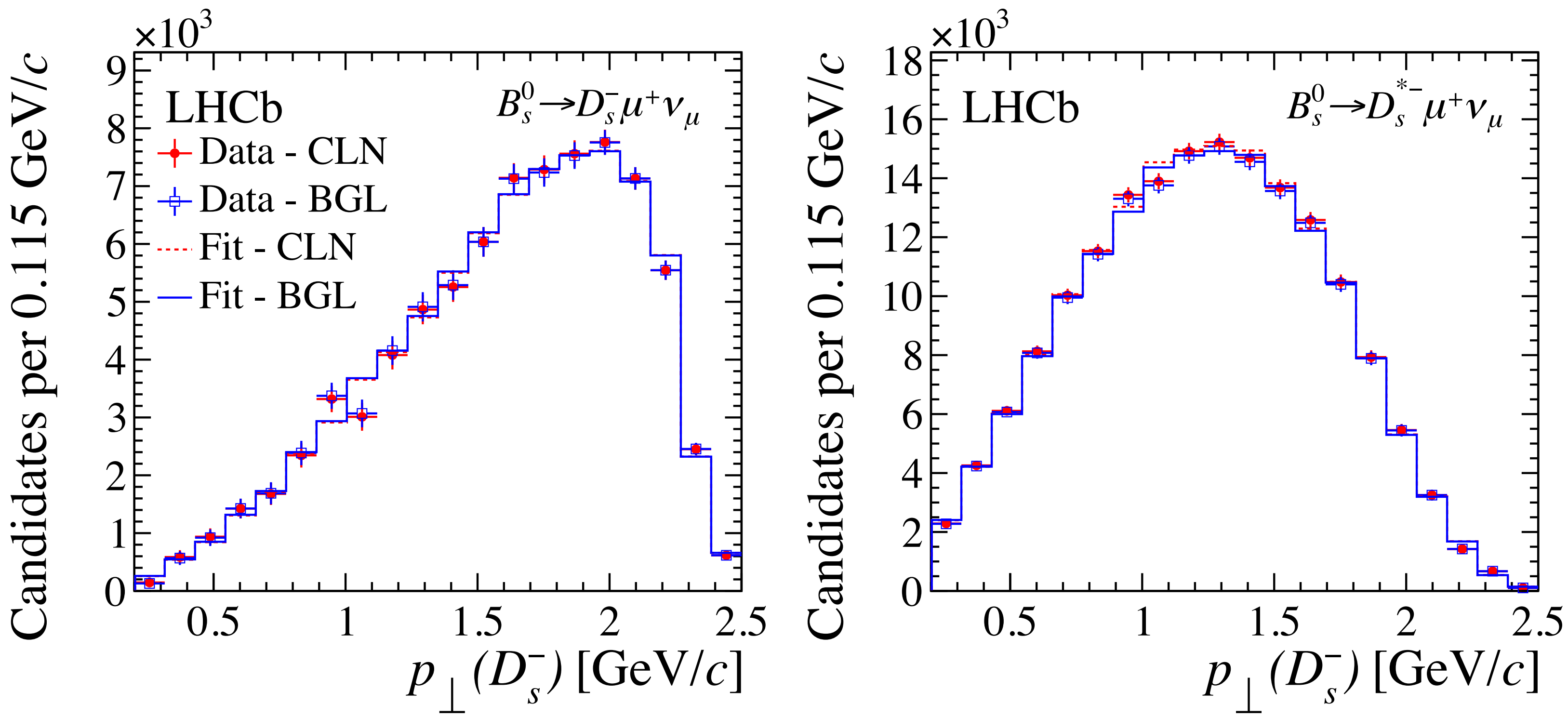


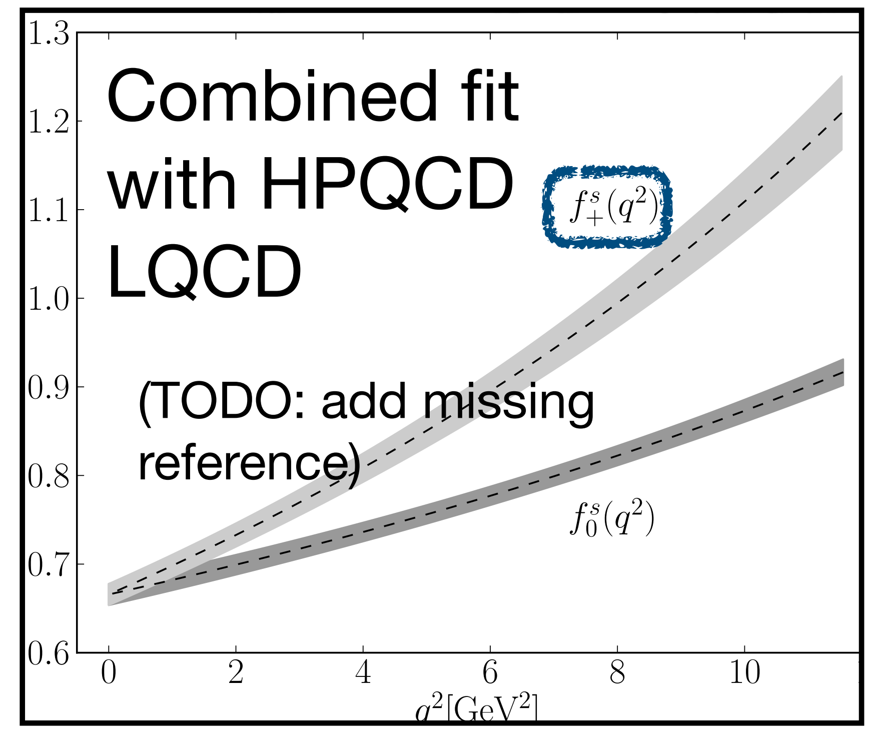


Measurement of $|V_{cb}|$ with $B_s \to D_s^{(*)}\mu\bar{\nu}_\mu$ decays

[Phys. Rev. D **101**, 072004, arXiv:2001.03225]

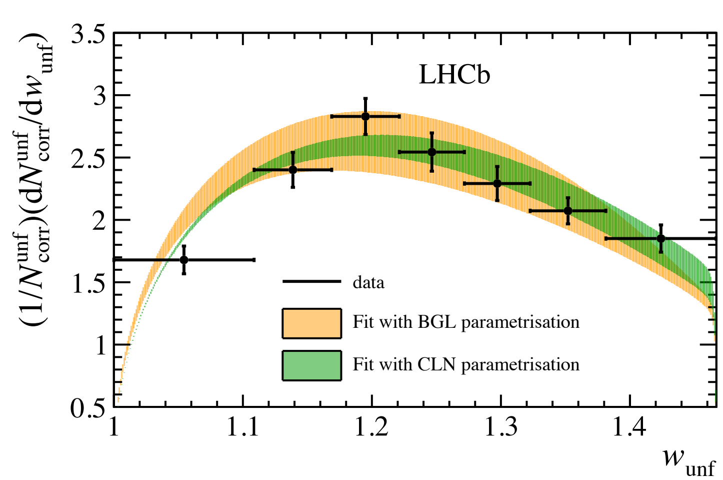
Background subtracted and **fitted** distributions:





$$\longrightarrow \quad |V_{cb}|_{\mathrm{BGL}} = (41.7 \pm 0.8(\mathrm{stat}) \pm 0.9(\mathrm{syst}) \pm 1.1(\mathrm{ext})) \times 10^{-3}$$

Also provide unfolded w spectrum for $B_s \to D_s^* \mu\bar{\nu}_\mu$

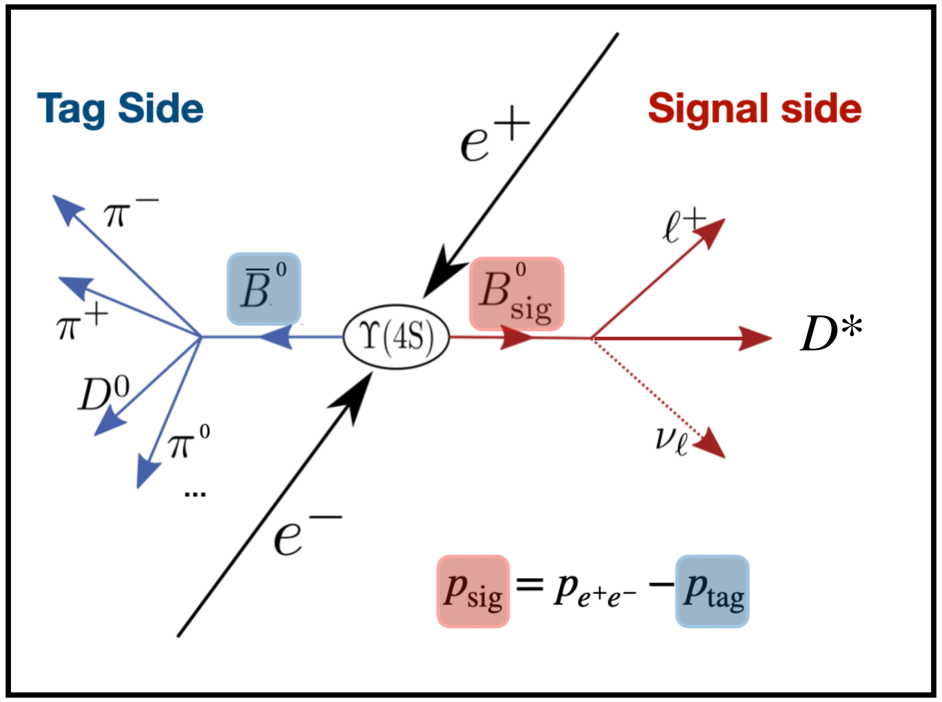


2.

First glimpse at $|V_{cb}|$ in $B \to D^{(*)}\ell\bar{\nu}_\ell$ with Belle II data

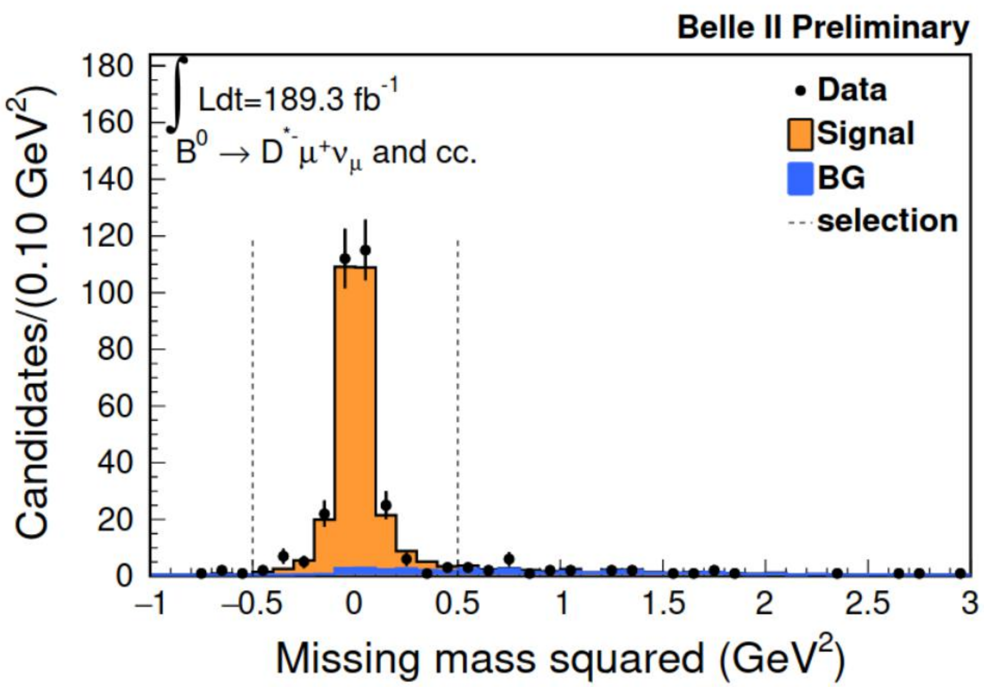
[Preliminary]

Reconstructed with hadronic tagging and using 189.3/fb



With hadronic tagging can reconstruct

$$m^2_{miss} = (p_{sig} - p_{D^*} - p_\ell)^2 \sim p^2_\nu = 0$$



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[Preliminary]

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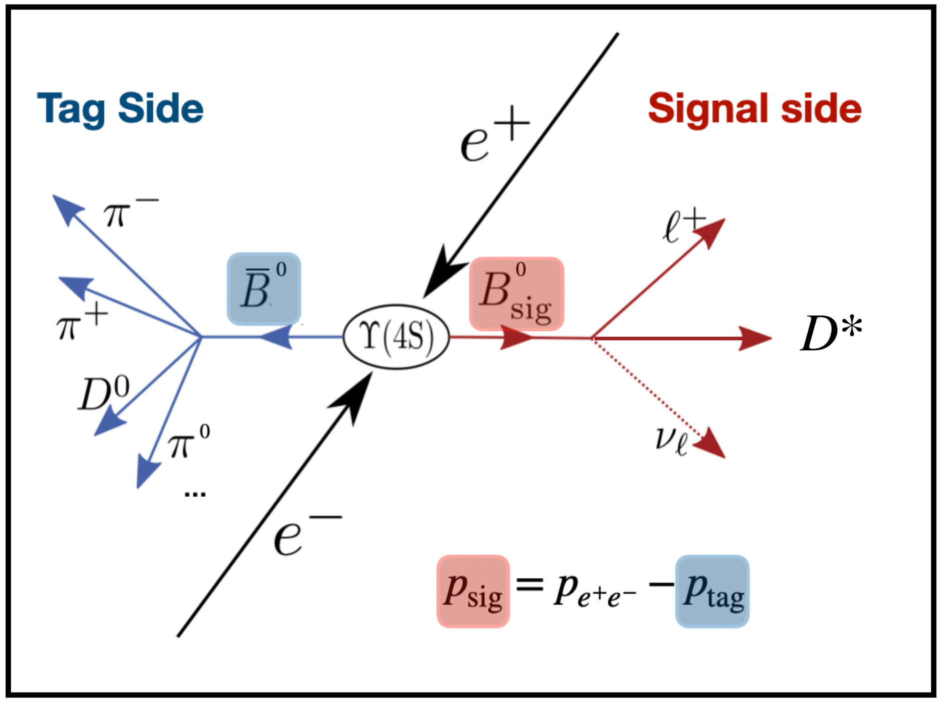

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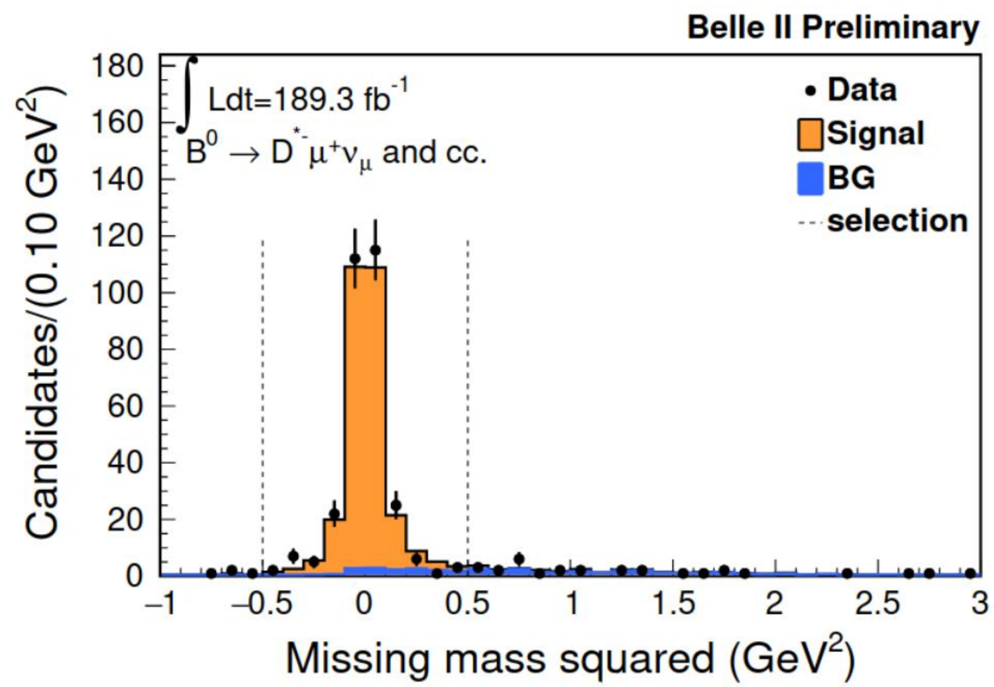
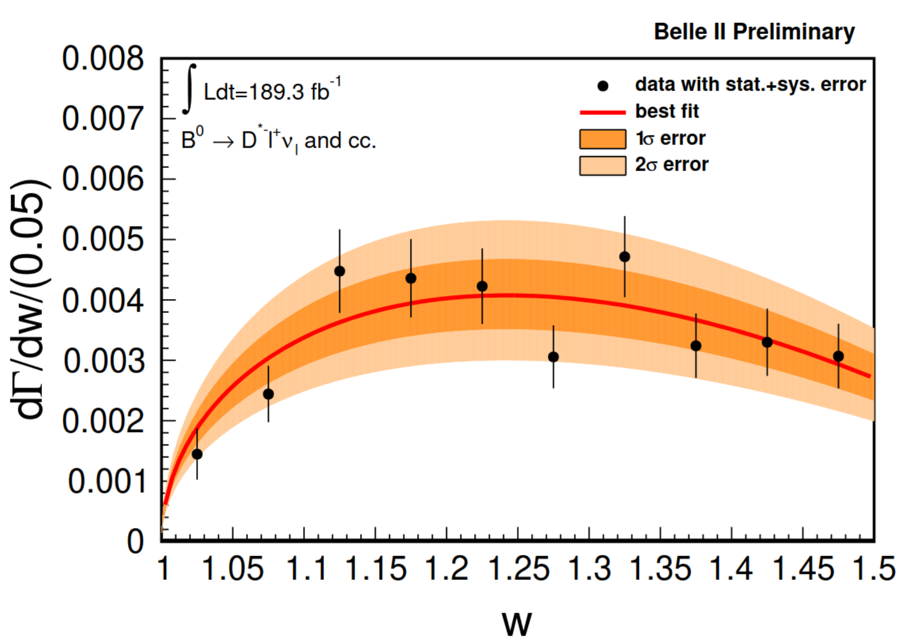
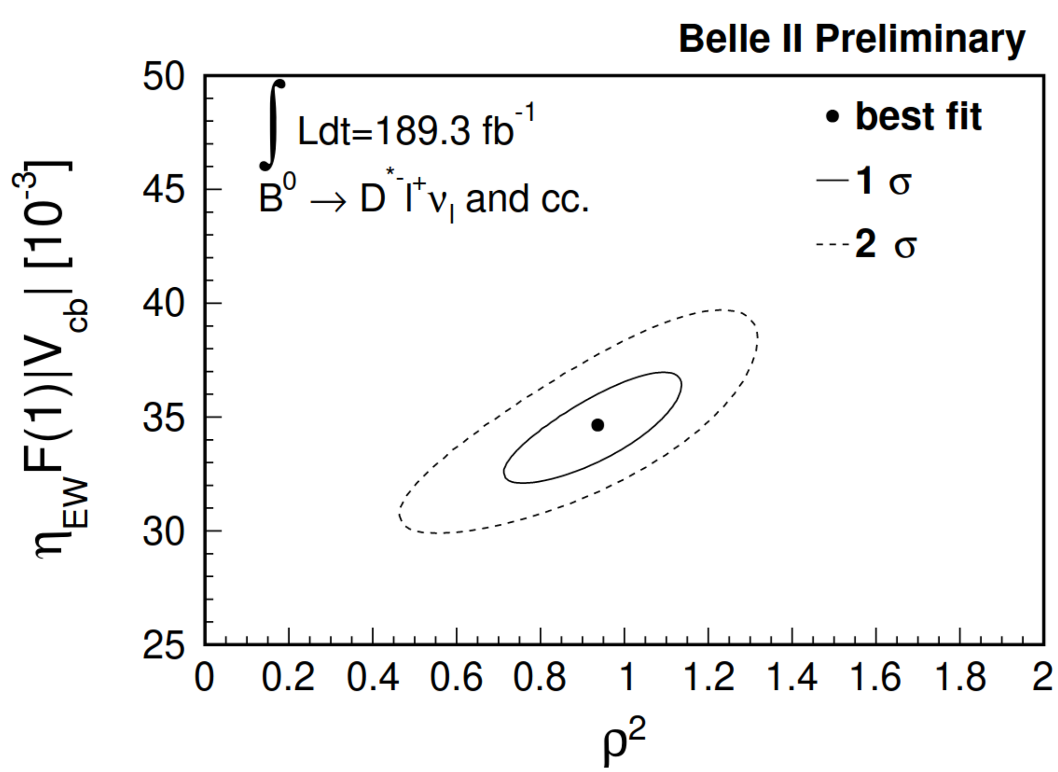


Background subtracted & unf. w spectrum



→



Determined $|V_{cb}|$:

$$|V_{cb}| = (37.9 \pm 2.7) \times 10^{-3}$$

Exclusive $|V_{ub}|$

- …

Inclusive $|V_{ub}|$

$B \to X_u \ell \bar{\nu}_\ell$

+ Fermi Motion / Shape Function

Inclusive $|V_{cb}|$

$B \to X_c \ell \bar{\nu}_\ell$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q\,\ell\,\bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \dots \right]$$

Exclusive $|V_{ub}|$

$B \to \pi, \rho, \omega\, \ell \bar{\nu}_\ell$, $\Lambda_b \to p \mu \bar{\nu}_\mu$

$B_s \to K \mu \bar{\nu}_\mu$

Exclusive $|V_{cb}|$

$B_{(s)} \to D^{(*)}_{(s)} \ell \bar{\nu}_\ell$

$$\mathcal{B} \propto |V_{qb}|^2 f^2$$

Form Factors

$$\langle B | H_\mu | P \rangle = (p + p')_\mu f_+$$

New Developments in exclusive $|V_{ub}|$

First measurement with $B_s \to K\mu\bar{\nu}_\mu$

LHCb presented a year ago a spectacular first measurement of exclusive $|V_{ub}|/|V_{cb}|$ from B_s decays

Small taste of what there is to come from both experiments !

Exclusive

1. First observation of the decay $B_s^0 \to K^-\mu^+\nu_\mu$ & meas. of $|V_{ub}|/|V_{cb}|$
[Phys.Rev.Lett. 126 (2021) 8, 081804, arXiv:2012.05143]

2. First glimpse at $|V_{ub}|$ in $B^0 \to \pi^-\ell^+\nu_\ell$ with Belle II data
[Preliminary]

1.

First observation of the decay $B_s^0 \to K^-\mu^+\nu_\mu$ & meas. of $|V_{ub}|/|V_{cb}|$

[Phys.Rev.Lett. 126 (2021) 8, 081804, arXiv:2012.05143]

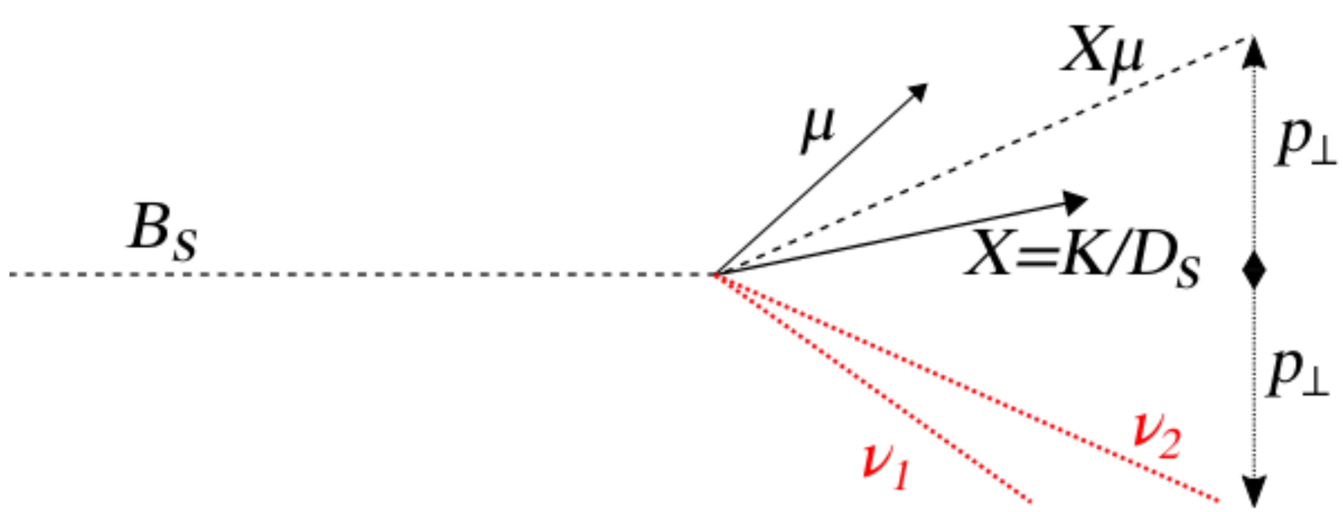
Directly aim to measure $|V_{ub}|/|V_{cb}|$ via the ratio

$$\mathscr{R} = \frac{\mathscr{B}(B_s^0 \to K^-\mu^+\nu_\mu)}{\mathscr{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)} = \frac{N_K}{N_{D_s}} \frac{\epsilon_{D_s}}{\epsilon_K} \times \mathscr{B}(D_s^- \to K^+K^-\pi^-)$$

efficiency ratio

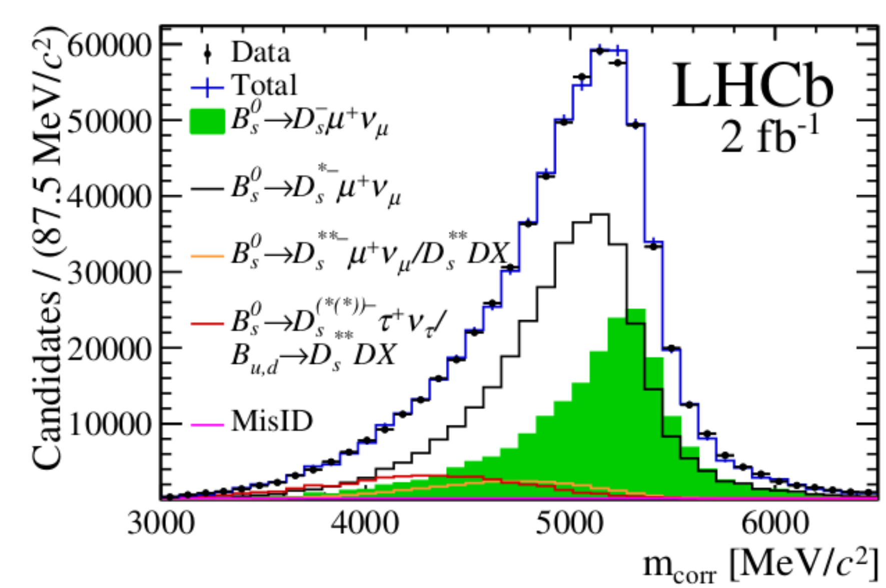
# of signal / normalization events

Again use *corrected* mass m_{corr} to separate signal from background and normalization:



$$m_{\text{corr}} = \sqrt{m^2(Y\mu) + p_\perp^2(Y\mu)} + p_\perp(Y\mu) \qquad \text{with} \quad Y = K^-, D_s^-$$

$B_s^0 \to D_s^- \mu^+\nu_\mu$



$N(B_s^0 \to D_s^- \mu^+\nu_\mu) = 201450 \pm 5200$

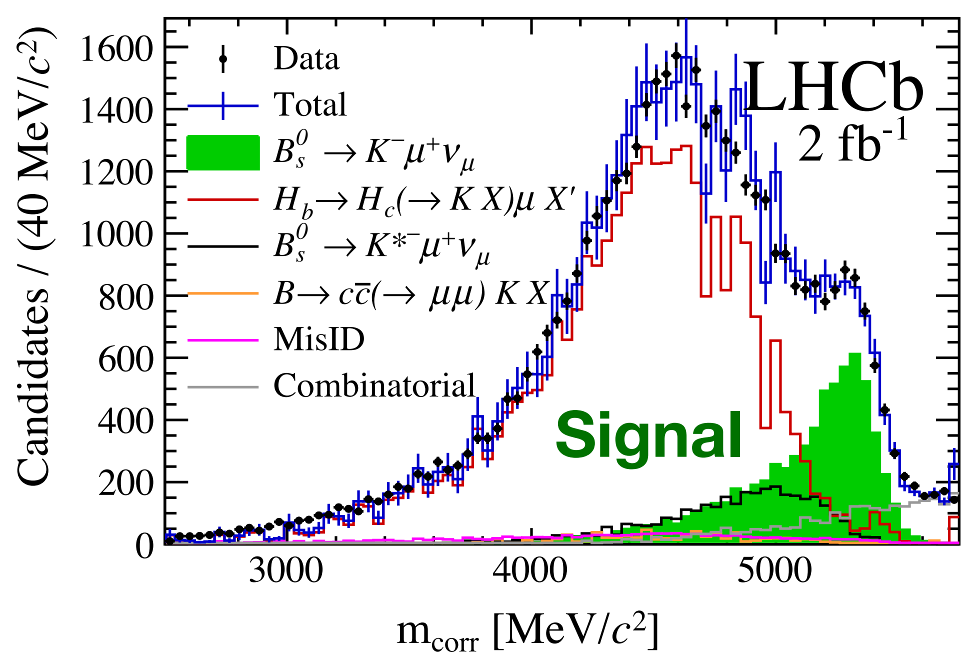
1.

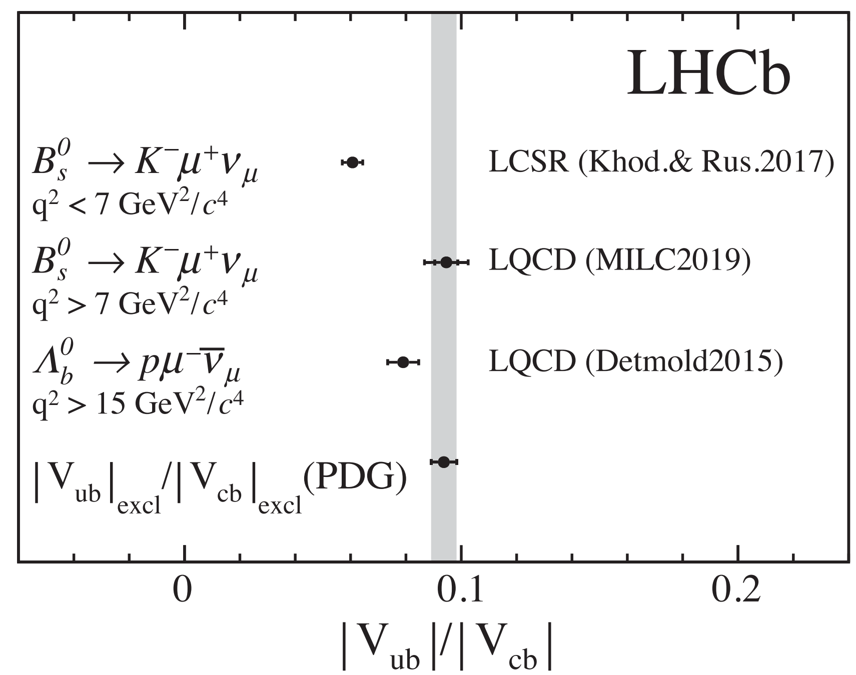
First observation of the decay $B_s^0 \to K^- \mu^+ \nu_\mu$ & meas. of $|V_{ub}|/|V_{cb}|$

[Phys.Rev.Lett. 126 (2021) 8, 081804, arXiv:2012.05143]

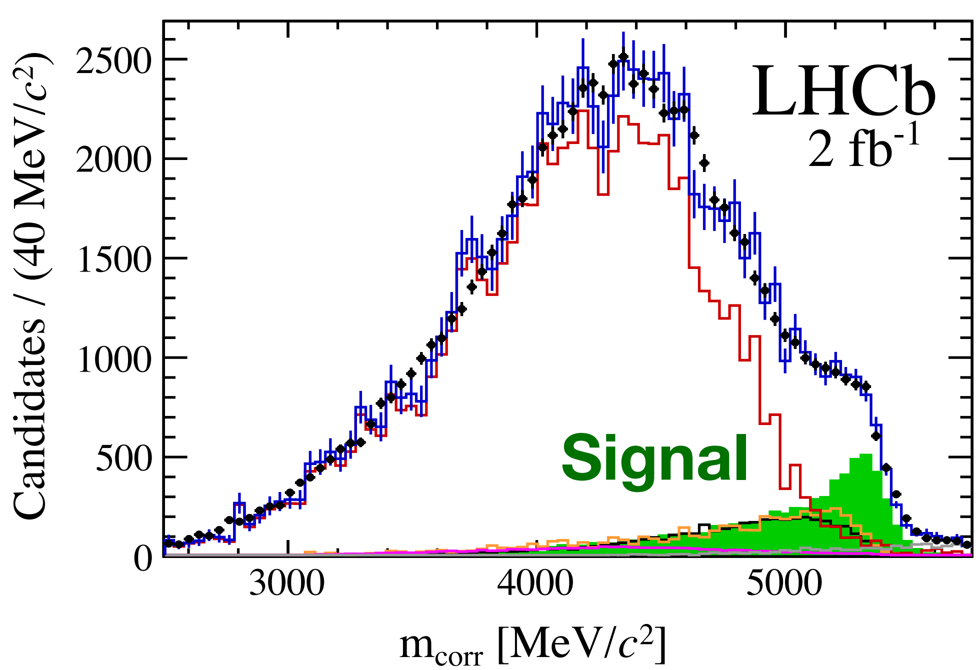
Extract $\mathscr{R}$ at low and high $q^2 = (p_B - p_K)^2$

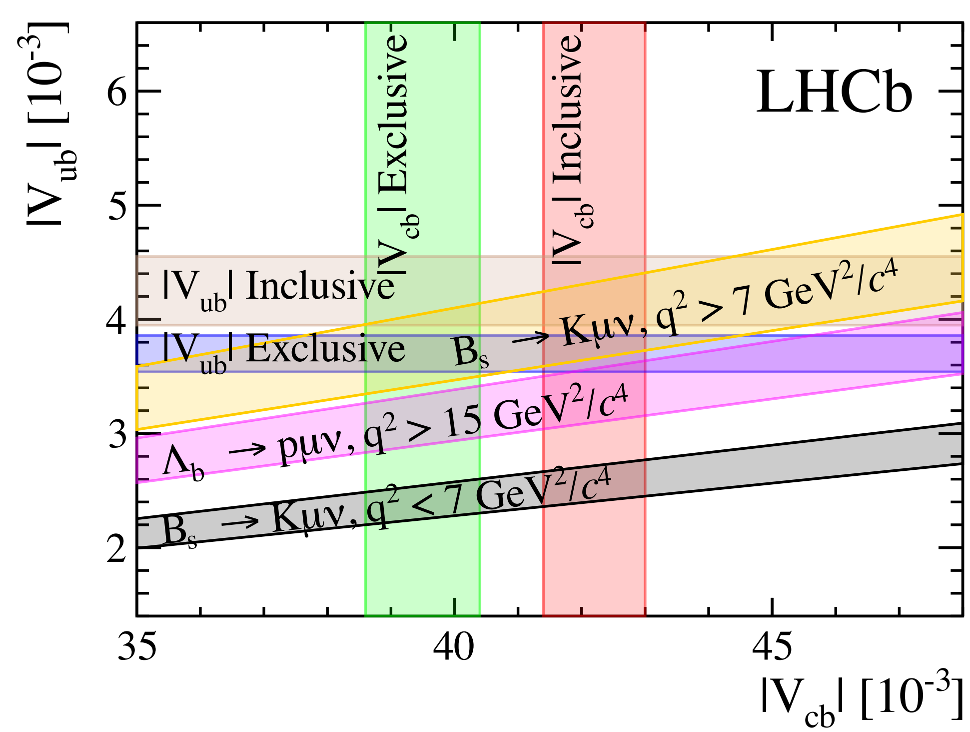
$q^2 < 7\,\text{GeV}^2$





$q^2 > 7\,\text{GeV}^2$

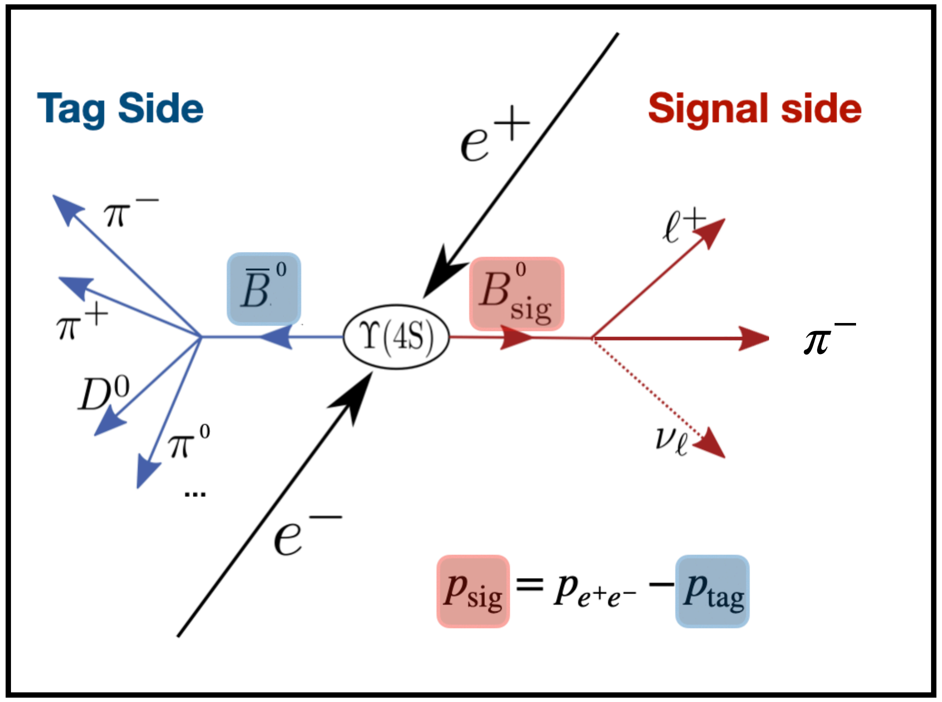




First glimpse at $|V_{ub}|$ in $B^0 \to \pi^- \ell^+ \nu_\ell$ with Belle II data

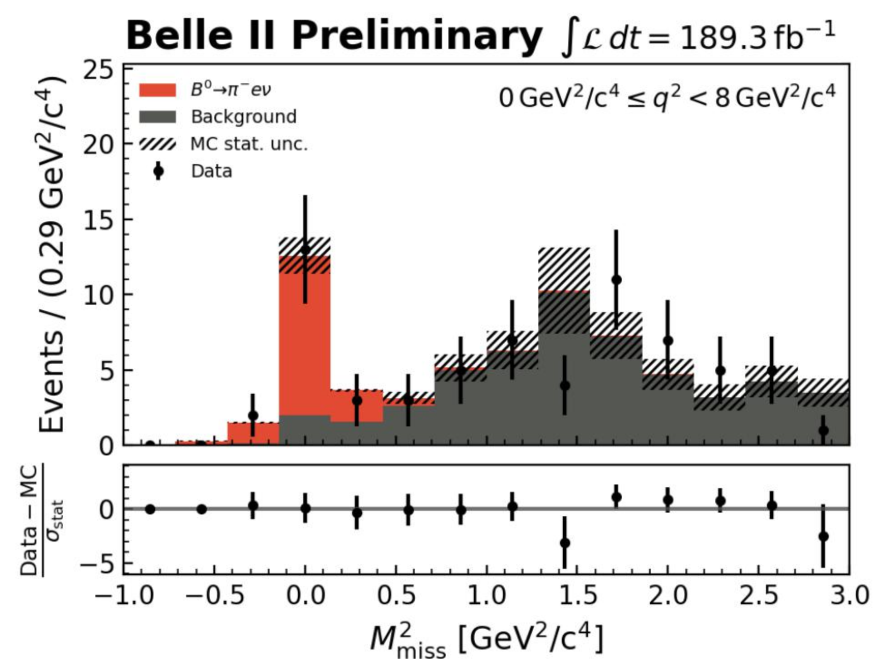
[Preliminary]

Reconstructed with hadronic tagging and using 189.3/fb



Fit $m^2_{\text{miss}} = (p_{\text{sig}} - p_\pi - p_\ell)^2 \sim p_\nu^2 = 0$

in **3 bins** of q^2 to separate signal from background

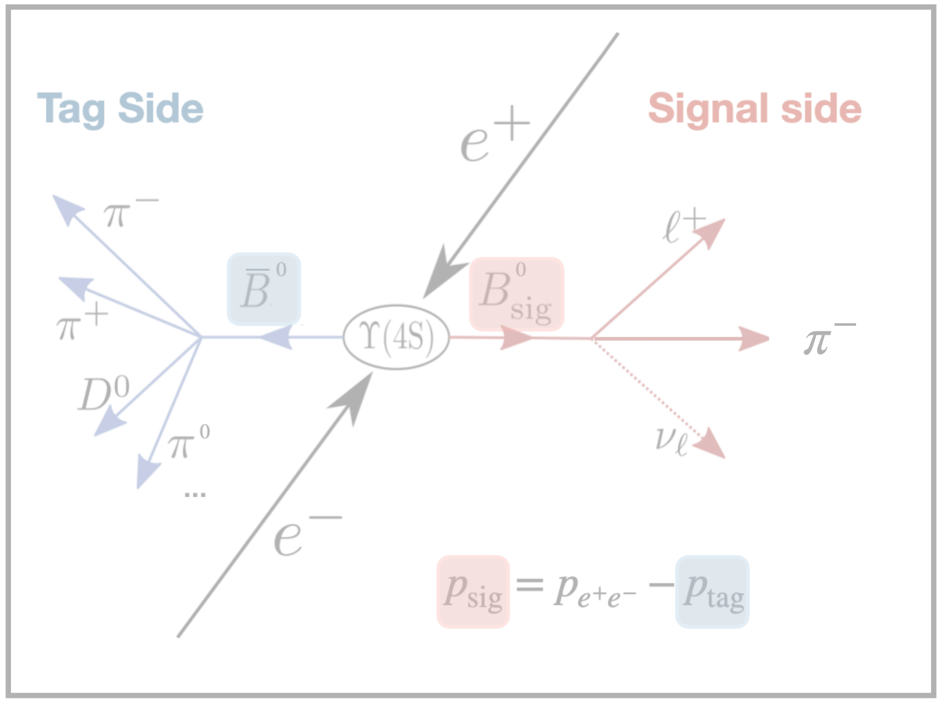


2.

First glimpse at $|V_{ub}|$ in $B^0 \to \pi^- \ell^+ \nu_\ell$ with Belle II data

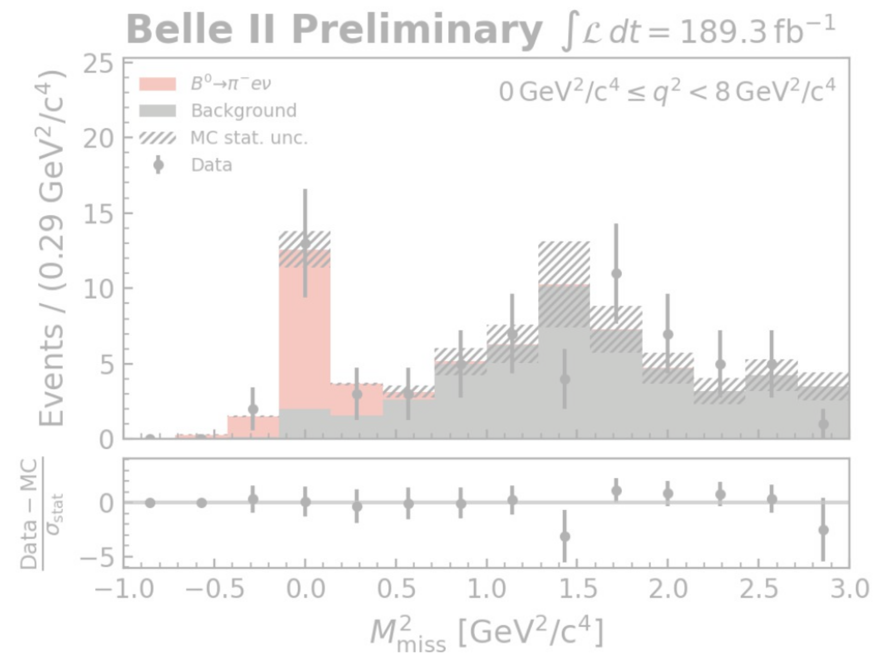
[Preliminary]

Reconstructed with hadronic tagging and using 189.3/fb

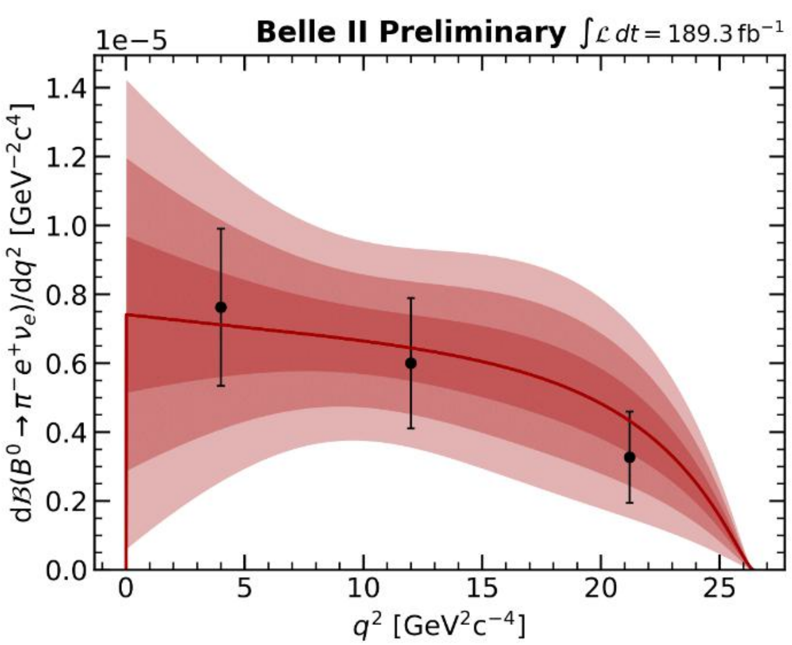


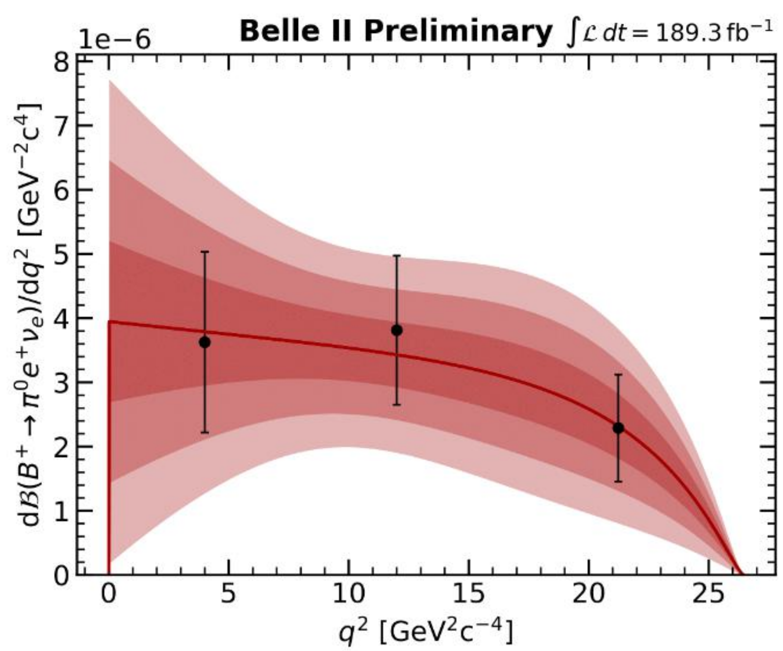
Fit $m^2_{\rm miss} = (p_{\rm sig} - p_\pi - p_\ell)^2 \sim p^2_\nu = 0$

in **3 bins** of q^2 to separate signal from background



Form Factor & $|V_{ub}|$ fit:





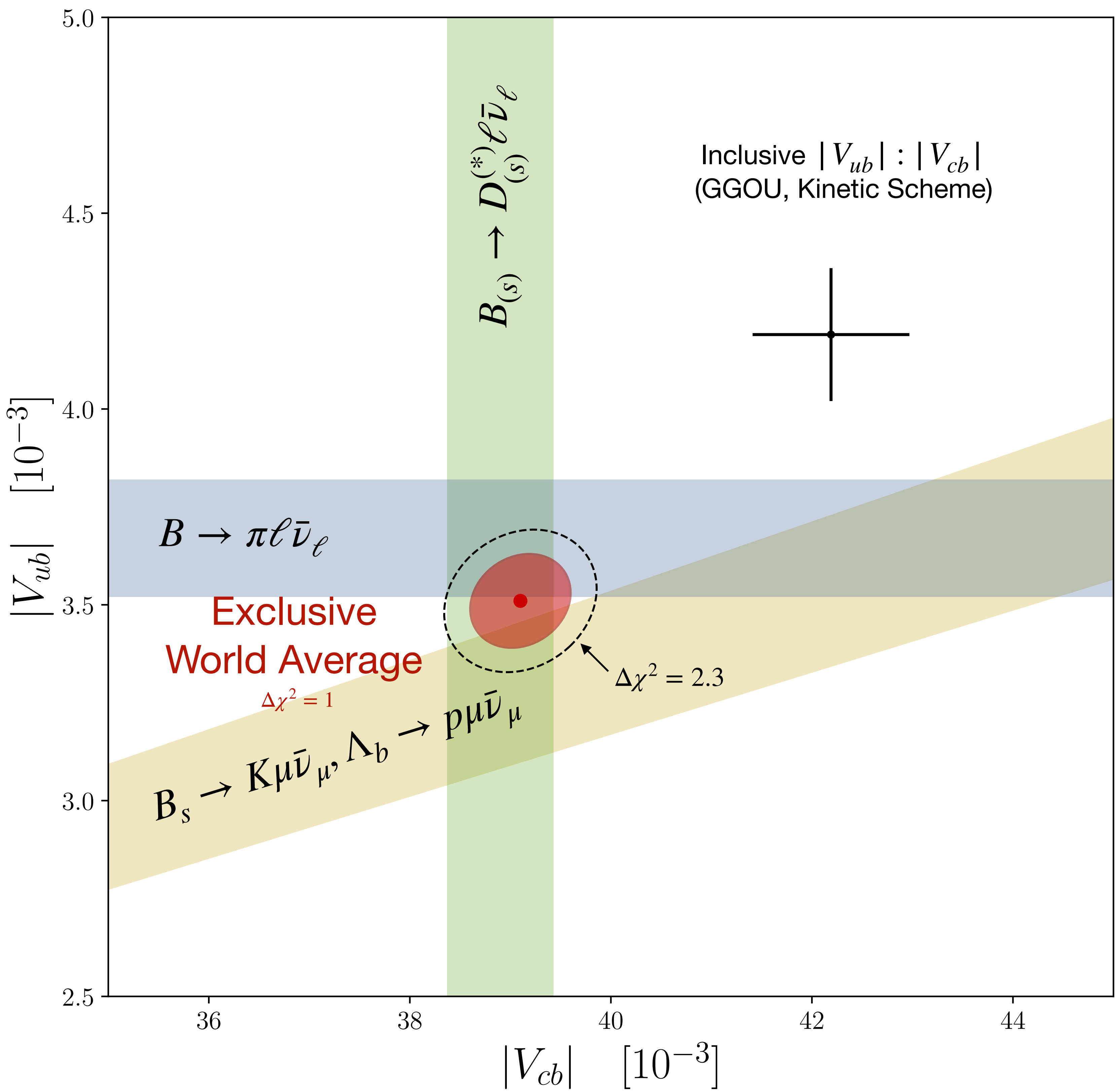
$\longrightarrow$ $|V_{ub}| \times 10^3 = 3.88 \pm 0.45$

with LQCD data from FNAL/MILC
Phys.Rev.D 92 (2015) 1, 014024, [arXiv: 1503.07839]

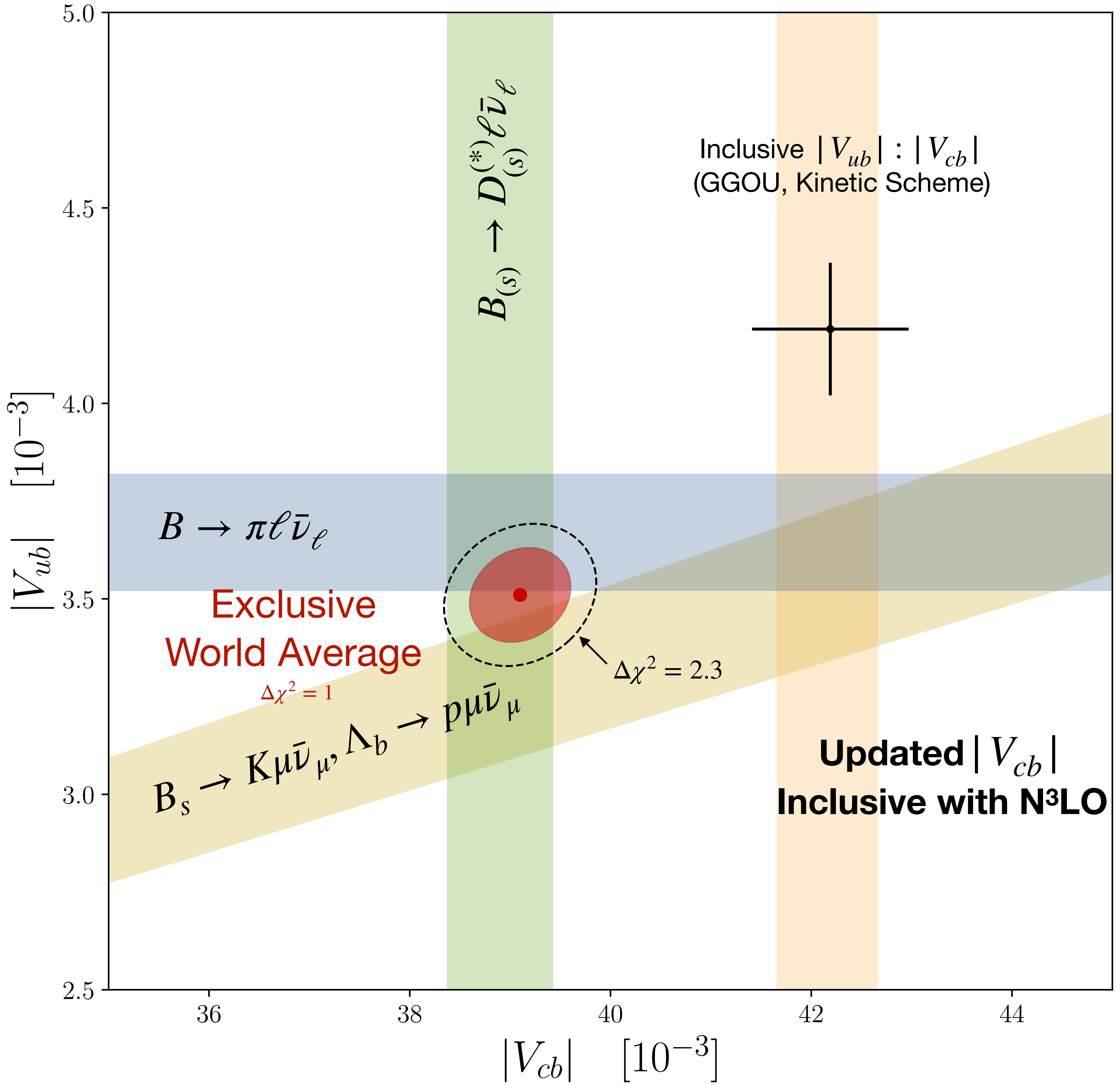
Summary & Outlook

Summary

Numbers from new HFLAV 2021 report
(will appear soon)

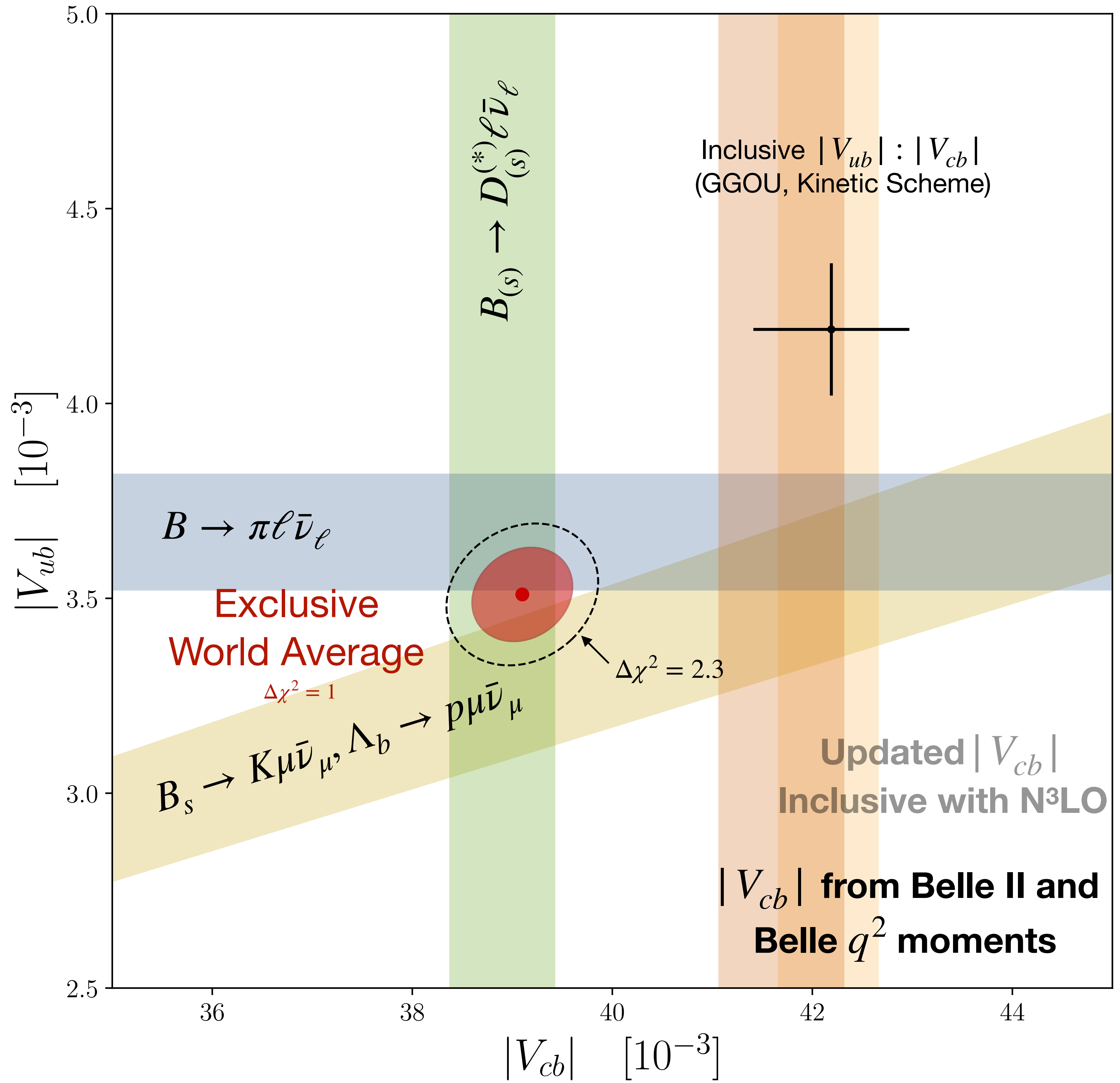


Summary



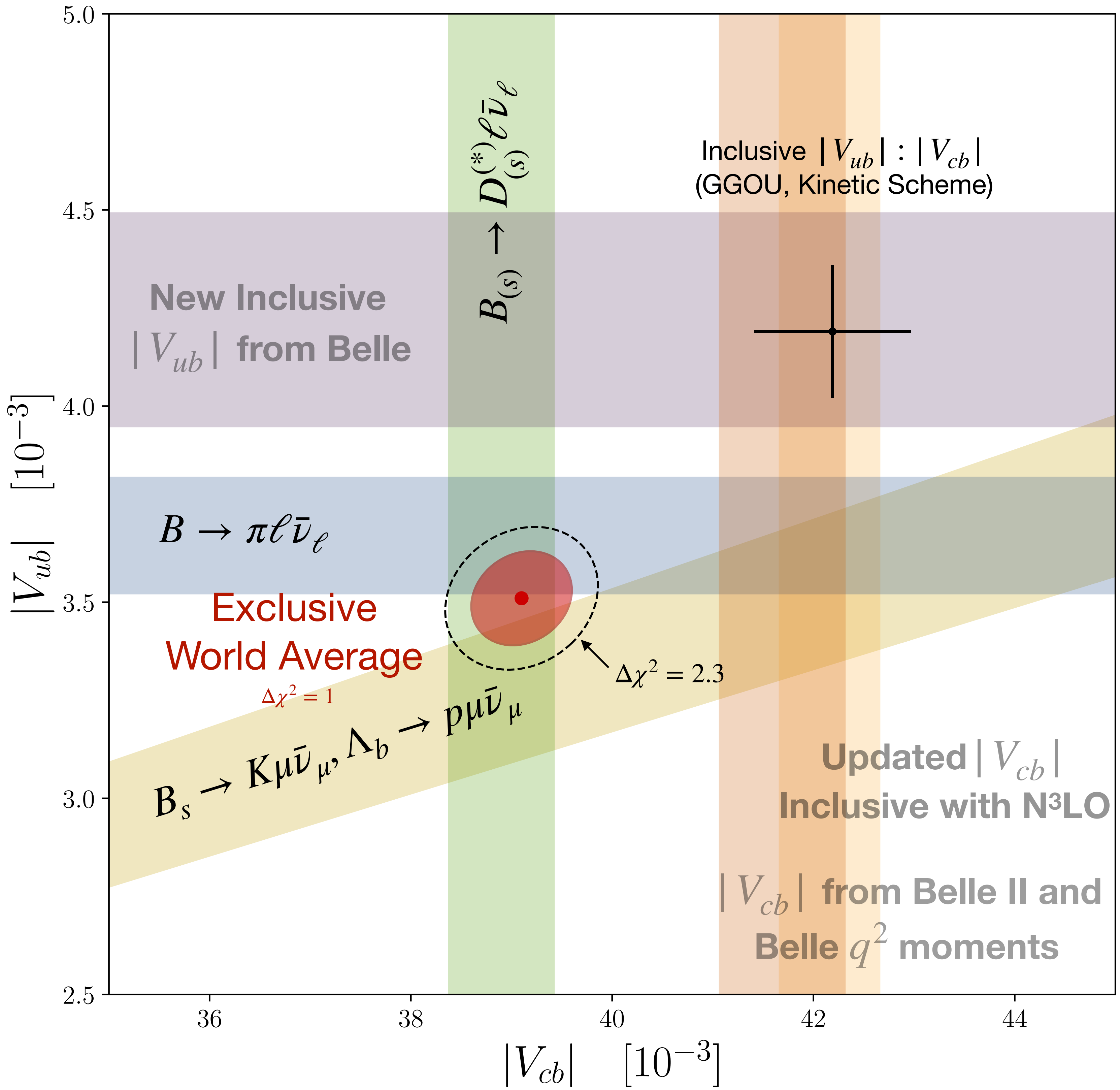
5.0
4.5
4.0
3.5
3.0
2.5
36
38
40
42
44
$|V_{ub}|$ $[10^{-3}]$
$|V_{cb}|$ $[10^{-3}]$
$B_{(s)} \to D^{(*)}_{(s)} \ell \bar{\nu}_\ell$
Inclusive $|V_{ub}|:|V_{cb}|$
(GGOU, Kinetic Scheme)
$B \to \pi \ell \bar{\nu}_\ell$
Exclusive
World Average
$\Delta\chi^2 = 1$
$\Delta\chi^2 = 2.3$
$B_s \to K\mu\bar{\nu}_\mu, \Lambda_b \to p\mu\bar{\nu}_\mu$
Updated $|V_{cb}|$
Inclusive with N³LO

Summary



$B_{(s)} \to D^{(*)}_{(s)} \ell \bar{\nu}_\ell$
Inclusive $|V_{ub}|:|V_{cb}|$
(GGOU, Kinetic Scheme)
$B \to \pi \ell \bar{\nu}_\ell$
Exclusive
World Average
$\Delta\chi^2 = 1$
$\Delta\chi^2 = 2.3$
$B_s \to K\mu\bar{\nu}_\mu, \Lambda_b \to p\mu\bar{\nu}_\mu$
Updated $|V_{cb}|$
Inclusive with N³LO
$|V_{cb}|$ from Belle II and
Belle q^2 moments
$|V_{ub}|$ $[10^{-3}]$
$|V_{cb}|$ $[10^{-3}]$
5.0
4.5
4.0
3.5
3.0
2.5
36
38
40
42
44

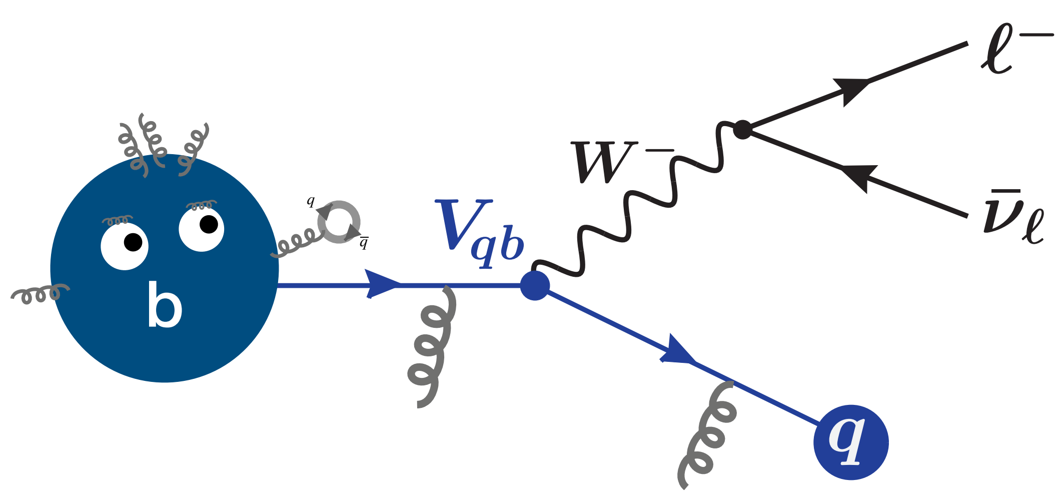
Summary



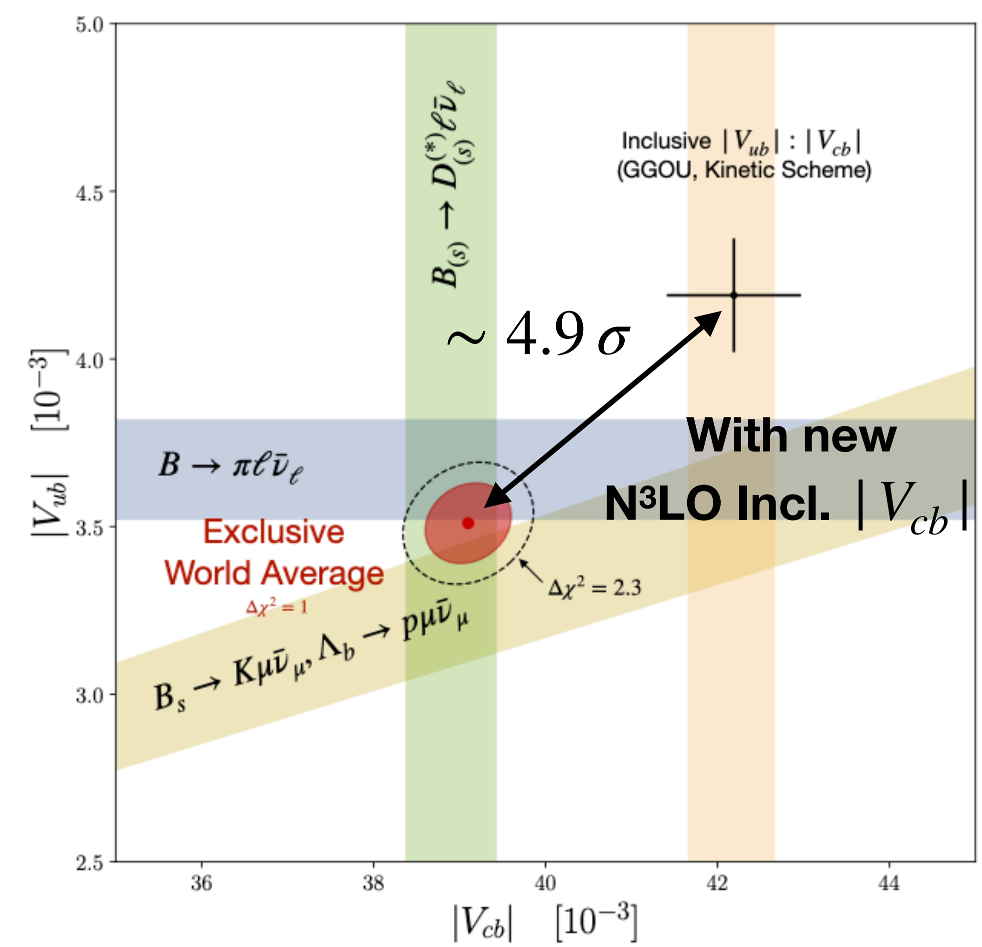
5.0
4.5
4.0
3.5
3.0
2.5
36
38
40
42
44
$|V_{ub}|$ $[10^{-3}]$
$|V_{cb}|$ $[10^{-3}]$
$B_{(s)} \to D^{(*)}_{(s)} \ell \bar{\nu}_\ell$
Inclusive $|V_{ub}|$: $|V_{cb}|$
(GGOU, Kinetic Scheme)
New Inclusive
$|V_{ub}|$ from Belle
$B \to \pi \ell \bar{\nu}_\ell$
Exclusive
World Average
$\Delta\chi^2 = 1$
$\Delta\chi^2 = 2.3$
$B_s \to K \mu \bar{\nu}_\mu, \Lambda_b \to p \mu \bar{\nu}_\mu$
Updated $|V_{cb}|$
Inclusive with N³LO
$|V_{cb}|$ from Belle II and
Belle q^2 moments

Summary

He may look cute, but that might be deceiving…



… the long-standing discrepancy is **not going away**



We need to tackle this problem:

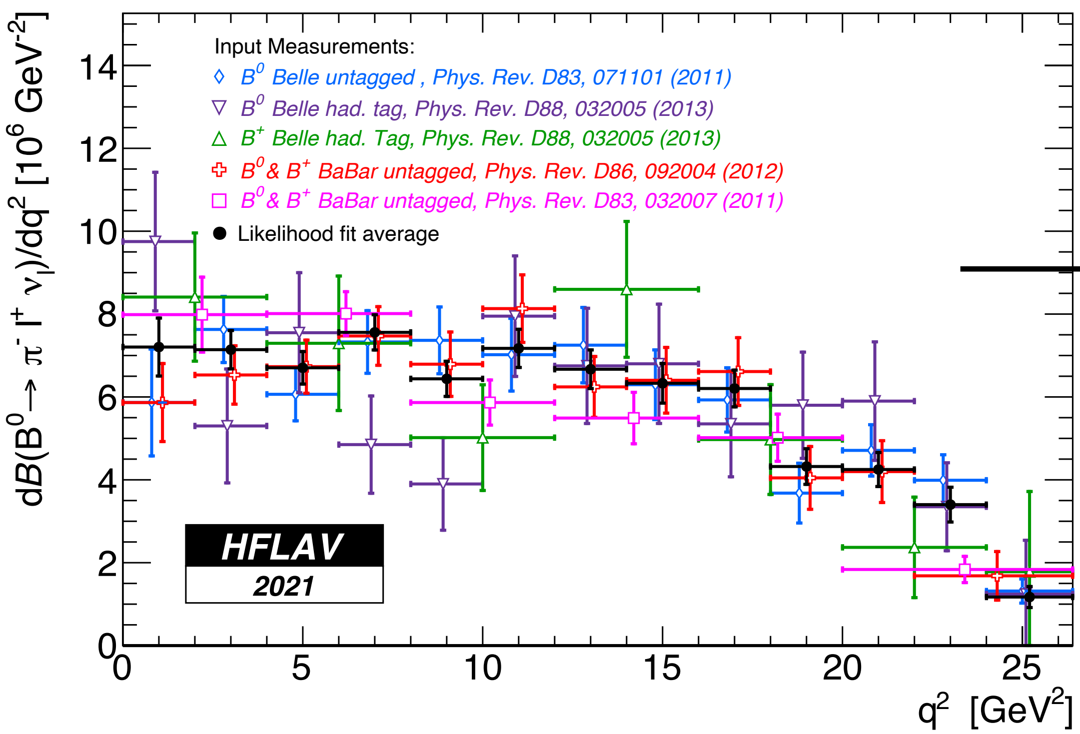
- There are three culprits that can cause this:
  - **Experimental Problem** / **Theory Problem** / **New Physics**

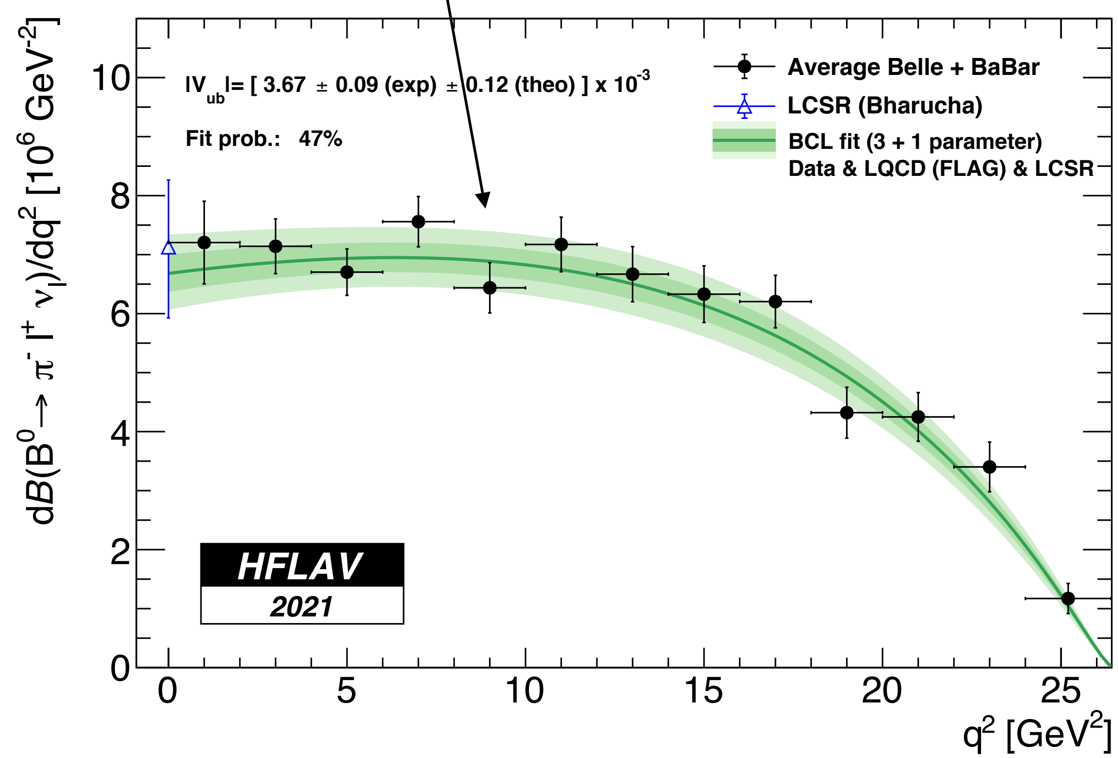
We need new experimental and theory results that challenge what we think we know

Backup slides

Exclusive $|V_{ub}|$

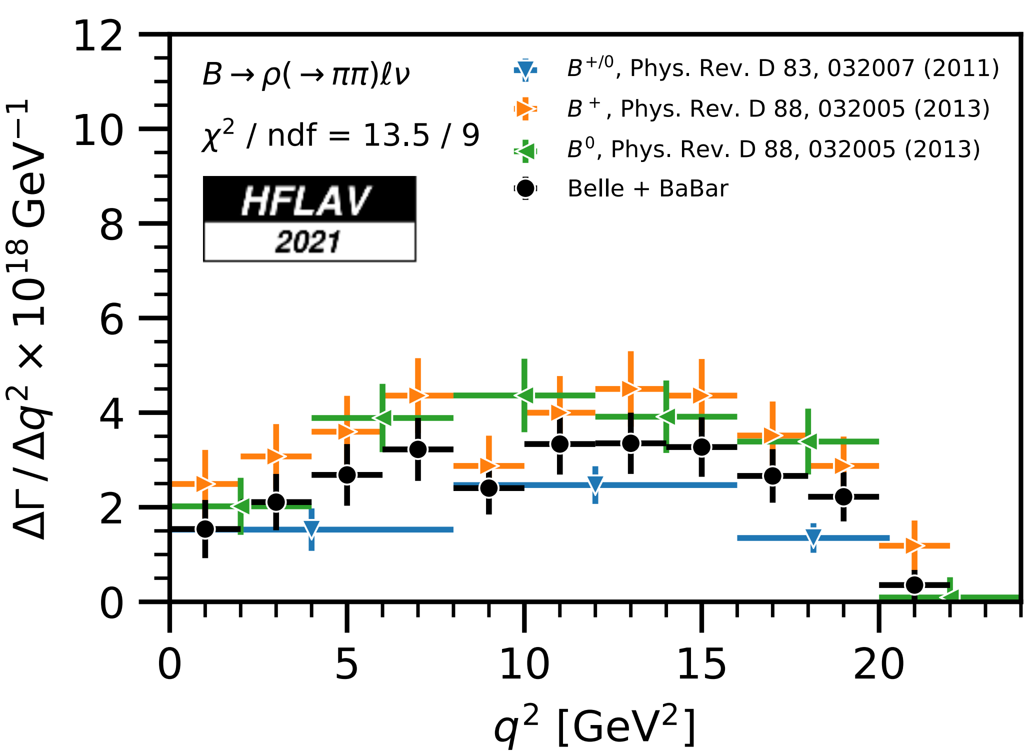
Likelihood combination with
systematic Nuisance Parameters
of all measurements

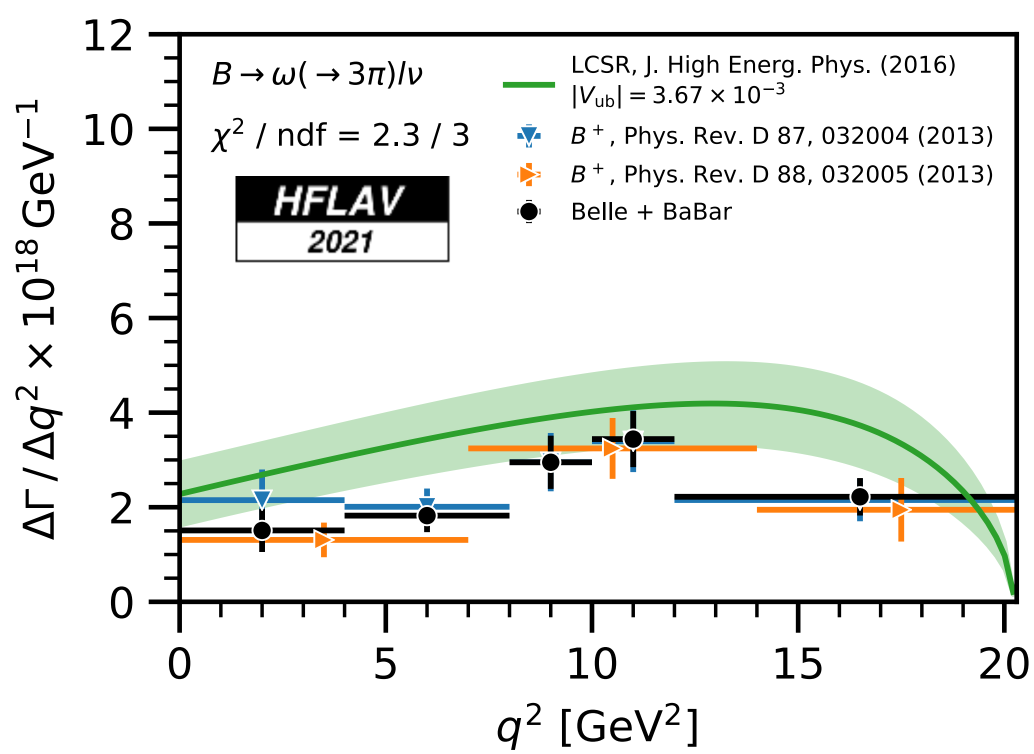




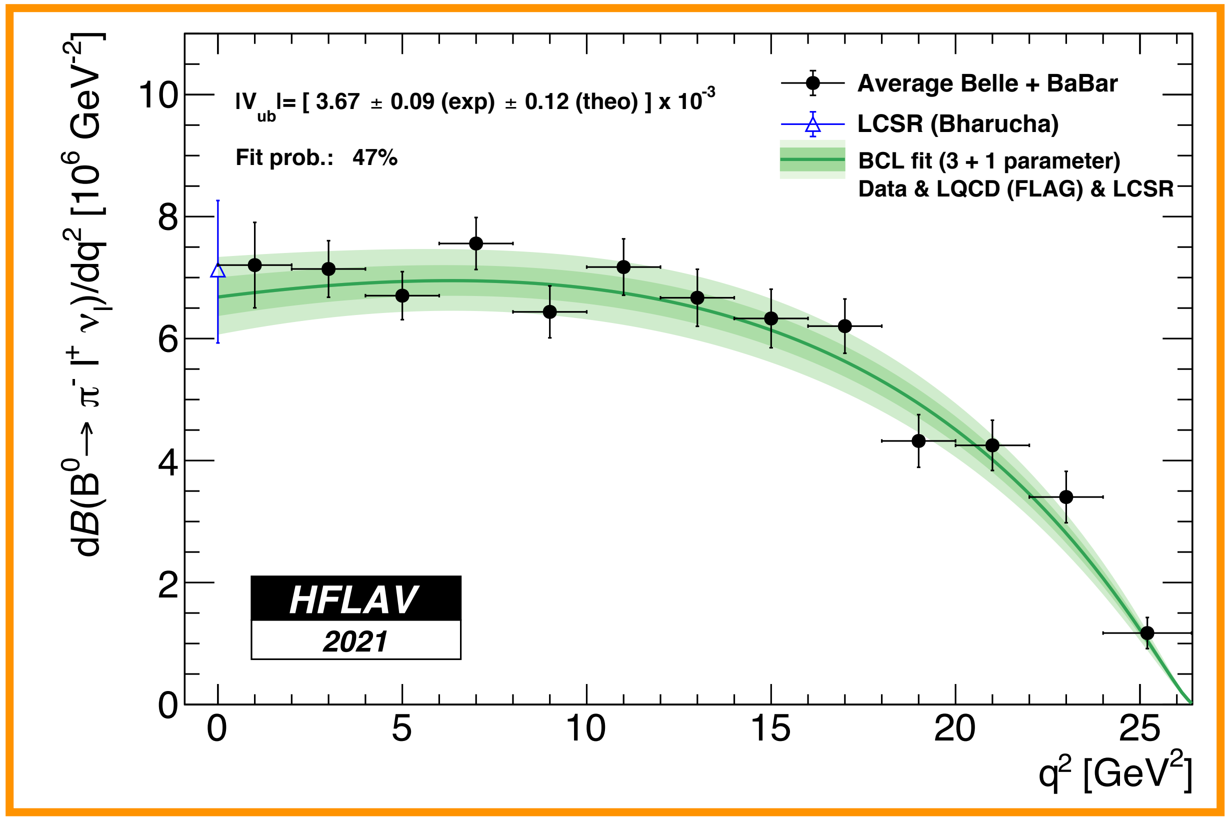
Now also available for $B \to \rho/\omega \ell \bar{\nu}_\ell$:

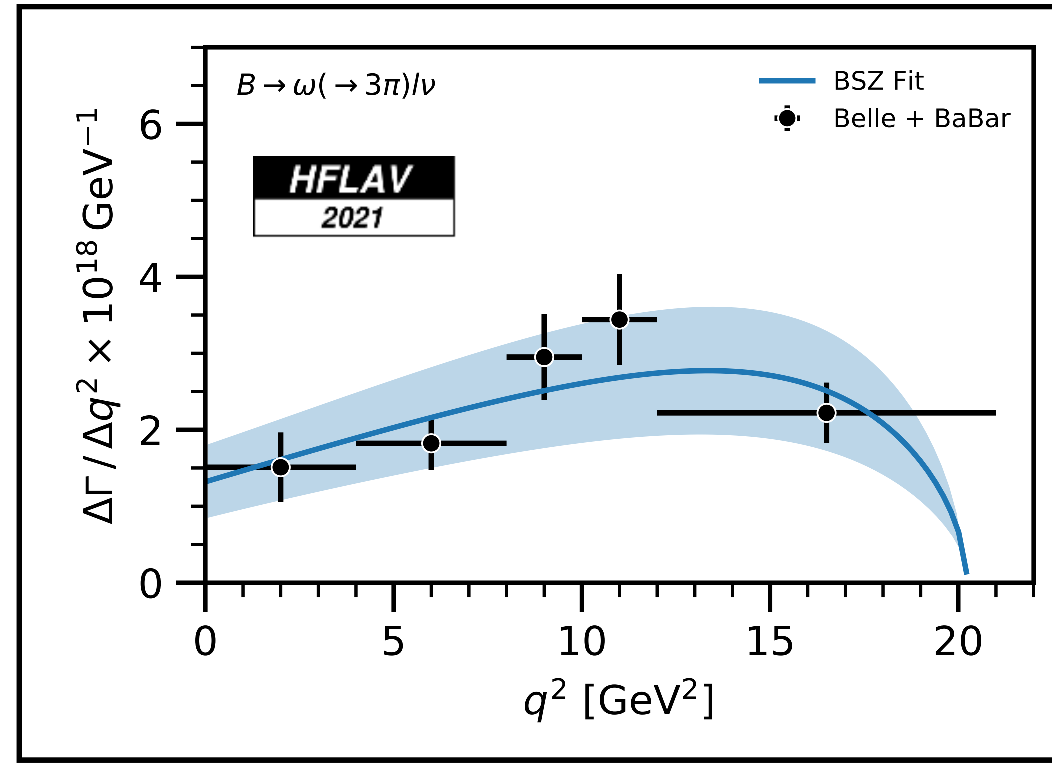
Plan to release public code for all of these

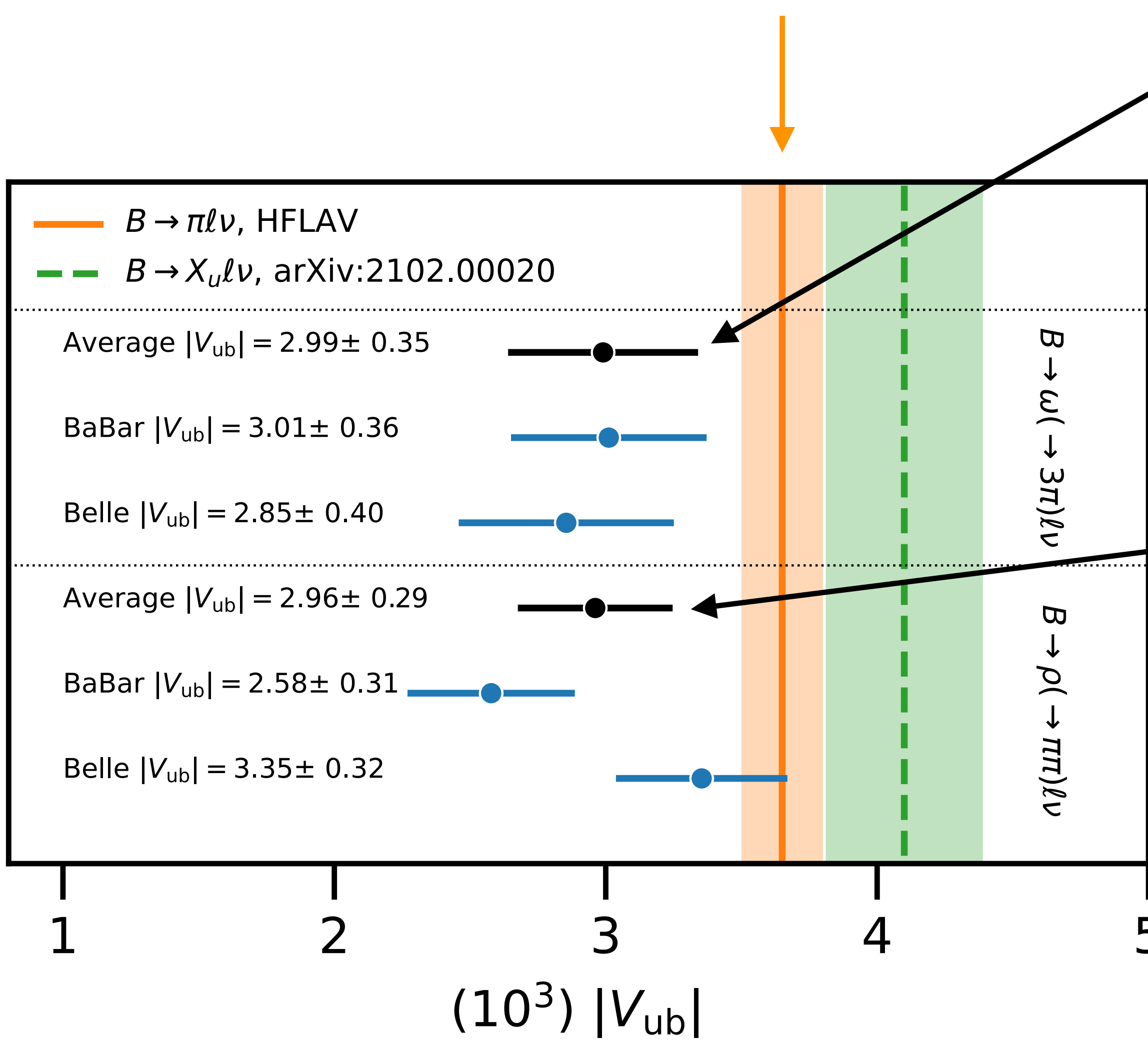


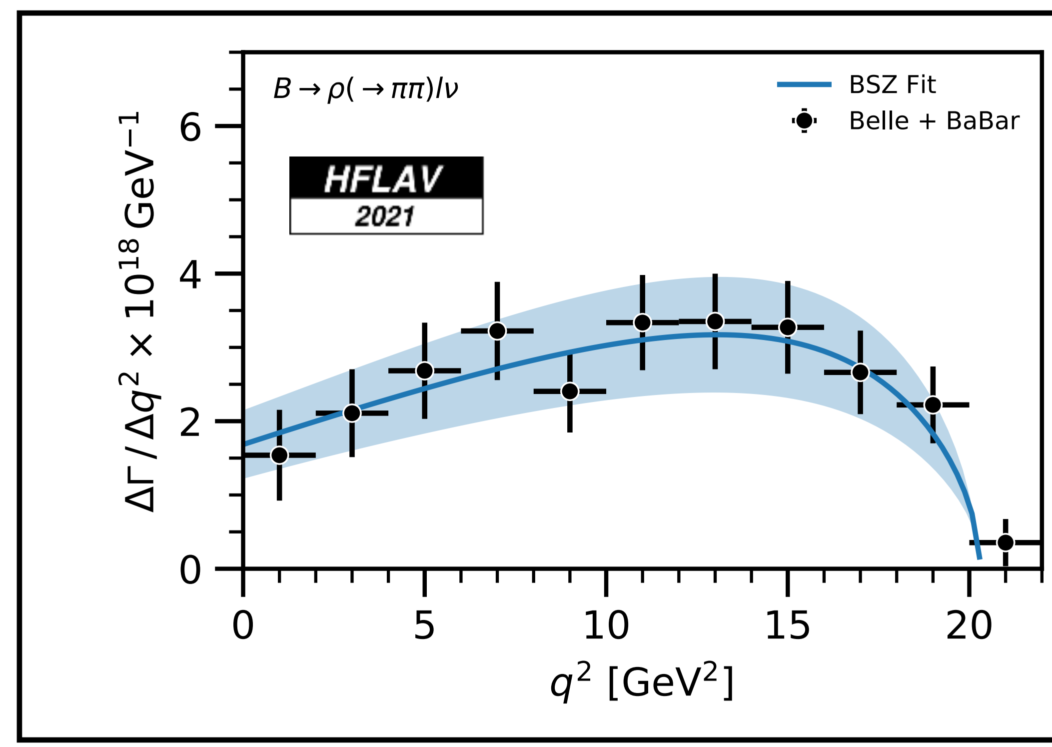


BSZ: J. High Energ. Phys. (2016) 2016: 98







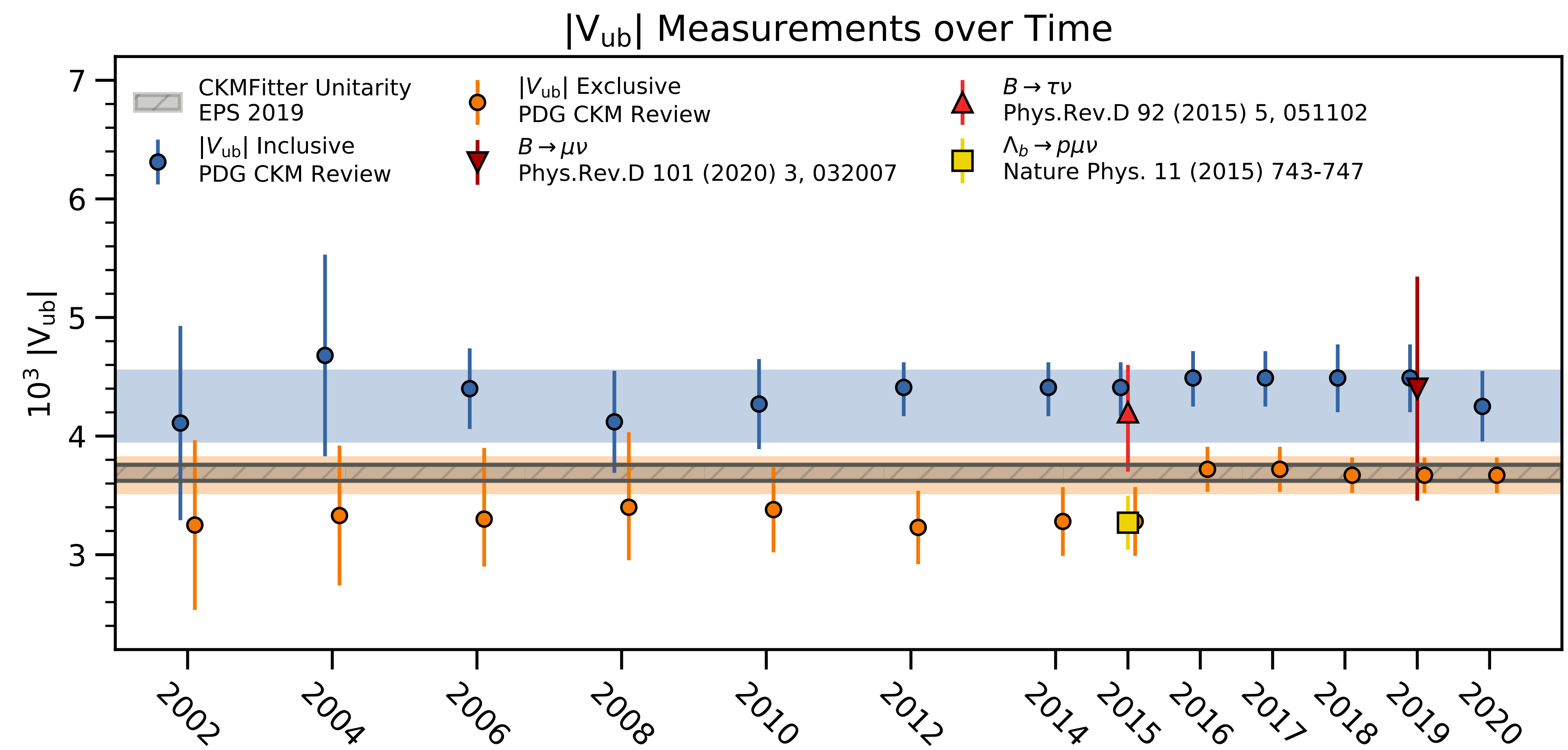


ρ/ω prefer much lower values of $|V_{ub}|$ …

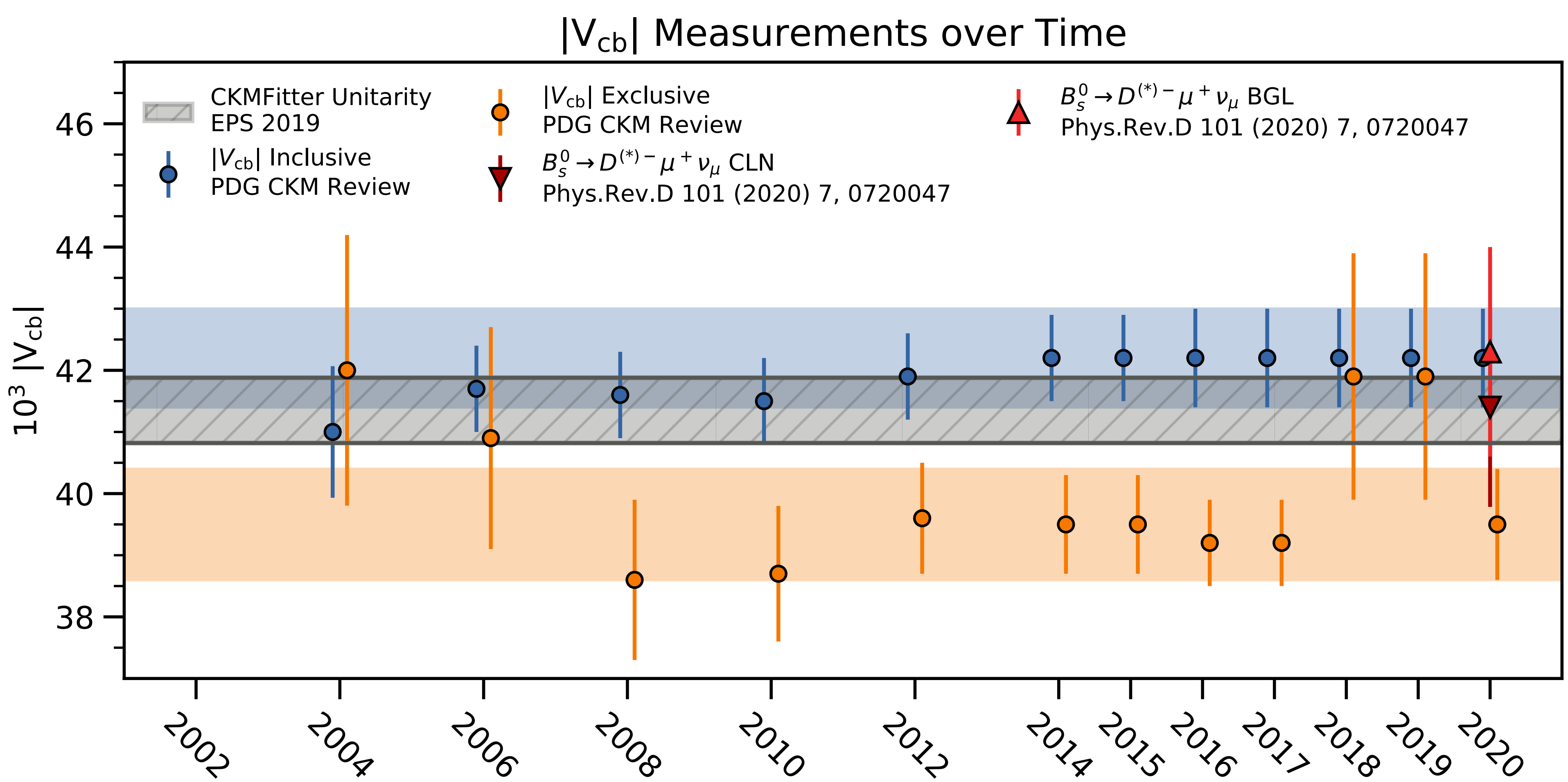
See also [FB, Markus Prim, Dean Robinson, Phys. Rev. D 104, 034032 (2021)]

	$\mathcal{B}(B \to X \ell \bar{\nu}_\ell)$ (%)	$\mathcal{B}(B \to X_c \ell \bar{\nu}_\ell)$ (%)	In Average
Belle [62] $E_\ell > 0.6\,\mathrm{GeV}$	-	10.54 ± 0.31	✓
Belle [62] $E_\ell > 0.4\,\mathrm{GeV}$	-	10.58 ± 0.32	
CLEO [64] incl.	10.91 ± 0.26	10.72 ± 0.26	
CLEO [64] $E_\ell > 0.6$	10.69 ± 0.25	10.50 ± 0.25	✓
BaBar [61] incl.	10.34 ± 0.26	10.15 ± 0.26	✓
BaBar SL [63] $E_\ell > 0.6\,\mathrm{GeV}$	-	10.68 ± 0.24	✓
Our Average	-	10.48 ± 0.13	
Average Belle [62] & BaBar [63] ($E_\ell > 0.6\,\mathrm{GeV}$)	-	10.63 ± 0.19	

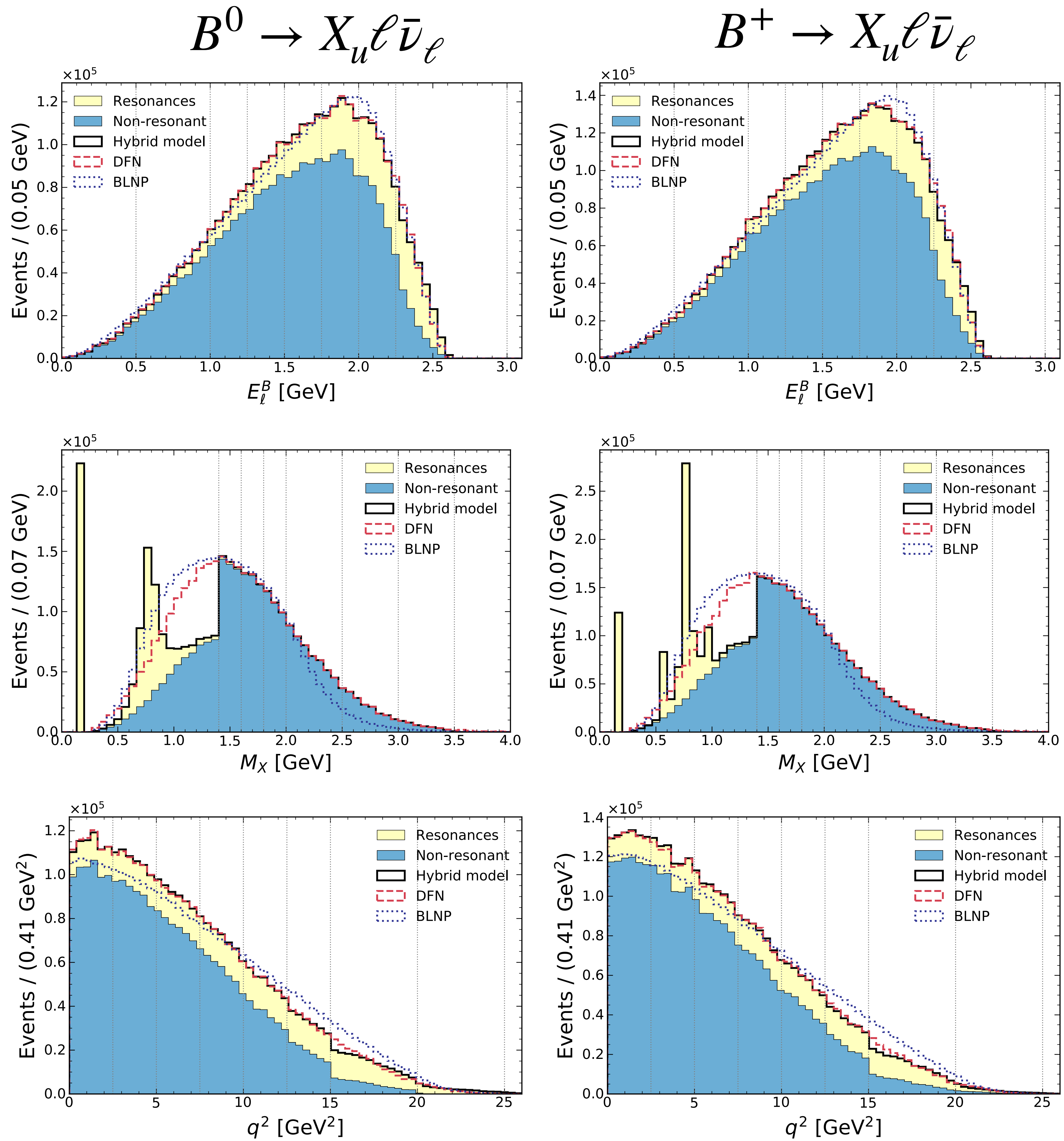
Table 2: Available measurements of the inclusive $B \to X \ell \bar{\nu}_\ell$ and $B \to X_c \ell \bar{\nu}_\ell$ branching fractions, extrapolated to the full region using the correction factors in (34). The χ^2 of our average with respect to the included measurements is 2.2, corresponding to a p-value of 52%. We do not include [65], as the analysis does not quote a partial branching fraction corrected for FSR radiation.



$|V_{ub}|$ Measurements over Time
CKMFitter Unitarity
EPS 2019
$|V_{ub}|$ Inclusive
PDG CKM Review
$|V_{ub}|$ Exclusive
PDG CKM Review
$B \to \mu\nu$
Phys.Rev.D 101 (2020) 3, 032007
$B \to \tau\nu$
Phys.Rev.D 92 (2015) 5, 051102
$\Lambda_b \to p\mu\nu$
Nature Phys. 11 (2015) 743-747
$10^3\ |V_{ub}|$
7
6
5
4
3
2002
2004
2006
2008
2010
2012
2014
2015
2016
2017
2018
2019
2020



$|V_{cb}|$ Measurements over Time
CKMFitter Unitarity
EPS 2019
$|V_{cb}|$ Inclusive
PDG CKM Review
$|V_{cb}|$ Exclusive
PDG CKM Review
$B_s^0 \to D^{(*)-}\mu^+\nu_\mu$ CLN
Phys.Rev.D 101 (2020) 7, 0720047
$B_s^0 \to D^{(*)-}\mu^+\nu_\mu$ BGL
Phys.Rev.D 101 (2020) 7, 0720047
$10^3\ |V_{cb}|$
46
44
42
40
38
2002
2004
2006
2008
2010
2012
2014
2015
2016
2017
2018
2019
2020



$B^0 \to X_u \ell \bar{\nu}_\ell$
$B^+ \to X_u \ell \bar{\nu}_\ell$
Resonances
Non-resonant
Hybrid model
DFN
BLNP
Events / (0.05 GeV)
E_ℓ^B [GeV]
Events / (0.07 GeV)
M_X [GeV]
Events / (0.41 GeV^2)
q^2 [GeV^2]
×10^5

Raynette Van Tonder's talk

$\bar{B} \to X_c \ell \bar{\nu}_\ell$ modelling

- **Update** excl. branching ratios to PDG 2020 and the masses and widths of D** decays
- **Generate** additional MC samples to fill the **gap** between the exclusive & inclusive measurement (assign 100% BR uncertainty in systematics covariance matrix)

	BR	B^+	B^0
	$B \to X_c \ell^+ \nu_\ell$		
D, D*	$B \to D \ell^+ \nu_\ell$	$(2.5 \pm 0.1) \times 10^{-2}$	$(2.3 \pm 0.1) \times 10^{-2}$
	$B \to D^* \ell^+ \nu_\ell$	$(5.4 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
D**	$B \to D_0^* \ell^+ \nu_\ell$ ($\hookrightarrow D\pi$)	$(0.420 \pm 0.075) \times 10^{-2}$	$(0.390 \pm 0.069) \times 10^{-2}$
	$B \to D_1^* \ell^+ \nu_\ell$ ($\hookrightarrow D^*\pi$)	$(0.423 \pm 0.083) \times 10^{-2}$	$(0.394 \pm 0.077) \times 10^{-2}$
	$B \to D_1 \ell^+ \nu_\ell$ ($\hookrightarrow D^*\pi$)	$(0.422 \pm 0.027) \times 10^{-2}$	$(0.392 \pm 0.025) \times 10^{-2}$
	$B \to D_2^* \ell^+ \nu_\ell$ ($\hookrightarrow D^*\pi$)	$(0.116 \pm 0.011) \times 10^{-2}$	$(0.107 \pm 0.010) \times 10^{-2}$
	$B \to D_2^* \ell^+ \nu_\ell$ ($\hookrightarrow D\pi$)	$(0.178 \pm 0.024) \times 10^{-2}$	$(0.165 \pm 0.022) \times 10^{-2}$
	$\rho(D_2^* \to D^*\pi, D_2^* \to D\pi) = 0.693$		
Gap	$B \to D_1 \ell^+ \nu_\ell$ ($\hookrightarrow D\pi\pi$)	$(0.242 \pm 0.100) \times 10^{-2}$	$(0.225 \pm 0.093) \times 10^{-2}$
	$B \to D\pi\pi \ell^+ \nu_\ell$	$(0.06 \pm 0.06) \times 10^{-2}$	$(0.06 \pm 0.06) \times 10^{-2}$
	$B \to D^*\pi\pi \ell^+ \nu_\ell$	$(0.216 \pm 0.102) \times 10^{-2}$	$(0.201 \pm 0.095) \times 10^{-2}$
	$B \to D\eta \ell^+ \nu_\ell$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
	$B \to D^*\eta \ell^+ \nu_\ell$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
	$B \to X_c \ell^+ \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

BR	B^+	B^0
$B \to D_0^* \ell^+ \nu_\ell$ ($\hookrightarrow D\pi\pi$)	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \to D_1^* \ell^+ \nu_\ell$ ($\hookrightarrow D\pi\pi$)	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \to D_0^*\pi\pi \ell^+ \nu_\ell$ ($\hookrightarrow D^*\pi\pi$)	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \to D_1^*\pi\pi \ell^+ \nu_\ell$ ($\hookrightarrow D^*\pi\pi$)	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \to D_0^* \ell^+ \nu_\ell$ ($\hookrightarrow D\eta$)	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
$B \to D_1^* \ell^+ \nu_\ell$ ($\hookrightarrow D^*\eta$)	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$

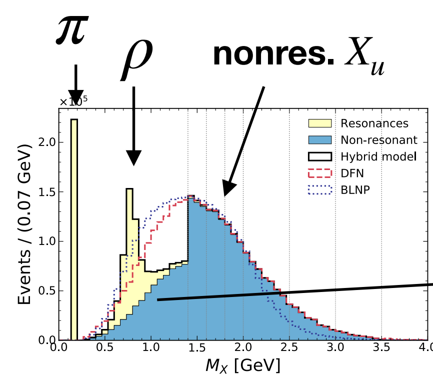
Fit for partial BFs

Subtraction of bkg in fit with coarse binning to minimize X_u modelling dependence

(low m_X, high q^2)

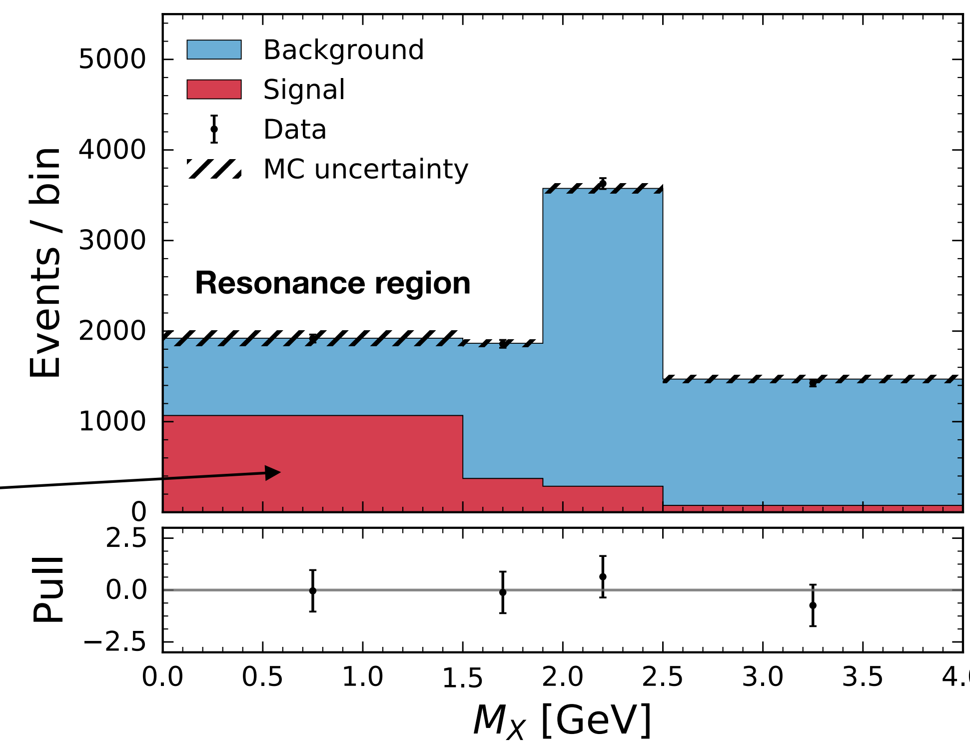
$$\mathcal{L} = \prod_i^{\text{bins}} \mathcal{P}\left(n_i; \nu_i\right) \times \prod_k \mathcal{G}_k \,,$$

Signal and Bkg shape errors included in Fit via NPs



W/o detector smearing

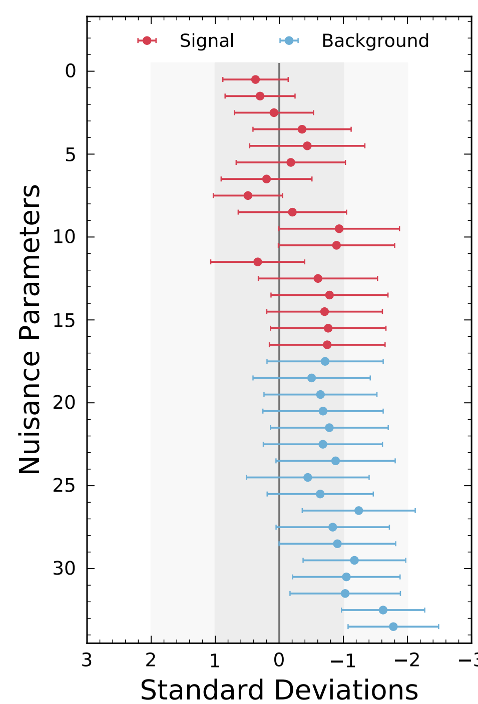
Projections of 2D fit in m_X : q^2

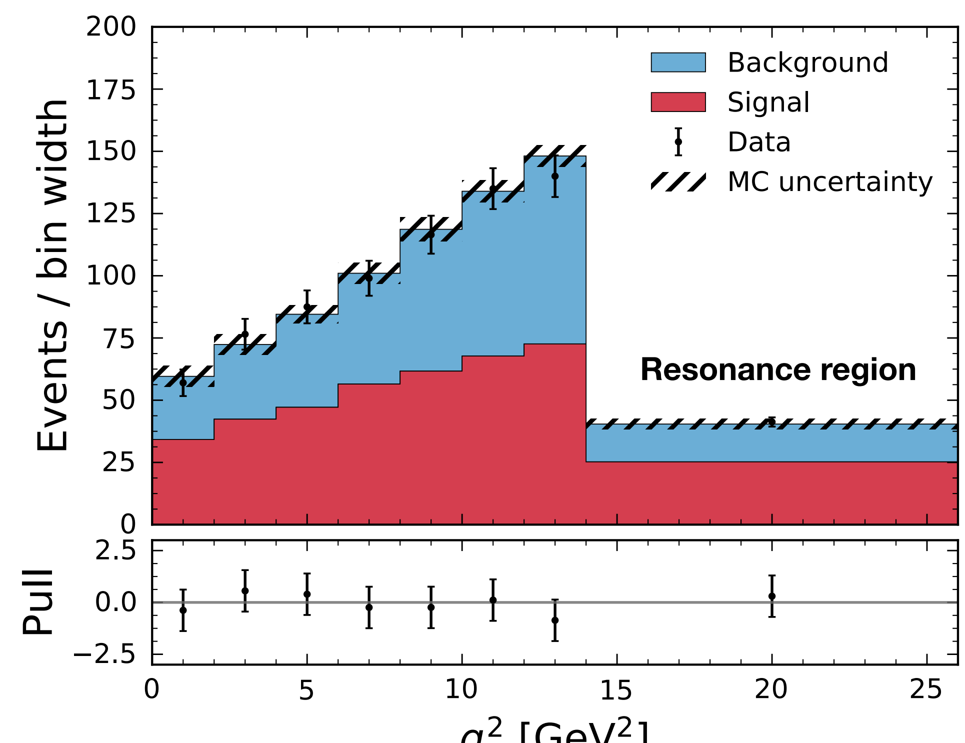


Unfold measured yields to **3 phase-space** regions:

Phase-space region
$M_X < 1.7\,\text{GeV}$
$M_X < 1.7\,\text{GeV},\ q^2 > 8\,\text{GeV}^2$
$E_\ell^B > 1\,\text{GeV}$

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u\,\ell^+\,\nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \to X_u\,\ell^+\,\nu_\ell)}}$$

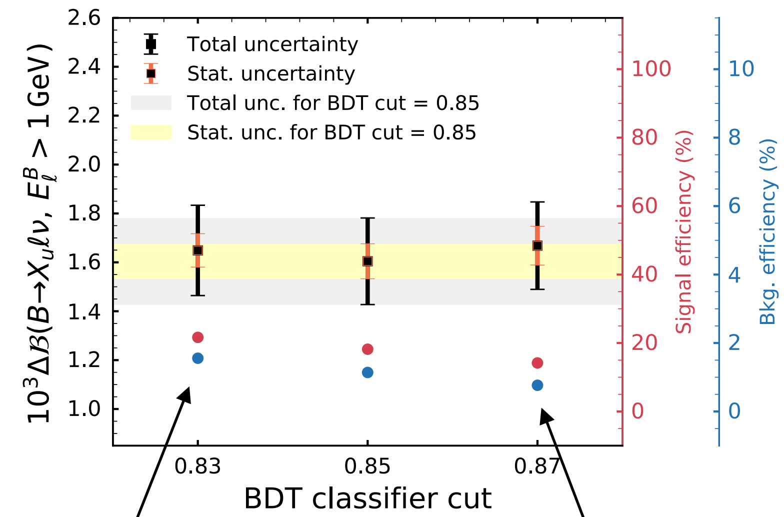




$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell^+ \nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \to X_u \ell^+ \nu_\ell)}}$$

Fit kinematic distributions and measure partial BF

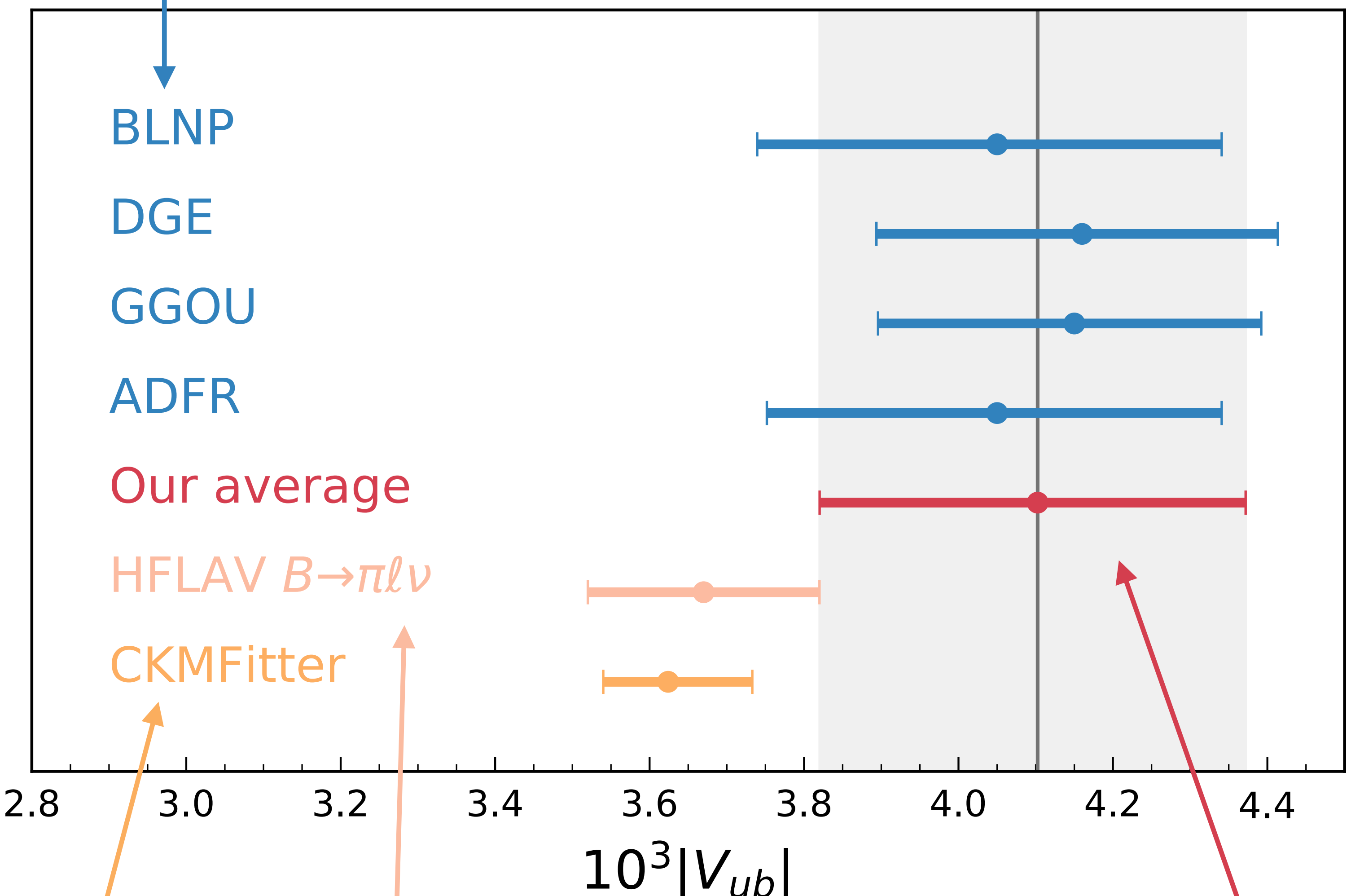
Stability as a function of BDT cut:



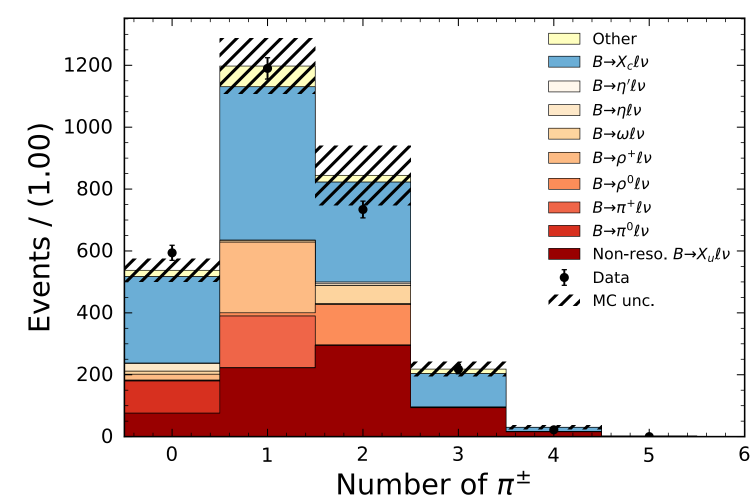
33% more Bkg

37% less Bkg

4 predictions of the partial rate



Post-fit N_{π^+} distribution:



Exclusive Average for $B \to \pi\ell\bar{\nu}_\ell$:
$|V_{ub}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}$

Arithmetic average:
$|V_{ub}| = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}$

CKM Unitarity:
$|V_{ub}| = (3.62^{+0.11}_{-0.08}) \times 10^{-3}$

Into the tool shed: EvtGen & Pythia8

Many analyses need generic B-Meson decay samples

* Pythia8 hadronized modes make up ca. **48%** (!) of all simulated decays

```
#  Lam_c X  / Sigma_c X      4.0 %
#
0.010520663 anti-cd_0 ud_0                                    PYTHIA 23;
0.021041421 anti-cd_1 ud_1                                    PYTHIA 23;

#  Xi_c X    2.5%
#
0.002869298 anti-cs_0 ud_0                                    PYTHIA 23;
0.005738595 anti-cs_1 ud_1                                    PYTHIA 23;

0.258091538 u       anti-d  anti-c  d                         PYTHIA 48;
0.043995612 u       anti-d  anti-c  d                         PYTHIA 13;
0.020084989 u       anti-s  anti-c  d                         PYTHIA 13;
0.017215691 u       anti-c  anti-d  d                         PYTHIA 48;
0.000860770 u       anti-c  anti-s  d                         PYTHIA 48;
#lange - try to crank up the psi production....
0.070775534 c       anti-s  anti-c  d                         PYTHIA 13;
0.005738595 c       anti-d  anti-c  d                         PYTHIA 13;
0.002869298 u       anti-d  anti-u  d                         PYTHIA 48;
0.003825730 c       anti-s  anti-u  d                         PYTHIA 48;
# JGS 11/5/02 This and similar a few lines above have been divided by two
# to solve a double-counting problem for this channel
0.001960649 u       anti-u  anti-d  d                         PYTHIA 48;
0.000066973 d       anti-d  anti-d  d                         PYTHIA 48;
0.000086068 s       anti-s  anti-d  d                         PYTHIA 48;
0.002104095 u       anti-u  anti-s  d                         PYTHIA 48;
0.001721541 d       anti-d  anti-s  d                         PYTHIA 48;
0.001434649 s       anti-s  anti-s  d                         PYTHIA 48;
0.004782163 anti-s  d                                         PYTHIA 32;
```

Modes for Matrix Element Processing

Some decays can be treated better than what pure phase space allows, by reweighting with appropriate matrix e signaled by a nonvanishing `meMode()` value for a decay mode in the particle data table. The list of allowed poss introduced, and most have been moved for better consistency. Here is the list of currently allowed `meMode()` co

- 0 : pure phase space of produced particles ("default"); input of partons is allowed and then the partonic con
- 1 : *omega* and *phi → pi+ pi- pi0*
- 2 : polarization in *V → PS + PS* (*V* = vector, *PS* = pseudoscalar), when *V* is produced by *PS → PS + V* or *F*
- 11 : Dalitz decay into one particle, in addition to the lepton pair (also allowed to specify a quark-antiquark pa
- 12 : Dalitz decay into two or more particles in addition to the lepton pair
- 13 : double Dalitz decay into two lepton pairs
- 21 : decay to phase space, but weight up *neutrino_tau* spectrum in *tau* decay
- 22 : weak decay; if there is a quark spectator system it collapses to one hadron; for leptonic/semileptonic d
- 23 : as 22, but require at least three particles in decay
- 31 : decays of type B → gamma X, very primitive simulation where X is given in terms of its flavour content spectrum is weighted up relative to pure phase space
- 42 - 50 : turn partons into a random number of hadrons, picked according to a Poissonian with average val new try with another multiplicity if the sum of daughter masses exceed the mother one
- 52 - 60 : as 42 - 50, with multiplicity between `code` - 50 and 10, but avoid already explicitly listed non-parto
- 62 - 70 : as 42 - 50, but fixed multiplicity `code` - 60
- 72 - 80 : as 42 - 50, but fixed multiplicity `code` - 70, and avoid already explicitly listed non-partonic channels
- 91 : decay to *q qbar* or *g g*, which should shower and hadronize
- 92 : decay onium to *g g g* or *g g gamma* (with matrix element), which should shower and hadronize
- 93 : decay of colour singlet to *q qbar* plus another singlet, flat in phase space (and arbitrarily ordered), whe
- 94 : same as 93, but weighted with *V-A* weak matrix element if the decay chain is of the type *neutrino \rarr;*
- 100 - : reserved for the description of partial widths of resonances

Combined Extractions

Interesting if heavy quark symmetry inspired Form Factors are used:

$$\hat{h}(w) = h(w)/\xi(w)$$ ⟵ Leading Isgur-Wise function

$B \to D\ell\bar{\nu}_\ell$

$$\hat{h}_+ = 1 + \hat{\alpha}_s\Big[C_{V_1} + \frac{w+1}{2}\,(C_{V_2} + C_{V_3})\Big] + (\varepsilon_c + \varepsilon_b)\,\hat{L}_1\,,$$
$$\hat{h}_- = \hat{\alpha}_s\,\frac{w+1}{2}\,(C_{V_2} - C_{V_3}) + (\varepsilon_c - \varepsilon_b)\,\hat{L}_4\,,$$

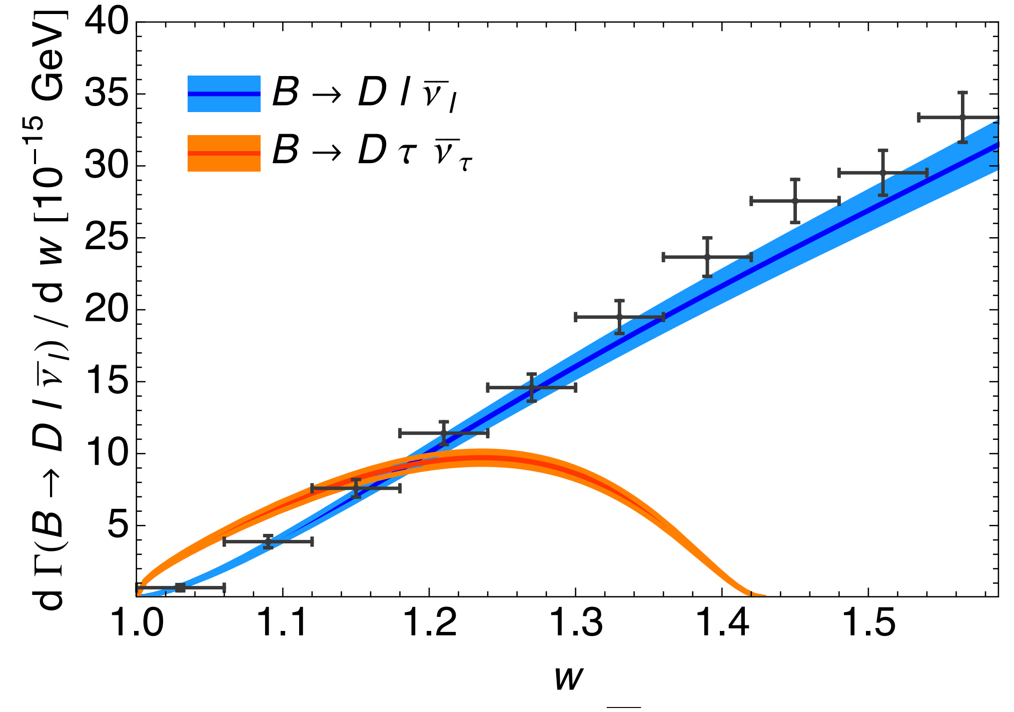
$B \to D^*\ell\bar{\nu}_\ell$

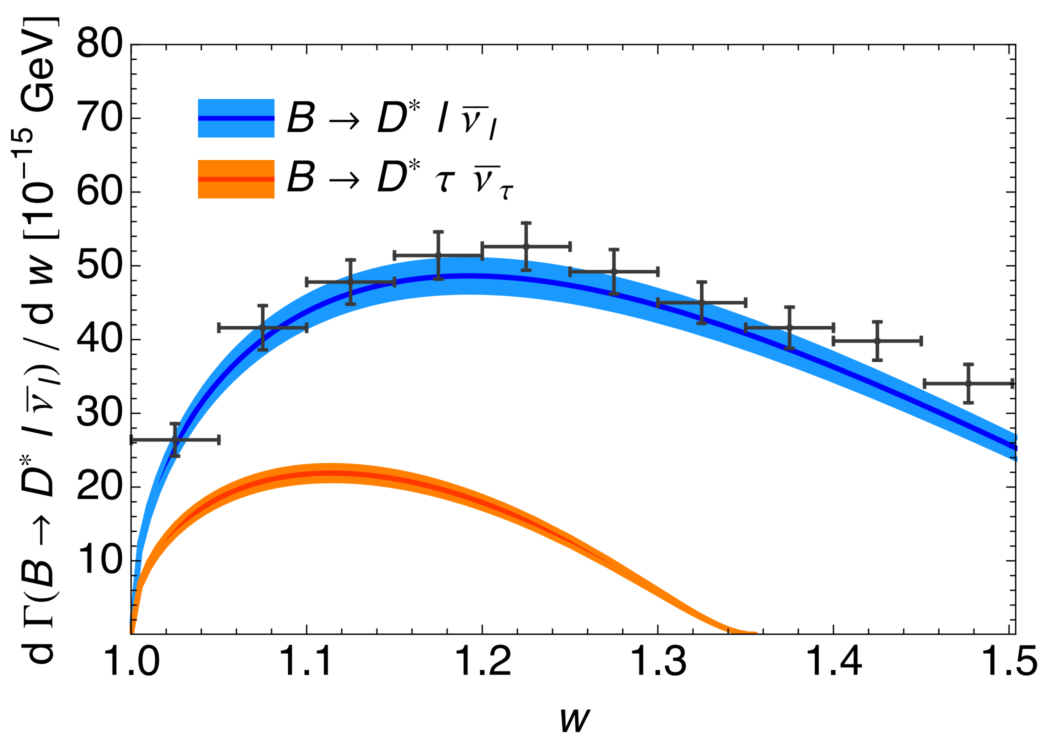
$$\hat{h}_V = 1 + \hat{\alpha}_s\,C_{V_1} + \varepsilon_c\big(\hat{L}_2 - \hat{L}_5\big) + \varepsilon_b\big(\hat{L}_1 - \hat{L}_4\big)\,,$$
$$\hat{h}_{A_1} = 1 + \hat{\alpha}_s\,C_{A_1} + \varepsilon_c\left(\hat{L}_2 - \hat{L}_5\,\frac{w-1}{w+1}\right) + \varepsilon_b\left(\hat{L}_1 - \hat{L}_4\,\frac{w-1}{w+1}\right),$$
$$\hat{h}_{A_2} = \hat{\alpha}_s\,C_{A_2} + \varepsilon_c\big(\hat{L}_3 + \hat{L}_6\big)\,,$$
$$\hat{h}_{A_3} = 1 + \hat{\alpha}_s\big(C_{A_1} + C_{A_3}\big) + \varepsilon_c\big(\hat{L}_2 - \hat{L}_3 + \hat{L}_6 - \hat{L}_5\big) + \varepsilon_b\big(\hat{L}_1 - \hat{L}_4\big)\,,$$

This links dynamics of $B \to D\ell\bar{\nu}_\ell$ & $B \to D^*\ell\bar{\nu}_\ell$

Example fit for leading IW function and sub-leading parameters

$\lvert V_{cb}\rvert \times 10^3$	38.8 ± 1.2
$\mathcal{G}(1)$	1.055 ± 0.008
$\mathcal{F}(1)$	0.904 ± 0.012
$\bar{\rho}_*^2$	1.17 ± 0.12
$\hat{\chi}_2(1)$	-0.26 ± 0.26
$\hat{\chi}_2'(1)$	0.21 ± 0.38
$\hat{\chi}_3'(1)$	0.02 ± 0.07
$\eta(1)$	0.30 ± 0.04
$\eta'(1)$	0 (fixed)
m_b^{1S} [GeV]	4.70 ± 0.05
δm_{bc} [GeV]	3.40 ± 0.02





LHCb Systematics

$B_s \to K\mu\bar{\nu}_\mu$

Uncertainty	All q^2	Low q^2	High q^2
Tracking	2.0	2.0	2.0
Trigger	1.4	1.2	1.6
Particle identification	1.0	1.0	1.0
$\sigma(m_{\text{corr}})$	0.5	0.5	0.5
Isolation	0.2	0.2	0.2
Charged BDT	0.6	0.6	0.6
Neutral BDT	1.1	1.1	1.1
q^2 migration	$\cdots$	2.0	2.0
Efficiency	1.2	1.6	1.6
Fit template	$^{+2.3}_{-2.9}$	$^{+1.8}_{-2.4}$	$^{+3.0}_{-3.4}$
Total	$^{+4.0}_{-4.3}$	$^{+4.3}_{-4.5}$	$^{+5.0}_{-5.3}$

$B_s \to D_s^{(*)}\mu\bar{\nu}_\mu$

Source	Uncertainty															
	CLN parametrization						BGL parametrization									
	$\lvert V_{cb}\rvert$ $[10^{-3}]$	$\rho^2(D_s^-)$ $[10^{-1}]$	$\mathcal{G}(0)$ $[10^{-2}]$	$\rho^2(D_s^{*-})$ $[10^{-1}]$	$R_1(1)$ $[10^{-1}]$	$R_2(1)$ $[10^{-1}]$	$\lvert V_{cb}\rvert$ $[10^{-3}]$	d_1 $[10^{-2}]$	d_2 $[10^{-1}]$	$\mathcal{G}(0)$ $[10^{-2}]$	b_1 $[10^{-1}]$	c_1 $[10^{-3}]$	a_0 $[10^{-2}]$	a_1 $[10^{-1}]$	$\mathcal{R}$ $[10^{-1}]$	$\mathcal{R}^*$ $[10^{-1}]$
$f_s/f_d \times \mathcal{B}(D_s^- \to K^+K^-\pi^-)(\times\tau)$	0.8	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	0.4
$\mathcal{B}(D^- \to K^-K^+\pi^-)$	0.5	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.3
$\mathcal{B}(D^{*-} \to D^-X)$	0.2	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.1	0.0	0.2	0.0	0.3	–	0.2
$\mathcal{B}(B^0 \to D^-\mu^+\nu_\mu)$	0.4	0.0	0.3	0.1	0.2	0.1	0.5	0.1	0.0	0.1	0.1	0.4	0.1	0.7	–	–
$\mathcal{B}(B^0 \to D^{*-}\mu^+\nu_\mu)$	0.3	0.0	0.2	0.1	0.1	0.1	0.2	0.0	0.0	0.1	0.1	0.3	0.1	0.4	–	–
$m(B_s^0)$, $m(D^{(*)-})$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	–	–
η_{EW}	0.2	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	–	–
$h_{A_1}(1)$	0.3	0.0	0.2	0.1	0.1	0.1	0.3	0.0	0.0	0.1	0.1	0.3	0.1	0.5	–	–
External inputs (ext)	1.2	0.0	0.4	0.1	0.2	0.1	1.2	0.1	0.0	0.1	0.1	0.6	0.1	0.8	0.5	0.5
$D_{(s)}^- \to K^+K^-\pi^-$ model	0.8	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.4
Background	0.4	0.3	2.2	0.5	0.9	0.7	0.1	0.5	0.2	2.3	0.7	2.0	0.5	2.0	0.4	0.6
Fit bias	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.2	0.4	0.2	0.4	0.0	0.0
Corrections to simulation	0.0	0.0	0.5	0.0	0.1	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0
Form-factor parametrization	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.0	0.1
Experimental (syst)	0.9	0.3	2.2	0.5	0.9	0.7	0.9	0.5	0.2	2.3	0.7	2.1	0.5	2.0	0.6	0.7
Statistical (stat)	0.6	0.5	3.4	1.7	2.5	1.6	0.8	0.7	0.5	3.4	0.7	2.2	0.9	2.6	0.5	0.5